EXTENDING AREA MORPHOLOGY TO MULTIVARIATE IMAGES

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ABSTRACT

Area morphology has proved a popular technique for image filtering and segmentation. This paper investigates its extension to multivariate images. By defining the extrema of a connected set of vectors as the vector that is furthest from all other vectors, measured using a norm, regional vector extrema can be identified. These extrema are then processed in a manner analogous to scalar area morphology. This is achieved by the incorporation of additional constraints to ensure that each flat zone is treated as a single entity and to process the additional extrema created. Results demonstrate the effectiveness of this approach for filtering motion fields and colour images.

1 INTRODUCTION

Since its inception in the 1960s, mathematical morphology has become a firmly established class of nonlinear techniques for the processing and analysis of binary and greyscale images. More recently, the development of area morphology [1, 2, 3] has eliminated the need for a fixed structuring element thus removing any shape bias from the filtering process. The result is morphological operators that dynamically adapt to the underlying image form. Vincent's work on area morphology was later extended to accommodate other attributes by Breen and Jones [4] and is also linked with connected operators [5] and granulometries [6].

The use of multivariate imagery such as that of motion fields, colour and multi- and hyper-spectral images is an area of rapid expansion and therefore it is desirable to be able to apply morphological techniques to data of this form. However, the absence of any natural ordering for multivariate data means that this process is not straightforward. This problem is also evident for other nonlinear filters such as the median. Here, it has been overcome by the vector median filter [7] which selects as its output the input vector that has the minimum sum of distance to all other inputs. Other recent approaches have used component-wise ordering, based on vector direction and magnitude [8] or the HSV components of colour images [9].

This paper presents an effective solution to the problem of extending area morphological operations to multivariate images. To control the filtering a scalar image of the sum of distances from each vector to its connected neighbours is calculated. Vector extrema (both maxima and minima) can then be identified as the maxima in the scalar image and processed in a manner similar to that of scalar area morphology.

The arrangement of this paper is as follows. A brief discussion of some recent approaches to ordering multivariate data is given in section 2. The vector area morphology (VAM) filter is described in section 3 and results showing its application to motion fields and colour images are given in section 4. Finally, some conclusions are drawn in section 5.

2 ORDERING IN MULTIVARIATE DATA

To extend mathematical morphology techniques from univariate to multivariate images the problem of ordering the data must be addressed. A simplistic approach is to apply standard morphological methods to each channel of the multivariate data. This approach can modify the spectral composition of the image and has been shown to produce jitter near edges for other nonlinear filters [7]. One attempt to overcome these problems was the development of vector dilations and erosions for colour image processing [10]. In this work, the morphological operations rely on a reduced ordering derived from a mapping of each vector to a scalar value, $d : R^p \rightarrow R$. Vector dilation and erosion ($\oplus_v$ and $\ominus_v$) are then defined by

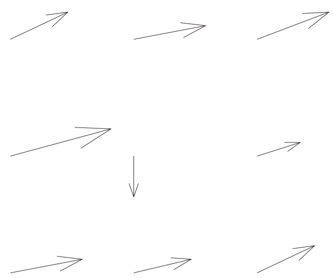


Figure 1: Example motion field

$$(\vec{f} \oplus_v H)(x,y) = \vec{a} \tag{1}$$

and

$$(\vec{f} \ominus_v H)(x,y) = \vec{b} \tag{2}$$

respectively, where $\vec{a}, \vec{b} \in \{\vec{f}(r,s) : (r,s) \in H_{x,y}\}$ and $d(\vec{a}) \geq d[\vec{f}(r,s)] \geq d(\vec{b}) \ \ \forall (r,s) \in H_{x,y}$. In [10], two metrics were proposed for reduced ordering based on the linear combinations of the tri-stimulus values and the Euclidean norm. However, any reduced ordering can result in extrema vectors being erroneously selected. For example, in figure 1 the vector with the largest Euclidean norm is in the centre left position but the central vector is clearly the natural outlier.

An alternative approach was adopted by the vector median filter [7] in which the vector median $\vec{x}_{vm}$ of a set of N vectors $\mathcal{V}$ is defined as the vector that has the minimum sum of distance to all other vectors,

$$\sum_{i=1}^{N} \|\vec{x}_{vm} - \vec{x}_i\|_p \leq \sum_{i=1}^{N} \|\vec{x}_j - \vec{x}_i\|_p \ \ \forall j \in \mathcal{V}. \tag{3}$$

In this filter, $\mathcal{V}$ is defined by a local window and therefore the ordering depends on the relationship between vectors in the window instead of an individual attribute. The vector directional filter takes a similar approach to (3) except that it first orders the vectors based on their directions and then uses their magnitudes to resolve any ties [8].

3 VECTOR AREA MORPHOLOGY

A scalar area morphology open-close of a greyscale image to an area of λ results in a filtered image inwhich all regional maxima and minima are flat zones of at least λ pixels. In this spirit, the result of a VAM open-close to area λ should therefore be a vector image in which all extrema regions are flat zones of minimum area λ. In multivariate imagery, a flat zone is a connected region of the identical vectors. The practical algorithm for the implementation of VAM proposed below follows the main stages of the scalar area morphology algorithm described in [3], but also includes additional steps to accommodate the multivariate data.

The first stage of the algorithm in [3] is to identify all regional maxima and minima. To identify true extrema vectors VAM adopts a similar philosophy to that of the vector median filter. That is, the ordering of a vector depends on the relationship between vectors within a local neighbourhood. Following (3) the vector extremum $\vec{x}_{ve}$ is defined as

$$\sum_{i=1}^{N} \|\vec{x}_{ve} - \vec{x}_i\|_p \geq \sum_{i=1}^{N} \|\vec{x}_j - \vec{x}_i\|_p \ \ \forall j \in \mathcal{V}. \tag{4}$$

This is the vector from the set $\mathcal{V}$ that is furthest from its neighbours. Referring again to figure 1, it can be seen that (4) successfully identifies the outlying vector in the set. Rather than applying this definition directly, a two stage process is proposed. Firstly, a mapping $d[\vec{x}_i]$ from a vector $\vec{x}_i$ to a scalar, given by

$$d[\vec{x}_i] = \sum_{j \in \mathcal{N}_i} \|\vec{x}_i - \vec{x}_j\|_p \,, \tag{5}$$

where $\mathcal{N}_i$ is the set of four or eight connected neighbours of $\vec{x}_i$, is used to form a scalar image from the multivariate image data. Vector extrema are then defined as the regional maxima of this image. Note that unlike scalar area morphology which has both maxima and minima, VAM only identifies outliers as extrema, without the requirement to differentiate between maxima and minima.

However, the mapping of (5) can result in identical vectors having different values in the scalar image. This is undesirable as area morphology works by successively enlarging those flat zones that are regional extrema. Therefore each flat zone should be processed as a single entity that can be preserved or enlarged but never destroyed. To ensure that this is the case for multivariate image data, after the vector to scalar transform has been performed all flat zones of area ≥ 2 are assigned the mean $d[\vec{x}_i]$ value of the flat zone. Extrema regions in this modified scalar image are identified and place in a list.

The next step in scalar area morphology is to increment the area size λ from 2 up to the desired area size and, at each area size, increase the area of all extrema regions with area $< \lambda$ to the current area size by adding the pixel whose intensity is closest to the current region intensity to the region and changing the intensity value of the region to that of the added pixel. The analogue for growing extrema in multivariate images is to change the vectors in each regional extremum to the closest vector in their connected neighbourhood, thus increasing the area of the region by 1. Here the closest vector is chosen to be the vector that has the minimum distance from the vector of the extremum region, measured by a norm such as the Euclidean distance.

Replacing the vectors in the extremum region results in changes in the scalar image produced by (5) which is used to identify the extrema. However, as the only positions to be affected are those of extrema regions that have changed and their neighbours, $d[\vec{x}_i]$ only needs to be recalculated for these positions, not the entire image. After updating $d[\vec{x}_i]$, those flat zones that are no longer regional extrema are removed from the list of extrema and any new extrema regions resulting from the modification of $d[\vec{x}_i]$ are added to the list. To accommodate the creation of new extrema, VAM continues processing the list of regional extrema until all its elements meet or exceed the current area size.

4 EXPERIMENTAL RESULTS

To illustrate the VAM technique, it is applied to two forms of multivariate data: motion fields and colour images. The original motion field in figure 2(a) shows a 10×10 image consisting of an underlying vector of $(0, 5)$ with three features represented by vectors $(5, -3)$, $(-3, -1)$ and $(-2, 2)$ of area 14, 6 and 4 respectively. The results of the VAM filter for areas 5 and 7 are also shown. For area 5 the VAM filter has removed the connected component that constitutes an extremum region with area 4 while perfectly preserving the larger connected features. The connected component of area of 6 is removed when the VAM area reaches 7 (see figure 2(c)) leaving the largest component which is only removed when the area reaches 15. This operation performs exactly how area morphology would intuitively be expected to for multivariate data.

The usefulness of VAM for reducing noise in motion fields is also investigated. Figure 3(a) presents a motion field of background vector $(5, -2)$ with a central 4×4 feature, vector $(3, -2)$, corrupted by additive Gaussian noise $\sigma = 1$ on both the x and y components. The filtered image, figure 3(b), is a good representation of the underlying, noise free image with minimal degradation of the feature boundary.

For colour image processing, a test image is created by adding Gaussian noise with $\sigma = 27.5$ to each channel of the Lenna image, see figure 4(a). The VAM results for areas of 2, 8 and 24 are shown in figures 4(b), (c) and (d) respectively. It can be seen that the variation due to noise is reduced with increasing area size, as flat zones of constant colour are created. As the image components are either

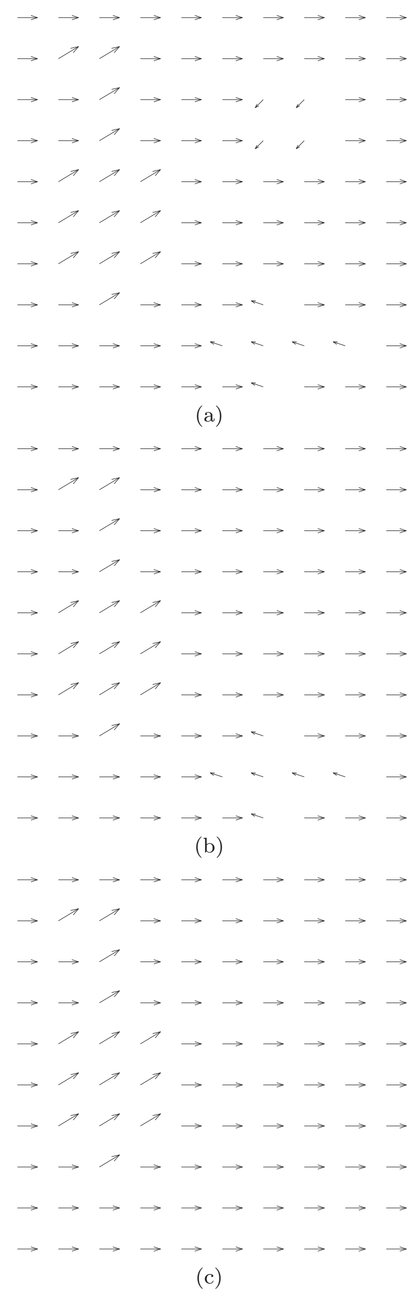
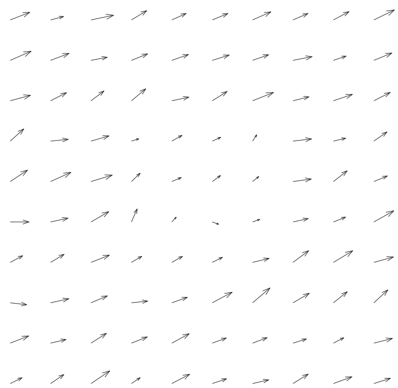
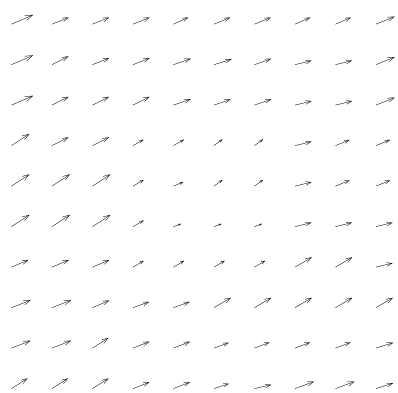


Figure 2: (a) Original motion field, (b) result of VAM algorithm for area 5 and (c) 7.



(a)



(b)

Figure 3: (a) Noisy motion field and (b) result of VAM algorithm for area 5.

completely removed or perfectly preserved, the more significant boundaries within the image are not degraded by the VAM filtering process. However, as increasingly larger extremum regions are removed, some of the more significant image information is lost, for example the whites of the eyes in figure 4(d).

5 CONCLUSIONS

VAM has been proposed as a new technique for applying area morphology to multivariate images. Although it does not demonstrate scale-space causality, the performance of VAM is in agreement with the intuitive extension of area morphology to multivariate images. Novel aspects of the method include the definition of the vector extrema, used to determine outlying vectors, and additional constraints for preserving flat zones in multivariate images.

The effectiveness of VAM for multivariate image segmentation and noise reduction has been demonstrated in application to motion fields and colour images. Although only area has been considered, the technique is easily extendable to other attributes. An area of further work is to investigate the suitability of recently proposed connect set algorithms for improving the computational efficiency of VAM.

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(a) (b)

(c) (d)

Figure 4: Colour images results. (a) original image corrupted with Gaussian noise $\sigma = 27.5$, (b) VAM result for area $= 2$, (c) area $= 8$ and (d) area $= 24$.