Letters

A Quantitative Comparison of the Refractive Index Structure Parameter Determined from Refractivity Measurements and Amplitude Scintillation Measurements at 36 GHz

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Abstract—**Results are presented of the determination of the atmospheric refractive index structure parameter C_n^2, deduced from amplitude scintillation measurements made on a 36-GHz radio link and also from refractivity measurements using a turbulence probe. These measurements were made in a town environment (central London) and covered a range of atmospheric conditions. The two methods of measurement give good agreement, particularly in convective conditions, and provide further experimental evidence of the applicability of Tatarski's wave propagation theory in a turbulent medium to millimeter wavelengths.**

I. INTRODUCTION

The use of millimeter wavelengths for civil and military purposes for both active and passive systems is receiving increasing attention and the ability to predict the effects of atmospheric turbulence on these systems is of considerable interest. The use of millimeter wavelengths for atmospheric remote sensing is also of interest to the meteorologist. Millimeter wave fluctuations depend on the refractive index structure parameter C_n^2, the wavelength of the signal, and the path length. It is useful to be able to correlate directly measured meteorological parameters with their effects on radio wave propagation although, in practice, it is often only possible to monitor these parameters directly at a single or a few locations along the path of a link.

This paper extends a previous preliminary study [1] to a range of atmospheric conditions and compares values of C_n^2, derived from the scintillations occurring at 36 GHz on a 4.1-km path across central London, with directly measured values of C_n^2 obtained at a single location at the receiving end of the link. The direct measurements were made by a turbulence probe [2] which provides simultaneous values of temperature, water vapour pressure, and wind speed and direction fluctuations.

II. THEORETICAL BACKGROUND

A. Measurement of C_n^2 *From the Amplitude Scintillations of an Em Wave*

The effect of atmospheric refractive index fluctuations on electromagnetic waves has been studied theoretically by Tatarski [3]. He assumed a locally homogeneous and isotropic medium between scale sizes l_0 (inner scale of turbulence) and L_0 (outer scale of turbulence). In the scale range l_0 to L_0, the refractive index will have a structure function given by:

$$D_n(r) = \overline{[n(r_1) - n(r_1 + r)]^2} = C_n^2 r^{2/3}, \quad l_0 < r < L_0 \tag{1}$$

where r is the separation between two points in the medium. Tatarski showed that the log amplitude variance (σ^2) of the amplitude scintillations of a plane wave propagating through a turbulent medium, in which the refractive index fluctuations obey the two-thirds law, is given by

$$\sigma^2 = 23.39 C_n^2 k^{7/6} L^{11/6} (db^2), \quad L_0 > \sqrt{\lambda L} > l_0 \tag{2}$$

where λ is the wavelength, L the path length, $\sqrt{\lambda \cdot L}$ is the first Fresnel zone size of the link, and k is the wave number $2\pi/\lambda$. The validity of the above inequality was tested for all the experiments by calculating, from the refractivity temporal spectra, the minimum frequency of the inertial subrange which approximately represents the input scale of the turbulence. To avoid possible errors in the calculation of the input scale (L_0), the autocorrelation function method [4] was used. The two methods, however, produced similar estimates.

B. C_n^2 *Measurement Using an Atmospheric Probe*

Assuming the two-thirds law (inertial subrange), the one-dimensional spectral density function of the refractive index is

$$V_n(K) = 0.124 C_n^2 K^{-5/3} \tag{3}$$

where K is the spatial wavenumber. In order to relate the one-dimensional spectral density function to the temporal spectral density function $W_n(f)$, which is measured in practice, use is made of Taylors hypothesis of "frozen turbulence" [5]. The temporal spectral density function of the refractive index can then be expressed as

$$W_n(f) = 0.0367 C_n^2 f^{-5/3} \bar{V}^{2/3} \tag{4}$$

where f is the spectral frequency and $\bar{V}$ is the mean wind velocity of the path. In practice the one-sided spectral density function $S_n(f)$ is calculated and this is related to $W_n(f)$ by

$$S_n(f) = 2W_n(f), \quad 0 < f < \infty. \tag{5}$$

Hence

$$C_n^2 = 13.62 \left(\frac{f}{\bar{V}}\right)^{2/3} f S_n(f). \tag{6}$$

The refractivity N is related to the refractive index n and the basic atmospheric parameters by [6]

$$N \equiv (n - 1).10^6 = \frac{77.6}{T}\left(P + \frac{4810e}{T}\right) \tag{7}$$

where T is the temperature in degrees Kelvin, P is the atmospheric pressure in mbars, and e is the partial water vapour pressure in mbars. e is normally calculated using the difference between the dry and wet bulb temperature ($T - T'$) from the relation

$$e = 10\left(9.4051 - \frac{2353}{T'}\right) - 0.666\,(T - T') \tag{8}$$

assuming that the atmospheric pressure is 1000 mbars.

Manuscript received on May 10, 1982; revised September 21, 1982. The radio propagation part of this work was financed by the Science and Engineering Research Council through a grant to University College London.

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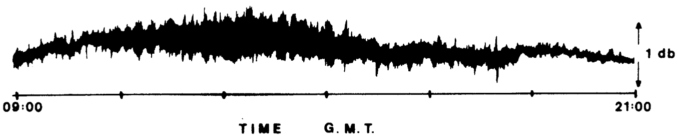

Fig. 1. Chart record of the 36-GHz link amplitude scintillations for the period between 09.00 and 21.00 on October 7, 1979.

TABLE I

Run	Date and Time (GMT)	Conditions	Averaged wind speed m/sec	Mean wind component reference to the link path	Wind speed standard dev. σ_v m/sec.	Averaged value of C_n^2	
						Probe $\times 10^{-14}$ $m^{-2/3}$	Link $\times 10^{-14}$ $m^{-2/3}$
1	27-8-79 14:31÷16:05	convective	3.5	transverse	1.4	8.8	6.2
2	1-10-79 18:15÷19:10	stable	1.0	transverse	0.3	2.1	1.2
3	5-10-79 18:38÷19:30	stable	3.6	along	0.9	1.2	0.6
4	7-10-79 14:04÷14:56	convective	4.1	along	1.3	6.2	6.6
5	22-10-79 16:35÷17:28	weak convective	7.6	along	2.1	0.9	4.2
6	20-12-79 18:45÷19:36	stable	2.4	transverse	1.2	5.4	0.3

III. Experimental Arrangement and Results

The vertically polarized 36-GHz millimeter wave link is 4.1 km long and is over central London with almost one-third over a park and the rest over and between tall buildings with a minimum path clearance in excess of the first Fresnel zone of 5.8 m. The average height of the link is 50 m above ground level. Details of the system used are given elsewhere [7]. The direct meteorological measurements were obtained from an atmospheric probe located at the receiving end of the link at University College. In this work the probe was used to measure the dry and wet bulb temperatures, with a frequency response of approximately 1 Hz, wind speed, and wind direction. The full details of the probe are given by Asimakopoulos *et al.* [2]. Data processing was carried out on line with the Modular I Computer and also, as a back-up facility, data was recorded on a multichannel FM tape recorder. Spectra were obtained using a Fast Fourier Transform (FFT) over 24 segments each of 256 data points sampled at 0.5-s intervals.

To produce an estimate of C_n^2 using (6), it is necessary to calculate the spectral density of refractive index at a frequency in the inertial subrange. In practice an average value of C_n^2 was calculated from estimates obtained from the values of the refractive index spectral density at four different frequencies within the inertial subrange.

The refractive index structure parameter C_n^2 varies significantly depending on the time of day and season. Solar heating of the ground, which initiates convective heating, causes C_n^2 to increase to a maximum value at early afternoon. Values of C_n^2 and hence the amplitude scintillations reach their maximum during the summer. A chart record of the diurnal variations of the scintillations are shown in Fig. 1, where the effect of this behavior on the amplitude scintillations can be clearly seen.

The comparison studies reported in this paper were carried out over a five month period during the summer and winter of 1979, under different meteorological conditions. Mean values of the probe and link estimates of C_n^2, together with some details of the prevailing meteorological conditions are given for a few typical examples in Table I. For convenience the results are classified into two groups corresponding to convective and stable conditions.

For all of the experiments, frequency spectra of temperature, water vapor pressure, and refractive index variations were obtained. These clearly showed, in both convective and stable atmospheric conditions, a $-5/3$ frequency dependence (e.g., Fig. 2). The outer scale of turbulence estimated from the refractive index spectrum is $\simeq 100$ m, which is much greater than the first Fresnel zone size of 5.8 m ($=\sqrt{\lambda L}$), which is a necessary condition for the application of (2). This spectrum will be discussed further in the following section. The atmospheric stability was determined using the σ_D^2 method [8].

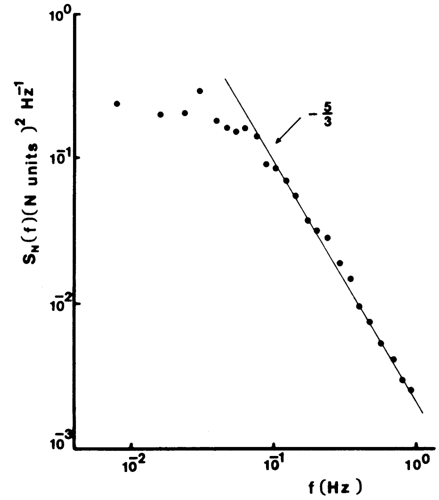

Fig. 2. Average spectrum of the refractive index for weak convective conditions on October 22, 1979 (Run 5).

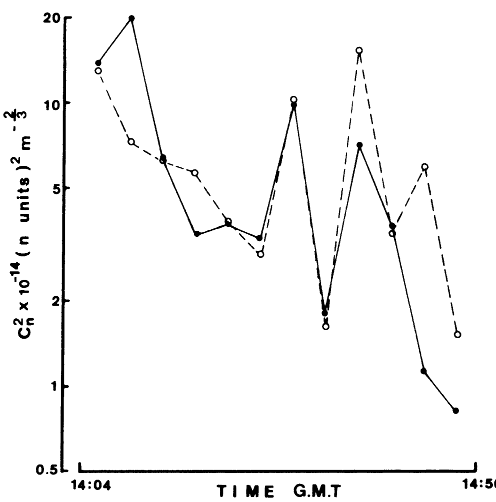

Fig. 3. Time histories of C_n^2 estimates for convective conditions from the turbulence probe (•) and 36-GHz link (○) between 14.04 and 14.56 GMT on October 7, 1979 (Run 4).

IV. Convective Conditions

An example of an experiment under convective conditions is illustrated in Fig. 3, (Run 4, Table I), which shows the time series comparison of C_n^2 estimates, obtained from the probe and link measurements. The mean wind speed was of the order of 4 m/s with a direction along the propagation path. The figure clearly shows that the estimates of C_n^2 in each case follow similar trends and are in reasonable agreement, with mean values of 6.6×10^{-14} $m^{-2/3}$ from the link and 6.2×10^{-14} $m^{-2/3}$ from the probe respectively. It should be noted that the values obtained from the link are a path integrated measurement, while the values from the probe are a point measurement. Fig. 4 shows the average spectrum of the link scintillations for Run 4 (Table I). The $-8/3$ slope at the high frequency end of the scintillation spectrum which characterizes the scattering mechanism in a locally homogeneous and isotropic medium [4] is clearly illustrated. The refractive index fluctuations spectrum for this case is similar to the spec-

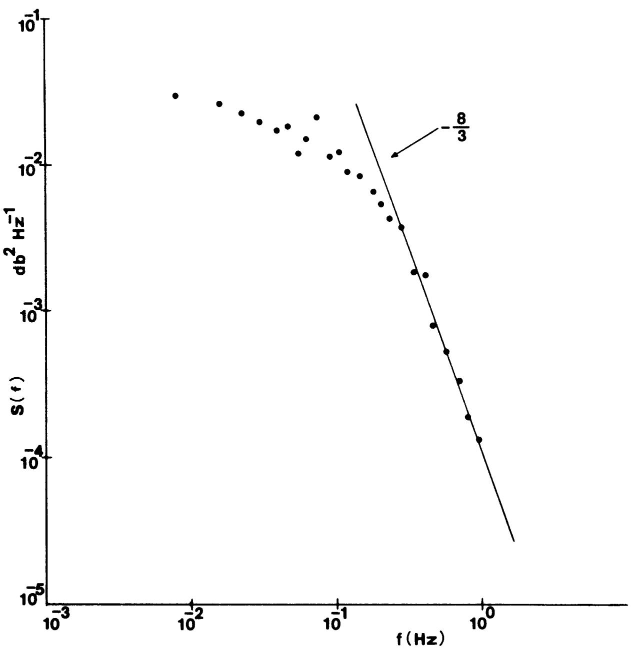

Fig. 4. Average spectrum of 36-GHz link scintillations for convective conditions on October 7, 1979 (Run 4, see Fig. 3).

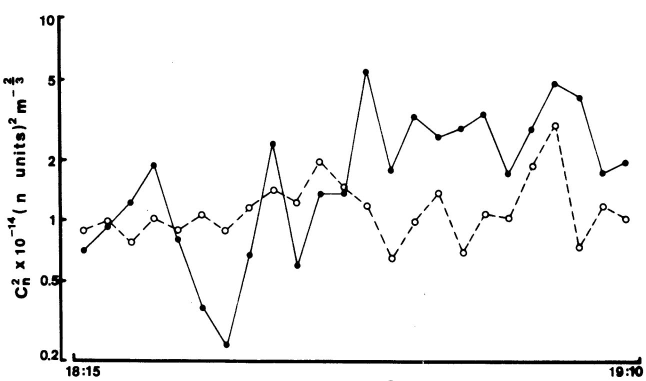

Fig. 5. Time histories of the C_n^2 estimates for stable conditions from turbulence probe (•) and 36-GHz link (○) for the period between 18.15 and 19.10 GMT on October 1, 1979 (Run 2).

trum shown in Fig. 2. Similar results were obtained in the other convective cases studied.

V. Stable Conditions

It is apparent from Table I that the agreement between probe and link measurements of C_n^2 is not as satisfactory as the agreement obtained in the case of convective conditions. An example is given in Fig. 5 (Run 2, Table I), where the wind is transverse to the path. Here the C_n^2 values from the link and the probe are 1.2×10^{-14} and 2.1×10^{-14} $m^{-2/3}$, respectively. The variability with time observed by the probe is much greater than that observed by the link. This is probably due to the patchy structure of the turbulence which occurs in a stable layer. In this case the probe follows the local inhomogenities while the link integrates over the whole path. Similar results were obtained in all the experiments made under stable conditions. In all cases the refractive index spectra exhibited a $-5/3$ slope (Fig. 6) and the scintillation spectra a $-8/3$ slope (Fig. 7). Fig. 6 shows a departure from the $-5/3$ frequency dependence of the refractive index spectrum at 0.4 Hz which was due to the low wind conditions which results in a slower response of the probe sensors, especially of the wet bulb.

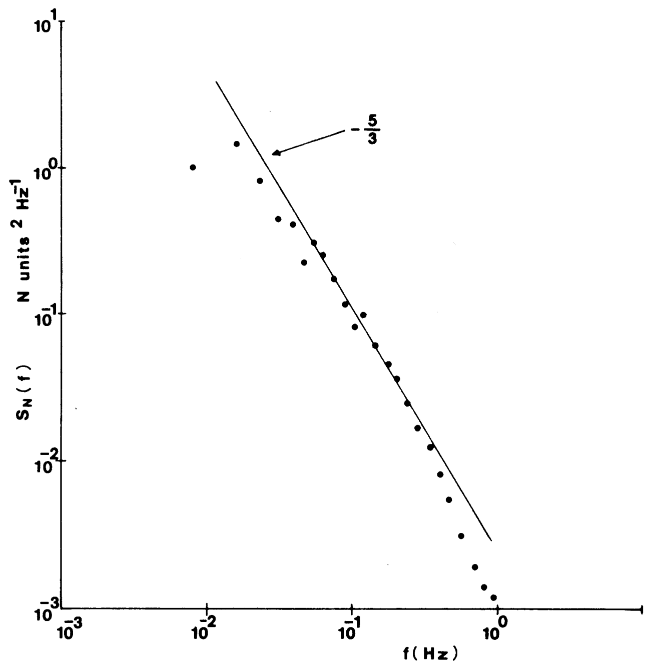

Fig. 6. Average spectrum of the 36-GHz link scintillations for stable conditions on October 1, 1979 (Run 2, see Fig. 5).

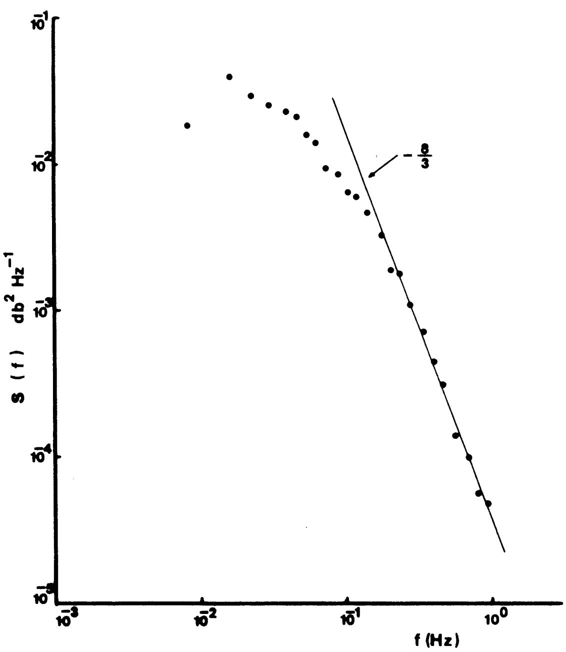

Fig. 7. Average spectrum of the 36-GHz link scintillations for stable conditions on October 1, 1979 (Run 2, see Fig. 5).

VI. Anomalous Conditions

Occasions were experienced when the results did not fall into either of the two categories previously discussed and the discrepancies between the probe and link estimates of C_n^2 were very large. This occurred in one experiment (Run 5, Table I) with a relatively high wind speed having a mean value of 7.6 m/s along the path. The value of C_n^2 obtained from the link was four and a half times the value given by the probe. The probe refractive index spectrum, clearly displayed the inertial subrange with a $-5/3$ slope (see Fig. 2). However, the frequency spectrum of the link scintillations did not show the customary well defined $-8/3$ slope. This behavior was attributed to the presence of a building crane on the path, about

1.5 km from the receiver, which was swaying in the wind thus enhancing the scintillations due to varying reflections from the crane structure.

On another occasion (Run 6, Table I), it was the probe which gave the high value of C_n^2 compared with a very low value indicated by the link. This was caused by the steam output of a heating system in the building located some 10 m away from the receiving antenna. The wind direction was such that hot moist air was blown towards the probe and the receiving antenna. The effect of this, was immediately detected by the probe giving a value of C_n^2 some ten times higher than its previously recorded value. The link measurement, however, showed no change, thus confirming the prediction by Lee and Harp [9] that the region near the receiver contributes little to the amplitude scintillations.

VII. Conclusions

This work has demonstrated that although the probe gives a point measurement and the link a path integrated measurement of C_n^2, they produce average values of the same order of magnitude for the path considered. This indicates that the turbulence "seen" by the probe at a point, is a good representation of the turbulence along the whole path in this experiment, provided no localized man-made perturbations are present. The experiments carried out under convective conditions, when the atmosphere can be considered to be isotropic and homogeneous, show the best agreement as might be expected. Under stable conditions this is not so, resulting in discrepancies, although not very large, between the two methods of measurement. Finally two cases of anomalous propagating conditions are presented which were the result of man-made interference on the ray path and show a considerable disagreement between the two methods of measurement.

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