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Intensity scintillation index and mean apparent radar cross section on monostatic and bistatic paths

E. J. Fremouw

Northwest Research Associates, Bellevue, Washington

Akira Ishimaru

Department of Electrical Engineering, University of Washington, Seattle

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We develop an expression herein for the intensity scintillation index on a two-way (radar) path, in terms of the one-way index and the correlation between scintillations produced on the uplink and downlink. The expression is appropriate for monostatic (fully correlated) and bistatic (totally or partially uncorrelated, or anticorrelated) paths whose links are statistically similar and obey Nakagami m statistics. A companion expression for the mean apparent radar cross section (RCS) in the presence of scintillation describes enhancement on monostatic paths and energy-conserving depletion of mean apparent RCS on small-angle bistatic paths. The companion expression, which does not depend upon Nakagami m statistics nor require statistical similarity, is consistent with more detailed calculations by previous authors. Special cases of both expressions are consistent with recent monostatic measurements.

1. INTRODUCTION

In seeking a simple expression for the intensity scintillation index on a two-way path through a randomly structured medium, we encountered a surprise: such a channel cannot display reciprocity and conserve energy simultaneously. The average power received on a two-way path in the presence of scintillation, given reciprocity, is greater than that received in the absence of scintillation. A monostatic radar channel is reciprocal, however, and resolution of the apparent dilemma lies in accounting for energy preferentially scattered into it from small bistatic angles. Such a resolution is consistent with calculations by de Wolf [1971] and by Kravtsov and Saichev [1982, 1985]. The enhanced apparent radar cross section (RCS) is caused by correlation between the reflected wave field and the refractive index irregularities it encounters on the return path. Recently, the enhancement in apparent RCS has been observed experimentally by Knepp and Houpis [1991].

Herein we first present expressions for the mean apparent RCS. We then extend the theory to obtain the scintillation index (fractional standard deviation of intensity) to be expected in monostatic and bistatic radar measurements through random media. The latter (but not the former) is limited to channels whose one-way paths (1) are statistically similar and (2) yield Nakagami [1960] intensity statistics. We present our general development in section 2 and apply it to Nakagami m statistics in section 3. In section 4 we present conclusions that may be of some practical value, at least on transionospheric channels (for which Nakagami m statistics provide a useful signal characterization).

2. GENERAL CONSIDERATIONS

Consider a radar link described by the operating wavelength λ; the antenna gain and aperture, G and A, respectively; the transmitted power P_t; and the target RCS and range, σ and R, respectively. In the absence of scintillation the power intercepted by the target would be

$$P_a = \frac{GP_t}{4\pi R^2}\sigma \quad . \tag{1}$$

In the presence of scintillation the intercepted power P_i consists of a mean value $< P_i >$ and a fluctuating part. Energy conservation requires that the mean

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value equal the undisturbed value P_a. If we express the instantaneous power fluctuation produced on the uplink as a fraction δ_u, of the mean value, we have

$$P_i = < P_i > (1+\delta_u) = \frac{GP_t}{4\pi R^2}\sigma(1+\delta_u) \quad . \tag{2}$$

If there were no further scintillation produced on the downlink, the received power would be

$$P_r = \frac{AP_i}{4\pi R^2} = \frac{\lambda^2 GP_i}{(4\pi R)^2} \quad . \tag{3}$$

In the presence of scintillation, however, the power P actually measured at the receiver consists of P_r plus an additional fractional fluctuation δ_d, produced on the downlink. That is, employing equations (3) and (2), we have

$$P = P_r(1+\delta_d) = \frac{\lambda^2 G^2 P_t}{(4\pi)^3 R^4}\sigma(1+\delta_u)(1+\delta_d) \quad . \tag{4}$$

We now express P in terms of its undisturbed value and a fractional fluctuating part δ. The undisturbed value P_0 is given by the standard radar equation, and it is just the factor in front of the right-hand side of equation (4)(including σ). That is, we write

$$P = P_0(1+\delta) = P_0\sigma_m/\sigma \quad , \tag{5}$$

where we have denoted the instantaneously measured (apparent) RCS of the target by

$$\sigma_m = \sigma(1+\delta) = \sigma(1+\delta_u)(1+\delta_d) \quad . \tag{6}$$

Thus the measured RCS consists of its true value plus a fluctuating part, given fractionally by

$$\delta = \delta_u + \delta_d + \delta_u\delta_d \quad . \tag{7}$$

Herein lies our apparent dilemma. The cross term in equation (7) is important and, when averaged, represents the correlation between intensity scintillations produced on the uplink and those produced on the downlink. Energy conservation on each one-way path requires that δ_u and δ_d each be a zero-mean random variable. Accordingly, averaging (denoted by $< \ >$) of equation (7) produces

$$< \delta > = < \delta_u\delta_d > \quad . \tag{8}$$

Thus we cannot have both energy conservation on the two-way path ($< \delta > = 0$) and reciprocity of the two one-way links ($\delta_d = \delta_u$ and therefore $< \delta_u\delta_d > = < \delta_u^2 > = < \delta_d^2 >$), except in the case of no scintillation ($< \delta_u^2 > = < \delta_d^2 > = 0$). Reciprocity requires that the average power on the two-way path in the presence of scintillation be different from that in the absence of scintillation. In fact, equation (8) describes the surprising conclusion that the mean RCS observed is enhanced by scintillation on reciprocal links, which we expect a monostatic radar path to comprise. As counterintuitive as this conclusion may be, it has now been definitively confirmed experimentally [Knepp and Houpis, 1991] on a monostatic path with apparently reciprocal links.

For such links the right-hand side of equation (8) is just the fractional variance of intensity experienced each way. that is, the square of the intensity scintillation index usually denoted by S_4 [Briggs and Parkin, 1963]. Generically, we will call the one-way fractional variance $S_m^2 = m^{-1}$, in preparation for using the notation of Nakagami [1960] in the next section. When specification of the link direction is useful, we will use S_u and S_d, respectively, for the uplink and downlink scintillation indices. Using this nomenclature, equation (8) for the general (partially correlated) two-way channel becomes

$$< \delta > = \rho S_u S_d \quad , \tag{9}$$

where ρ is the correlation coefficient between intensity scintillations produced on the uplink and those produced on the downlink. (For statistically similar links, $S_d = S_u = S_m$, even though the two links may not be deterministically reciprocal.) Averaging equation (6) and inserting equation (9) produces

$$< \sigma_m > = \sigma(1+\rho S_u S_d) \tag{10}$$

as the general expression for the mean observed RCS. In the special case of Rayleigh scatter ($S_m = 1$) on fully correlated paths ($\rho = 1$) it describes a twofold (3-dB) enhancement in mean apparent RCS ($< \sigma_m > = 2\sigma$).

Formally, $< \ >$ denotes ensemble averaging. The amount of enhancement depends upon the degree of correlation that spatial structures in the medium and temporal changes therein permit between intensity scintillations developed on the uplink and on the downlink. In some situations, for instance radar astronomical situations, temporal changes may contribute to decorrelation of uplink and downlink scintillations. In those cases, enhancement on a monostatic path would be decreased. (One also could concoct geometries in which bistatic paths would display full enhancement due to drift of "frozen-in" structures.) In many practical cases of interest, including transionospheric propagation from surface to, say, geostationary orbit, temporal changes are negligible, as they apparently were in the measurements of Knepp and Houpis [1991].

In interpreting their (solely monostatic) observations of the enhancement, Knepp and Houpis followed the procedure established by Yeh [1983] for computing the mutual coherence function of the returned signal. To Yeh's spatial lag they added temporal and spectral lags as arguments to this correlation function of the complex signal, which depends primarily on phase relationships between different (multidimensional) lag points. While the mutual coherence function is affected also by the degree of intensity correlation, the formalism does not address correlation between uplink and downlink intensity scintillations per se. Indeed, the procedure invokes reciprocity by requiring the uplink and downlink channel transfer functions to be identical to one another for all lags. Thus the scintillations (of both intensity and phase) produced on the two links are restricted to being fully correlated. The cumulative-forwardscatter/single-backscatter (CFSB) formulation introduced by de Wolf [1971] and extended by Yeh does not permit accurate integration over all (monostatic and bistatic) scattering angles. Thus Knepp and Houpis were not able to interpret the apparent lack of energy conservation encountered on their strictly monostatic channel.

To inquire where the extra energy measured by Knepp and Houpis came from, one must consider bistatic as well as monostatic paths within the system. Indeed, the development culminating in equation (10) is quite consistent with far more detailed calculations by Soviet authors [Kravtsov and Saichev, 1982, 1985] and with the original CFSB calculations of de Wolf [1971]. Those calculations indicate that, while energy must be conserved on the totality of paths, correlations between the returning wave field and irregularities in the medium can scatter energy preferentially into the monostatic two-way channel from small bistatic angles. Equations (9) and (10) suggest that the angles from which the energy is scattered are those for which $\rho < 0$.

While the foregoing prediction that scintillation enhances the mean RCS observed on a monostatic path is not new, we think that the simplicity with which we have shown it to arise may provide new clarity. In particular, there is no need to invoke saturation of the scintillation so that the scintillating signal may be treated as a complex Gaussian variate, as did Knepp and Houpis. Moreover, it is not necessary that the signal approach such behavior as a special case of Rice [1945] statistics nor even that it obey a generalization thereof, such as Nakagami m statistics. Rather, the enhancement and (energy-conserving) depletion of mean apparent RCS described by equation (10) are fundamental behaviors of scintillation-producing two-way channels. They are not merely incidental consequences of some statistical description of signal behavior; indeed, they place constraints on admissible statistical treatments.

We are not aware of detailed propagation calculations of the variance of intensity on two-way paths, which would involve the fourth moment of field strength in the presence of correlation between it and the refractive index irregularities it encounters on the return trip. Equation (10) provides a ready tool for evaluating the relationship between mean apparent RCS and the one-way scintillation index in both monostatic and bistatic channels. (Parameterization in terms of bistatic angle, however, would require knowledge of the spatial correlation function of the medium). We now turn to the relationship between the two-way scintillation index S_4 and its one-way counterpart S_m.

We begin by defining the positive, real, normalized, random voltage e_u (produced on the uplink) and e_d (produced on the downlink) as follows:

$$e_u^2 = (1 + \delta_u) \ , \tag{11}$$

$$e_d^2 = (1 + \delta_d) \ , \tag{12}$$

and noting that $< e_d^2 > = < e_u^2 > = < e_m^2 > = 1$, where we are employing e_m as a direction-independent symbol for the normalized random voltage produced on a one-way path. Now recall that the square of the one-way scintillation index is defined as

$$S_m^2 \equiv \frac{< (e_m^2 - < e_m^2 >)^2 >}{< e_m^2 >^2} \equiv m^{-1} \ . \tag{13}$$

Accordingly,

$$S_m^2 = < e_m^4 > -1 \ . \tag{14}$$

We now define the positive, real, normalized, random voltage e, observed on the two-way path as follows:

$$e^2 = (1 + \delta) \ , \tag{15}$$

and note from equation (6) that

$$e = e_u e_d \ . \tag{16}$$

By definition, the square of the scintillation index S_4 for the two-way link is the ratio of the variance of P to its mean square value. Thus from equations (5), (15), and (16) we have

$$S_4^2 \equiv \frac{< (P - < P >)^2 >}{< P >^2} = \frac{< e_u^4 e_d^4 >}{< e_u^2 e_d^2 >^2} - 1 \ . \tag{17}$$

For fully reciprocal paths ($e_m = e_d = e_u$) the foregoing reduces to

$$S_4^2 = \frac{< e_m^8 >}{< e_m^4 >^2} - 1 \ . \tag{18}$$

We see that in the simple case of reciprocal paths, calculation of the two-way scintillation index involves the eighth and fourth moments of field strength on the one-way link. For this case the two-way voltage distribution is identical to the one-way power distribution, as is obvious from equation (16). Indeed, the latter equation directly answers the deterministic question that prompted this work: namely, "If a one-way transionospheric communication link undergoes a scintillation fade of x dB, how deep will the fade be on the corresponding (monostatic) radar link?" (G. Bishop, private communication, 1990). Equation (16) supports the reciprocity-based intuitive answer, $2x$ dB. As we have seen, however, the same intuition applied statistically leads to an apparent dilemma over energy conservation.

For partially correlated links, mixed moments of fourth and second order are required to compute the two-way scintillation index. Computing these correlations directly from propagation theory for a medium prescribed statistically probably would be a formidable task. If the statistics of amplitude on the one-way path are well described, however, useful results can be obtained with relative ease. In the next section we illustrate this approach by means of the Nakagami

m distribution, which provides a useful description of signal intensity on transionospheric paths [Whitney et al., 1972; Fremouw et al., 1980; Knepp and Houpis, 1991]. The two-way description is limited to statistically similar one-way links.

3. APPLICATION TO A NAKAGAMI m CHANNEL

If e_u and e_d are totally uncorrelated Nakagami m variables, then the distribution of the normalized electric field magnitude (voltage) sensed by the radar's receiving antenna, namely, e, is that given in equation (90) of Nakagami [1960] for the product of two such variables. In the event of partial correlation, Nakagami's equation (135) gives the distribution of e as a function of the correlation coefficient ρ. Moreover, his equation (137) may be used to compute moments of e as mixed moments of e_u and e_d, also as functions of ρ. In particular, the latter equation yields

$$< e_u^2 e_d^2 >= 1 + \frac{\rho}{m} = 1 + \rho S_m^2 \ , \qquad (19)$$

where we have employed formulas (14.4.1) and (6.1.15) of Abramowitz and Stegun [1972] to evaluate the hypergeometric function appearing in Nakagami's equation (137). This result is identical to that stated for enhancement of the mean apparent RCS on statistically similar one-way paths by equation (10), which does not depend upon applicability of Nakagami m statistics.

Similarly, we use Nakagami's equation (137) to compute the fourth-order mixed moment. Again, applying formulas (14.4.1) and (6.1.15) of Abramowitz and Stegun [1972] and some algebra, we obtain

$$< e_u^4 e_d^4 >= \frac{(m+1)^2}{m^2}[1 + \frac{4\rho}{m} + \frac{2\rho^2}{m(m+1)}] \ . \qquad (20)$$

For $\rho = 1$ the foregoing reduces to

$$< e_m^8 >= \frac{(m+3)(m+2)(m+1)}{m^3} \ , \qquad (21)$$

which also may be obtained as the eighth moment of the Nakagami m distribution from his equation (17) and the recursion relation for the gamma function.

Substituting equations (19) and (20) into (17), we obtain

$$S_4^2 = [\frac{S_m^2 + 1}{\rho S_m^2 + 1}]^2 [1 + \rho S_m^2 (4 + 2\frac{\rho S_m^2}{S_m^2 + 1})] - 1 \ . \qquad (22)$$

For a monostatic channel ($\rho = 1$) the foregoing reduces to

$$S_4^2 = 4 S_m^2 + 2\frac{S_m^4}{S_m^2 + 1} \ . \qquad (23)$$

For large bistatic angles ($\rho = 0$) it reduces to

$$S_4^2 = S_m^4 + 2 S_m^2 \ . \qquad (24)$$

Equation (23) for the monostatic special case has been derived independently in terms of m by Knepp and Reinking [1989]. The general expression, equation (22), and its wide-angle bistatic special case, equation (24), are new.

4. CONCLUSION

Equation (10) provides a general expression for the mean apparent RCS as a function of the one-way scintillation index and the correlation between scintillations developed on the uplink and the downlink. It is not restricted to Nakagami m statistics. Equation (22) provides a general expression for the scintillation index on a two-way Nakagami m path, as a function of the one-way index and the aforesaid correlation.

Nakagami m statistics admit values of S_m^2 from 0 to 2. Values greater than unity account for geometrical optics focusing and defocusing, which can arise from media rich in large refractive index structures (e.g., those characterized by steep power law spatial spectra). Thus equation (23) admits values of S_4 ranging from 0 to $\sqrt{5} = 2.24$ for Rayleigh fading to a maximum of 3.27. For the same range of conditions on a monostatic path, equation (10) admits enhancement of mean apparent RCS by a factor ranging from 1 to 2 (3.01 dB) to 3 (4.77 dB). The largest value of S_m that the authors have encountered in ionospheric scintillation is 1.3, for which equation (23) would yield a two-way S_4 value of 2.98 and for which equation (10) would yield a mean apparent RCS enhancement of 2.69 (4.30 dB).

For large bistatic angles (uncorrelated scintillations on the uplink and the downlink), equation (24) yields S_4 values from 0 to $\sqrt{3} = 1.73$ (for Rayleigh scatter) to a maximum of $\sqrt{8} = 2.83$. For $S_m = 1.3$ it would yield $S_4 = 2.5$. The mean RCS would show no enhancement on such channels.

Energy is conserved by a depletion in mean apparent RCS at small bistatic angles in the presence of scintillation, the behavior being governed by equation (10) with negative values of ρ. Since Fresnel filtering cuts off the spectrum of intensity scintillations even in the common case of refractive index structures described by a red, power law spatial spectrum, ρ probably oscillates as a function of bistatic angle in practical situations. To the extent that it becomes substantially negative, the mean apparent bistatic RCS may suffer substantial depletion, complementary to the monostatic enhancement.

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E. J. Fremouw, Northwest Research Associates, P.O. Box 3027, Bellevue, WA 98009.

A. Ishimaru, Department of Electrical Engineering, University of Washington, Seattle, WA 98195.