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3 The School Mathematics Project

Basic information

Beginnings The heads of mathematics of four public schools together with Bryan Thwaites, then Professor of Theoretical Mechanics in the University of Southampton, initiated the School Mathematics Project (SMP) in 1961 as a semi-private experimental venture. The intention was to reform the teaching of mathematics in the four schools to reflect modern developments in the subject and its wider usage, and to encourage other schools to do the same. The production of new GCE O- and A-level syllabuses acceptable to the universities was to be undertaken together with associated text books and teachers' guides. Four state grammar schools joined in to form the central group of schools responsible for shaping policy. Some two years later, the number of participating schools increased to 41.

Support The University of Southampton provided accommodation for the project within its Department of Mathematics and appointed a lecturer funded from external sources to assist Professor Thwaites. Supporting non-academic staff also were appointed. During the first few years, the financial support from industry and other sources was between £5,000 and £10,000 per annum. The project gained rapidly in momentum. By 1966, it had become clear that the role of the project was longer term and wider than originally envisaged. SMP was legally constituted a charitable trust in August 1967, with Dr. Bryan Thwaites, by then Principal of Westfield College, University of London, chairman of the Board of Trustees. The project now exists as a, voluntary, fully independent, self-supporting organisation whose annual expenditure runs into five figures. It is self-dedicated to curriculum development and research in mathematical education.

Production timetable The first experimental text, called *Book T*, for transition from the traditional O-level to the new projected course at the 13+ stage, was produced cooperatively by some 7 school teachers and the university editor, for use and testing in the schools in 1962–63. Modification and re-testing in 1963–64 led to a third version which was published by Cambridge University Press in July 1964. Meanwhile, an experimental draft of the continuation book for 14+ pupils, T4, had been written in time for the first SMP GCE O-level examination to be held in July 1964, with the Oxford and Cambridge Examinations Board acting as coordinator on behalf of the GCE Examining Boards. *Book T4* was published in June 1965.

The production of all SMP textbooks followed the same pattern: they were written by school teachers and published only after classroom trials and subsequent revision.

An experimental draft of the first book of a new projected five-year O-level course was ready for use by the grammar school 11+ pupils at the start of the session 1963–64. The completed sereis, *Books 1–5*, was written and published in stages over a period of some five years to 1969.

The first SMP A-level examination came in July, 1966, two years after the first O-level. Publication of the four A-level texts was completed by September, 1968, and of the Further Mathematics series, by 1971. The services of an advisory group of five university mathematicians were available to the teachers involved in the preparation of the materials. Before the syllabuses were drawn up, a questionnaire was issued to all university mathematics departments to ascertain their views on sixth form mathematics.

A modified version of *Books 1–5*, the lettered series, *Books A–H*, intended to be suitable for a CSE examination, was begun in 1966 and completed in stages by 1972. Three supplementary books, *X*, *Y* and *Z*, bringing the series to O-level standard, were published, one each year during 1972–74. *Books A–D* were subsequently issued in card form, *Cards I and II*, to facilitate the use of individualised and group learning techniques, and to extend the range of pupils with whom the material could be used.

The preparation of a primary and middle school series, SMP 7–13, a new venture for the project, was begun at a conference of interested persons held in April 1972. *Units 1 and 2* of the series, consisting of pupils' workcards and booklets, were published in March 1977, together with answer books, pupil's record sheet, assessment tests and teachers' handbook. The six unit series is to be completed by 1980. New versions of the A-level and Additional Mathematics syllabuses, permitting the use of non-programmable electronic calculators, came into operation in 1977.

Following an enquiry to the O-level schools, arrangements have been made for there to be two O-level syllabuses from 1977. The present 'non-calculator' syllabus will continue and an alternative 'calculator' syllabus has been drawn up. Teaching materials for the latter are in preparation.

An outline for possible N and F syllabuses and examinations has been prepared.

Evaluation Evaluation has been formative, relying on the subjective impressions of teachers using the trial materials and, later, the published texts.

Growth of interest

Table 3.1

Year (Summer)	*1964*	*1965*	*1966*	*1967*	*1968*	*1969*	*1970*	*1975*	*1977*
No. of O-level candidates	919	1,848	3,526	6,642	10,980	12,879	20,100	54,015	62,691*

*Some 20 per cent of the national entry.

The numbers of A-level candidates in 1975 and 1977 were respectively 5926 and 7024, the latter being about 10 per cent of the national entry.

The influence of the project has almost certainly been greater than these figures suggest. For example, when the CSE and other GCE examining boards instituted their own examinations in modern mathematics, the pattern had already been set by the SMP. More than half the schools in the country are said to be making some use of the SMP materials.

Headquarters The SMP office is at Westfield College, Kidderpore Avenue, London NW 3 7ST.

Full time staff Mr. John Hersee (Executive Director, appointed 1 January 1976), Dr. Alan T. Rogerson (Research Director), Mr. John Ling (Team Leader, SMP 11–16, appointed 1 September 1977), Mrs. Shirley Berry (Secretary).

Project teams Over ten working groups are currently functioning. Over one hundred teachers have been or are currently involved in writing materials.

Course materials A substantial body of classroom materials with teacher's guides has been produced in the attempt to provide for the mathematical and examinational needs of school pupils of various abilities from the age of seven upwards. There is also a guide for parents, a series of handbooks dealing with specific topics, and texts which look ahead to university mathematics. Eight English language and five foreign language editions of some of the courses are available or in preparation. Production continues and full details may be obtained from the publishers.

Publishers Cambridge University Press, PO Box 110, Cambridge CB2 3RL.

Materials in preparation A two-year O-level self-tuition course in conjunction with the National Extension College. Remedial cards for the 11–13 very slow learners. Materials for a course for one-year sixth form pupils. Supplementary material in revision of the Further Mathematics texts.

Plans for further materials A fundamental review of the SMP provision for the 11–16 age range is being carried out, bearing in mind the possibility of a common system of examination at 16+ and the changing educational scene.

A computing-in-mathematics group is investigating the implications for teaching of the electronic calculator and computer.

Teacher support Assistance to individual teachers is given on request by the members of the executive staff, the editors, authors and revisers of the various texts.

Residential teacher training conferences are arranged as required. Attendances over the past five years have totalled over 2,500 at some six to eight conferences per annum.

Other support The School Mathematics Project has given general help to the cause of mathematical education. For example, a joint Mathematical Association/SMP committee organises the National Mathematics Contest held in March each year which is open to persons under the age of 20 who have not proceeded to higher education. The contest is the first of a series of competitions to select a team of eight, sponsored by the SMP, to represent the UK in an International Mathematical Olympiad.

Background influences: developments in the USA

Amongst British projects in mathematical education, the SMP stands in a class of its own if only for its size and the range of its activity. Its remarkable growth is indicative of the support and approval it has received but there has been fundamental criticism of the type of course it has provided. This account of the SMP and its contribution centres on its O-level and main school courses.

The movement for modernising the teaching of mathematics by introducing concepts from the 'new mathematics' was becoming world-wide in the early 1960s. (Meder 1957, Smithies 1963). Considerable impetus came from developments in the USA following the launching of the first Russian sputnik in October 1957. Some six months later, the School Mathematics Study Group (SMSG) was in being. It was national in scope and funded from government sources to an amount of four million dollars during the first four years of its existence (Wooton 1965).

The SMSG course, determined by university mathematicians, presented mathematics as a series of logical structures. A rigorous deductive development based on the notion of set made mathematics appear remote-from-reality, self-generating and self-contained. Morris Kline (1966) pointed out also that too little attention had been given to pedagogical considerations and that teachers were insufficiently prepared for the change. In view of the social pressures in operation, mathematical applications were oddly neglected in the new course.

At the time, SMSG seemed generally to make good sense. Technology cannot advance without mathematics, and the US government had taken the advice of mathematicians. But it consulted pure mathematicians and they have tended to be inward-looking in relation to their subject this century following the discovery of non-Euclidean geometry around 1830 which directed their attention more particularly to structure, rigour and formal developments within mathematics. SMSG was certainly short on application, seeming to reflect G. H. Hardy's words, 'If he is consistent, a man of the mathematical school washes his hands of applications.' (Coulson 1973).

Other American projects adopted similar approaches to the SMSG (Thornton 1963; Nacome 1975; Sherman 1977) in particular, the Educational Services Incorporated African Mathematics Program (1961–70), funded to an amount of $3\frac{1}{4}$ million dollars, which produced complete courses for African primary and secondary school pupils and teachers-in-training. Its materials, or adaptations therefrom, were used by 2,000,000 primary, 200,000 secondary school pupils and 60,000 students-in-training. (Entebbe Mathematics Series 1971). The UNESCO Mathematics Project for the Arab States, launched in Cairo in 1969, came under the same influences. (UNESCO 1969a, 1969b, 1970–71).

Progress in Britain: the contribution of SMP

Teachers in Britain took longer than in the US to decide the school implications of modern mathematics. A feature of the educational system in England and Wales is that before curriculum change can be effected, teachers have to be convinced of its desirability. Innovation tends to come about slowly by a piecemeal process through the influence of textbook writers, teacher-educators, research workers and such agencies as the Mathematical Association, the Association of Teachers of Mathematics, LEAs and the Department of Education and Science.

The pressure to reform school courses built up gradually. (Rollett 1963, Pitt 1963). There were informed teachers who resisted, unconvinced of the advantages of modern mathematics for the classroom. The terminology seemed likely to hinder rather than help the learning process. (Goodstein 1962). The majority of teachers probably knew very little about the 'new mathematics' since the universities only began to modernise their undergraduate courses during the 1950s. Thus existing teachers were faced with the need to relearn their subject. Feelings of uneasiness, even apprehension, existed. A course of lectures by professors of mathematics from various universities at the University of Leicester in 1952 attracted unusually large audiences of teachers who seemed to be hearing about most of the topics for the first time. The position of mathematics staffs in the teacher-education institutions was similar.

The OEEC conference in Royaumont in 1959, (OEEC 1961), following a survey of the status of mathematics in member states, brought the need in Europe into sharp focus and several modern mathematics school projects were started in Britain during the 1960s. (Mathematical Association 1968). The SMP was one of the first off the mark. Professor Thwaites' involvement in school mathematical education became well-known if only because of the Southampton Mathematical Conference. (Thwaites 1961a, 62) and his inaugural lecture at Southampton in 1961 in which he drew forceful attention to the dire shortage of mathematics teachers which had reached crisis proportions. (Thwaites 1961b).

Widespread concern had already resulted in an upsurge of activity in mathematical education. For example, A. P. Rollett had convened the Whitelands conference of teacher-educators in 1955 and, as a result, the Mathematics Section of ATCDE was re-formed. E. E. Biggs was beginning her work in the primary schools. There was the Oxford Mathematical Conference in 1957 for teachers and users of mathematics to which the origins of the SMP have been ascribed. (Oxford Mathematical Conference 1957).

These activities were essentially exercises in personal and public relationships and a sense of mission was in evidence. The SMP was set up in this climate. The management of its affairs was outstandingly able (Clarke 1973).[1] Its immense programme was carried through on time. Institutions and people of standing were involved. The project rose rapidly to a unique position of strength, independence and influence whence it could challenge accepted positions. It received the widest publicity through its publications and its programme of lectures and conferences, the draft materials were readily available and an increasing measure of teacher-confidence was established.

Secondary schools were being offered a system of modern mathematical education – a continuous course from 11+ to 18+ with its own examinations including a single-subject A-level agreed by the universities, an advisory service and in-service training facilities. The materials captured the spirit, joy and fascination of modern professional mathematics and their presentation was polished. There was openness in the approach, humour, and a place for learning by discovery. In keeping with British practice, mathematical applications were not neglected, a wider range being introduced than in traditional courses. The materials took us nearer to the position 'where, in fine, all the branches of elementary mathematics, pure and applied, theoretical and experimental, are comingled at appropriate times, so that the mind sees and uses its mathematical conceptions and processes as a beautiful well-ordered and powerful whole, instead of a thing of shreds and patches.' (Branford 1908).

Teachers, many of whom were dissatisfied with existing practice, had gradually come to feel that the change to modern courses had to come and would be irreversible. The SMP materials were very well received (see, for example, Sturgess 1971). Perhaps the main contribution of the project has been to the personal re-education and support of established teachers including those involved in the writing, thereby facilitating the national acceptance of the need for syllabus reform. The textbooks presented mathematics in a form which could be widely understood and so helped to put teachers in the position to make valid judgements on the new courses.

Controversy in mathematical education

On the other hand, the modern conceptual trend in school mathematics met with resistance from the start (Hammersley 1968, Lyness 1969) and a firm reaction to the new syllabuses has since set in. The call for 'back to basics' in the USA has become sufficiently strong for it to have been countered at the third International Congress on Mathematical Education (1976) in two papers. (Hilton 1976).

In this country, it is said that modern mathematics courses place undue emphasis on pattern and structure to the neglect of the techniques and processes required by users of the subject and, in particular, that SMP pupils do not acquire the skills in arithmetic, algebra and analysis which are needed in other school subjects and industry. (*Maths in School 1975–76; Bulletin,* Institute of Mathematics and its Applications 1969–76; *Times Educational Supplement* 1974–76. *Int. J. Maths. Ed. Sc. Tech.* 1975–77). A policy of continuously reviewing its syllabuses has allowed SMP to respond. There were three major revisions of its courses during the first ten years. The A-level and Further Mathematics courses are continuing to be revised (Rogerson 1975) and materials containing manipulatory exercises supplementary to the O-level courses have been produced. But the controversy continues.

The issues are not resolved by questioning SMP teachers, their pupils, students-in-training or university teachers of mathematics. Some are keenly enthusiastic, some non-committal and others hostile.

What is clear is that standards of numeracy amongst school leavers have declined (Lindsay 1975; Parliamentary Committee 1978), the shortage of qualified teachers of mathematics in the schools continues, (Kerr 1977; Ollerenshaw 1977), and there has been a serious drop in the number of students opting for the subject at university (Griffiths 1975; McLane 1975).

Yet there has been progress (see Howson 1978). University departments and colleges of education, some of which were without mathematics lecturers twenty years ago, are now well staffed. The provision for in-service training has improved. Primary arithmetic, which was narrowly focussed on computation for instrumental purposes, has been enlivened by the introduction of a wide range of mathematical ideas coherently presented and there is more understanding about how children learn. Classrooms are happier places and children's attitudes to mathematics have consequently improved. Many teachers have been conspicuously successful using the newer methods. Despite such gains, the high expectations which accompanied the immense activity in the various fields of mathematical education during the 50s and 60s have not been fulfilled.

Various forces in our society have undoubtedly combined to delay the progress of mass education in recent years. The alleged decline has not been confined to one subject and it would be 'facile to attribute present low standards to a deterioration brought about by something called "modern" mathematics' (HMI 1977). Those who produced the SMP syllabuses were breaking new ground at the time with little previous experience to guide them. Their experimental venture, initially involving a small number of schools, became large scale and, in consequence, served as a focus for the dialogue on the place of modern mathematics in schools.

Towards an evaluation of SMP

CURRICULUM METHODS

The collection of director's reports serves as a main source of information about the SMP and its early development(Thwaites 1972a). In these, we find, 'of over-riding importance to us ... is that the syllabuses and the associated teaching methods should be developed as the practical outcome of teaching experience, rather than as a result of theoretical discussion round committee tables' (6). The syllabuses are 'an amalgam, though most carefully alloyed, of their [teachers'] own interests, preferences and prejudices which, in turn, have been moulded by their schools' traditions and characters' (7).

The recognition in practice by the SMP of the professional role of the teacher as the main arbiter, if not always the architect, of the curriculum, which is in keeping with the traditions of the Mathematical Association, must be counted as a major reason for the pedagogical quality and wide acceptance by schools of the course materials (see Howson 1974, 1975).

As the director's reports imply, practical experience rather than curriculum theory has been the guide. No attempt appears to have been made to set out the course objectives in detailed behavioural terms and, though the materials were tried out in schools, no summative evaluation has been carried out. To break down course objectives into minute detail, as some curriculum theorists have advocated, is hardly feasible in this subject and the creative element in mathematics learning often arises spontaneously. Mathematics teachers have tended to the distrustful of curriculum theory. In view of the long history of reform in the teaching of their subject, they are inclined to agree with Herbert M. Kliebard (1975) who, in reference to the a-historical perspective of curriculum theory, writes, 'Issues tend to arise *de novo*, usually in the form of a bandwagon, then quickly disappear in a cloud of dust. The field in general is characterised by an uncritical propensity for novelty and change rather than funded knowledge or a dialogue across generations.'

In fact, the general principles underlying the SMP course as set out in the director's reports are capable of various interpretations for classroom purposes. Looking back, it seems that, if the principles had been stated more comprehensively and in some specific detail with a view to eventual evaluation, maybe teachers would have been better helped, especially those unable to attend training conferences.

The SMP group of people seem not to have made advance preparation for the possible overall assessment of their projected materials. They could hardly have anticipated the extent of the project's development and impact and, in any case, one can well believe that they were so carried along by an optimistic, enthusiastic conviction of the rightness of what they were doing, the magnitude, interest and challenge of the task, the wide following they attracted, and their intention to revise their materials regularly in the light of classroom experience, that the thought of objective evaluation did not at first cross their minds. Each revision of the materials necessitated some form of evaluation and the project teams continued 'to rely on the subjective impressions of the classroom situation' (Thwaites 1972a, 98). 'We have to admit that no progress has been made during the year [1965–66] with the problems of assessing, in any absolute sense, the educational merits of the new curriculum' (115). Objective evaluation is discussed on two pages with special reference to its complexity. The suggestion is twice made that some national authority should undertake comprehensive, independent assessments of projects and their products. A specific proposal to the Schools Council was considered early in 1971 but it was felt that teachers would not find the results of evaluation of individual projects helpful in present circumstances.

The large number of variables in any educational situation, including subjective factors which do not lend themselves to measurement, makes research difficult to conduct and can render its findings inconclusive. Any curriculum project must rest on a set of values and function from a position of informed faith and belief. The SMP is no exception and evidently has proceeded largely on this basis. Curriculum innovation usually originates in the intuitive, experimental gropings of teachers and educational research follows as an instrument of validation.

In attempting to assess the worth of the SMP materials account must be taken of the needs and conditions then and now, the extensive dialogue, any independent evaluations, and any other relevant considerations. (Parlett and

Hamilton 1972). More evidence is needed, in particular, on the performance of ex-SMP students at university in their first and subsequent years when reading mathematics or another subject which requires mathematics.

The SMP materials were written by teachers for the use of teachers. The reader may care to consider the extent to which the publisher's sales figures – by January 1978, for example, 300,000 and 690,000 copies respectively of *Books I* and *A* including overseas sales – may be taken to reflect the soundness of the course, and so serve as a form of summative evaluation. The materials of some modern mathematics courses prepared during the 1960s have ceased publication.

It is appropriate to our study of SMP methods as a curriculum development project to note that, in common with other projects, some of its intentions – in this case, invariably fine intentions – have not fully been achieved, as the following examples indicate:

1. The director's personal conviction is 'that SMP (and other similar experiments) are only in the foothills of the mountainous changes in mathematical syllabuses which tower ahead' (Thwaites 1972a, 7). An SMP organisation 'with built-in mechanisms for evolution' is looked for. No special validity is claimed for the SMP syllabus which is but one of many possible and variety of syllabus is said to be desirable. It is not desirable that the 'SMP offerings should degenerate into a new classroom dogma'. Yet, despite the efforts taken to prevent it happening there are indications that, 'SMP maths' became a dogma for some people though, for obvious reasons, not to the same extent as 'Fletcher maths' in the primary schools.
2. 'The cardinal feature of the SMP is that it is a free association of school teachers of mathematics who have a common interest in improving the teaching of mathematics ... It is independent of all governmental and other official bodies; this not only gives the SMP valuable freedom in the conduct of its research but also ensures that there is no pressure on schools to adopt SMP materials against their will' (Thwaites 1972a, 195). At least one teacher considers he did not teach the SMP course from free choice but as a result of indoctrination. '... perish the thought that a mere teacher should dare to question the views of university professors, educational researchers *and* book publishers, who said they *must* change their thinking on mathematics in schools. ... I had been 'brainwashed' into thinking tradition is a dirty word. Today ... I see what an absolute fool I have been' (Hodgkinson 1976).
3. 'The project cannot possibly claim to know all the answers and yet in this extraordinary *laissez faire* system of English education the probability is emerging that what began as a private experimental scheme will be adopted before its results are fully assessed, by large numbers of the country's schools'. (p. 16, 1962–63 Report in Thwaites 1972a). This may

be a penetrating comment on the process of change in education but it also reflects a failure in intention. Initially, knowledge and experience of ordinary state schools may have been lacking. In any case, in the exciting, complex situation which existed, with the need for action paramount and so much at stake it must have been difficult to temper enthusiasm with reason in the application of professional safeguards. It is not clear that the results of the 'private experimental scheme' ever were 'fully assessed'.

4. The existence of a tendency towards weakness in arithmetical and algebraic skills amongst SMP students which the publication of the supplementary manipulatory exercises in 1977 seems to confirm, is an example of failure in an intention which seems not to have been stated explicitly.

The reader may care to consider the implications of these examples. How far do they reflect deficiencies in the English system of education and teacher-training, in individual schools and teachers? Where does the responsibility lie? To whom is an independent organisation operating within the state system accountable?

THE COURSE

Content and method

The invention of non-Euclidean geometries and of algebras other than the classical algebra are amongst the developments which, in a sense, changed the character of mathematics, giving it a different basis and an extended content. This is why the gap, which the SMP proposed to close had opened up between school and university mathematics.

The new SMP syllabuses departed radically from established practice, firmly and effectively confronting the situation. The O-level and main school materials introduce a variety of concepts and topics taken to varying depths, including the notions of set, mapping, group, vector, matrix and some topological ideas together with linear programming, transformation geometry (introducing motion into the subject), computers and programming, probability and statistics. Topics such as relation, function and number-systems are linked with the concept of set which is basic in the course and serves as a main unifying idea. The accent is on conceptual learning as is seen clearly in the treatment of area.

The standard method of teaching school mathematics is to move from topic to topic as often as is necessary to maintain interest, returning to the topics from time to time for consolidation and extension purposes. (IAAM 1957; NCTM 1953). Pupils easily forget what they have learnt and frequent revision is necessary. Learning thus proceeds spirally in a hit or miss, but directed, manner which more or less ensures that the necessary concepts, techniques and processes are learnt sufficiently well for examination purposes. If the order of the textbook treatment is not to the teacher's liking, no matter. With average competence, he does not have to follow it slavishly.

The SMP textbooks apparently take these teaching procedures for granted. The course progresses from topic to topic in lively fashion and there is natural development as in the emergence of matrices. The teacher will have to know what he wants to get from the course for his pupils in order to lay the emphasis correctly and organise the revision. More revision exercises would have been an advantage. The treatment generally seems more superficial than in a traditional course presumably because of the increased syllabus content. Learning must inevitably be more thinly spread. It is easy to understand how pupils may not be acquiring the arithmetical and algebraic skills considered requisite by users of mathematics. The supplementary manipulatory exercise materials will be helpful and they will serve to draw the teacher's attention to the need. But the problem of the time factor must remain as the course is so full.

An issue of principle is involved, for, as appears in the director's 1962–63 report (19, 20), '... we have constantly tried to shift the emphasis towards mathematical ideas and away from manipulative techniques. Considerable facility in manipulation is, of course, required by pupils who are hoping to become mathematicians, physicists or engineers, but it is the opinion of those in the project that the acquisition of these techniques is best left until the post O-level stage ... This will also free the pupil who stops at O-level from much unnecessary learning.' 'At O-level, then, we seek to convey something of the nature of various algebraic concepts rather than convey a definite body of knowledge.' (17). This view is maintained in the SMP pamphlet *Manipulative Skills in Mathematics* (1974). 'The syllabus has been constructed with the pupil who will do no mathematics beyond O-level primarily in mind' (16).

The course is thus intended to be an end in itself at that stage. The traditional stress on techniques is clearly to be avoided, but there exists the alternative possibility of shifting the emphasis too far in the direction of understanding high-level ideas and not facing the nitty-gritty of mathematics learning. Acquiring knowledge, skills and habits of thought as the outcome of sustained effort and the due practice is a 'way of life' in mathematics, not only for utilitarian reasons, but as a means of deepening understanding. The student who does not suitably meet this 'way of life' for himself can only arrive at a diminished view of the subject. Teachers generally find that the 'bite' of mathematics is more likely to lead to commitment than butterfly learning by itself, however pleasant.

Has the emphasis in the SMP course moved too far in the direction of mathematical ideas? This may be the case, as is indicated by the findings of M. Preston (1972) in an investigation into affective behaviour in CSE mathematics using a sample of 699 SMP, 116 Scottish Mathematics Group (SMG), 83 Midland Mathematics Experiments (MME) and 73 Westminster pupils. He identifies three factors of affective behaviour defined respectively as: (a) tending to see mathematics as an algorithmic, mechanical, somewhat stereotyped subject; (b) tending to see mathematics in an open-ended, intuitive heuristic setting; and (c) representing commitment, interest and willingness to work at mathematics. 'The results of pupils taking the SMP course do seem significantly different from the norm. The level of Factor B mean scores indicates these pupils see mathematics in a wider context of applications, that they have a more strongly developed sense of intuition and their approach to problems allows greater flexibility. The disappointing values emerging from Table 7 [the table of scores including those for Factor C] is the level of commitment and interest for the SMP pupils' (49, 67). The lower mean score was significant at the 1 per cent level. Preston also finds that 'the cognitive skill of a student is indirectly related to his score on the algorithmic scale and directly related to his interest and commitment' (45).

The SMP O-level course is examined by two papers, each of $2\frac{1}{2}$ hours duration. Paper 1 consists of two sections, one containing short-answer questions and the other objective type questions. In Paper II, section A questions are easier than those in section B which score about two-thirds of the marks. Candidates are allowed the use of tables which include comprehensive lists of formula and definitions. Since the examination can exercise a determining influence on the teaching, the reader may wish to consult some past papers.

Evolutionary or revolutionary change

It is clear from the textbooks that SMP has endeavoured to bring about a liberation in the secondary mathematics classroom equivalent to that which has been achieved in the teaching of the subject in some infant and junior schools. The project's strategy for effecting innovation has rightly been two-pronged because to have employed traditional methods in the teaching of modern mathematics would have defeated the educational purpose of the exercise. This suggests that the effective introduction of modern mathematics courses may be a more complicated business than it appears at first sight, especially in developing countries. The double transformation required must challenge the most knowledgeable and experienced teacher and its general implementation take years of continuing trial and experiment in constantly changing circumstances. If this is the case, there is a strong argument for evolutionary rather than revolutionary change. Yet the latter is needed if we are to keep pace with developments. This is the dilemma which the SMP has had to face.

Perhaps classroom mathematics should evolve from and be integrated with the traditional. Abrupt breaks with tradition, as in the skills versus ideas dichotomy, are to be avoided. Discarding Euclid's development in favour of transformation geometry, admirable though the SMP treatment of the latter is, both in its own right and as a focus of integration, could be another case in point. In its zeal, SMP may have been partially defeating its own purposes by adopting policies which are too radical.

A similar note of caution was sounded recently by Rene Thom (1973) when he spoke of genetic constraints he believes to be operative in the learning of mathematics.

> There is always a stage of necessary apprenticeship, genetic constraints to respect, in order to learn to walk, to speak, to read, to write, and it does not seem as if progress in psychology has been able to modify in any way the normal calendar which governs the acquisition of such knowledge. This is why one can legitimately ask whether the same kind of constraints are not operating in the learning of mathematics. If this is the case, then the hope of arriving, by means of a general reorganisation of curricula or methods, at an accelerated awareness of the great theories of contemporary mathematics, could well prove illusory . . . This is why it is not obvious that an advancement in recent knowledge must, of necessity, be reflected in syllabuses, especially at the elementary and secondary levels.

In the second of two articles, A. J. Malpas (1974) investigates the SMP claim that *Books A* to *H* are suitable for all pupils excepting those in the fourth quartile of the intelligence range. This he does by examining the cognitive demands of the books in relation to pupils 'in the middle 50 per cent of the intelligence range', using a Piagetian yardstick. The results indicate that a majority of these pupils may be expected to experience considerable difficulty with a substantial part of the work in the third and fourth years, and all the books from D onwards are likely to make severe demands on them. The work is 'within the intellectual compass of probably an average to good O-level-bound group.' This suggests that the SMP may have been in error in its view that the same basic type of course in modern mathematics can be modified by simplifying the language, using a card system to facilitate individual learning, and the like, so as to be suitable for three quarters of the school population.

The treatment of number

The director of the SMP writes 'that this first English experiment in cooperative mathematical teaching has not been as radical, so far, as some of the larger experiments in the USA and in Europe.' (Thwaites 1972a, 44) Though we have suggested, in effect, that the SMP experiment may already have been too radical for school pupils, teachers and employers, it may be pointed out that having gone so far in that direction, to have moved slightly further would have enabled a distinctly more satisfying treatment of number to be undertaken.

The *Teacher's Guide* for *Book 1* states that 'directed numbers traditionally cause many difficulties and are often as much misunderstood by the teacher as the pupil' (139). The SMP textbooks provide evidence for this in that they mix directed number with natural number. In *Book 4*, 198, we find '. . . the counting numbers, that is, the positive whole numbers, together with zero . . . The union of the set of counting numbers with their inverses and zero under addition is a new set of numbers called *integers*. This set is the set of "directed" whole numbers' (*sic*). But the counting numbers do not have inverses; otherwise there would be no need to invent directed numbers.

In *Book 1* (chapters 1 and 2), number is introduced as counting number and the accent is on the ordinal concept. Cardinals are taken for granted as in:

Pick out from the following questions the one whose answer is in a counting number:

(a) If 5 men eat 4 loaves each, how many loaves are needed?
(b) If 18 loaves are divided equally between 6 men, how many does each man get?
(c) How many loaves are left if 4 loaves are eaten from a batch of 20?
(d) If 3 loaves are divided equally between 4 children, how much does each child get?

Directed numbers are introduced in chapter 12. 'Corresponding to every counting number (for example, 3), we associate a *negative shift number* (3 paces back) and a *positive shift number* (3 paces forward)'. These are termed negative three ($^-3$) and positive three ($^+3$) respectively and 'calculating rules' for the new 'numbers' are derived. Later in the chapter appears: 'So far in this chapter, care has been taken to distinguish between the counting and the positive numbers. However, it will be clear by now that the positive numbers, when added and subtracted, behave just like the counting numbers. This means that there should be no confusion if we leave out the + sign. We shall therefore write 3 instead of +3' (205). By the time *Book 4* is reached, the counting numbers and the positive integers are mentally equated and the current misconception of the number concept extension process is fully apparent. To have entitled chapter 12 'Negative Numbers' instead of 'Directed Numbers', the title used in the lettered series, giving the impression that the negative integers are numbers in a class apart, different in type from the positive integers, prepares the way for the misconception.

In the circumstances, would it not have been better to have approached number as the property of a set in *Book 1*, Chapter 2, bringing in notions of correspondence and equivalence to explain ordering and counting? The integers could then have been discussed as an extension of the number concept in illustration of an important aspect of the way mathematical thinking proceeds.

Mathematical applications

In the first of his two articles, Malpas (1974) investigates the objectives of the SMP main course. He finds that 73 of the 93 chapters of *Books A* to *H* make 'substantial reference to the external world', about half being concerned with abstracting mathematical concepts and developing the languages, and two-fifths with applying the mathematics so learnt. The complete mathematical modelling process involves deriving from a verbalisation of the real situation under investigation a pictorial diagram leading to a symbolic representation (the abstraction/language aspect) followed by the same steps in reverse (the application aspect) (see Figure 3.1). (For an earlier account of the Modelling Cycle, see Ormell 1973a.)

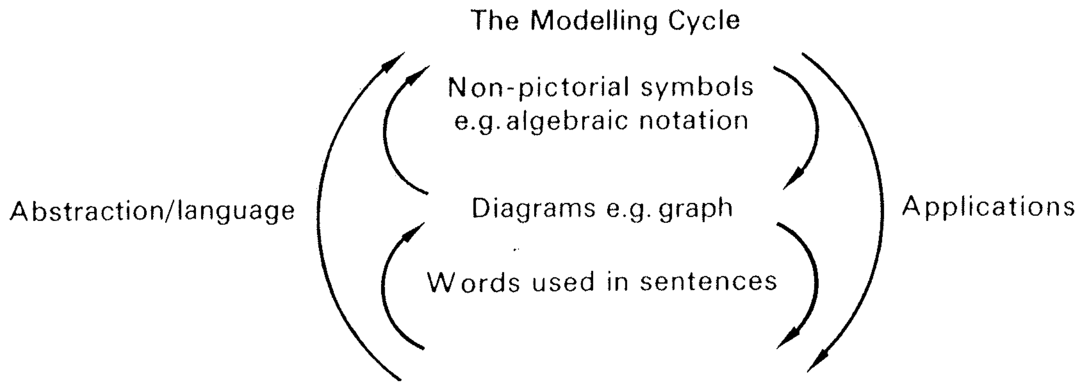

Figure 3.1

Pupils like to apply their knowledge as soon as it is acquired, and for learning to be well-motivated and effective, the abstraction/application cycle should be completed in toto. In only 9.5 per cent of the 73 chapters does this happen. The majority deal only with half the cycle, usually abstraction/language in the earlier chapters and application much later. It is inferred from this pattern that 'it was abstraction and not the situation which really interested the authors'. Pupils would probably sense this and their interest could be diminished.

This raises a basic issue in the teaching of mathematics. The claim, often heard in the classroom, that mathematics is useful on account of its wide applicability in the real world, may be accepted by the student on the teacher's authority but it is not usually borne out in his immediate personal experience. Much of what he learns lacks apparent relevance, and therefore, meaning, unless he generally enjoys mathematics for its own sake when problems of motivation do not arise. Otherwise, the concrete examples the student is called upon to work do not have sufficient interest of themselves to hold him. There is usually a degree of artificiality about them and they are used only to give practice in the working of the appropriate mathematical rule, or so it seems; but when the mathematics has been learnt, what is it for? Various writers have suggested or implied that the teaching of mathematics is in difficulty at this point. (Niss 1977; Kendall 1977; Lighthill 1972). (See also chapter 13, 'The Sixth Form Mathematics Curriculum Project'.)

LOOKING AHEAD

Extensive and rapid change has been a salient feature of the educational scene during the fifteen years which have passed since the SMP courses were thought out. A period of stimulating expansion has given way to one of retrenchment. Increasing emphasis is being placed on public accountability and the utilitarian purposes of schooling. There has been little time for adaptation and teachers of mathematics are still learning how to deal with their subject in a modern setting. Further changes seem undesirable but there is no standing still. The director of the SMP stated in 1972 that he hoped 'to see all existing books torn up, burnt or otherwise disposed of by 1985 at the latest'. (Thwaites 1972b)

The original SMSG texts are no longer in production and the fact must be faced that the SMP materials may now largely have served their initial classroom purposes. It is time to be thinking of the second-generation series and how best to effect their introduction: an SMP team leader for the courses for 11–16 year-olds took over his duties in September 1977. The present materials will have to be appraised in the existing situation in order to plan any new course. Maybe the method of this appraisal would serve as a basis for making provision in advance for the evaluation of the new materials when the time comes, if this is considered desirable.

Modern mathematics courses in schools tend to reflect the viewpoint of the pure mathematician in being oriented towards the subject and its structure. A crisis of meaning arises for some students, and the user of mathematics is severely critical of the orientation. Would it be possible to redress the balance by injecting into existing courses exercises suitable for the development of the understanding and translation skills which successful modelling requires? Maybe, but probably not with the SMP course which is more or less wedded to the outlook in question and already full. An alternative also to be considered is the production of a course with a different main emphasis, one in which the accent is on applicability, realistic modelling and the relevant mathematical knowledge.

The world we live in is both Euclidean and Newtonian for all workaday, scientific and technological purposes, and well within the limits of accuracy possible in measurement. Does this imply a reconceptualisation of the nature of mathematics for the purposes of the envisaged re-centred course, away from the formalist towards a Platonic view? (Thom 1970. See also Ormell 1973b). What would be the content of the course? One would expect the unifying terms, 'set', 'mapping' and 'relation', to be introduced in order to further understanding and assist explanation, but how many such notions would be considered helpful and necessary? We are told by Jerome S. Bruner in his *Process of Education* that the way to ensure that a student's learning is both remembered and of use to him in the future is to communicate an understanding of the fundamental structure of the subject being studied, together with any implied habits and skills. Would it be sufficient to stress the elementary patterns of sequential development in mathematics – number to counting to addition to multiplication? Or would it be desirable also to link number with sets and explain cardinals, ordinals, numerals, the commutative, associative and distributive laws, the extension of the number concept . . . ? Would the course be less or more suited to the future purposes of any potential mathematician? The demands it would make would be different but no less exacting.

The reader will wish to make up his own mind on the various issues in curriculum development and mathematical education which have been raised in relation to the SMP and its materials. Perhaps one may be allowed to suggest that yesterday's challenge to the SMP has been effectively met and is now over except in the field of in-service training where the need continues unabated.

One of the present challenges is to devise trial courses for the 11–16 age group, deriving in part from stated theoretical principles. They must be suitable for classroom and teacher use, place due emphasis on structural learning and eliminate the dichotomy between mathematics and its applications. Such courses could have the advantage of being less radical in their mathematical approach but more searching, in real terms, of understanding and competence. They could also be sufficiently different and demanding as to hold the teacher's interest and help maintain the trend for development in mathematical education.

Any new SMP courses will doubtless be built on the primary and middle school series, SMP 7–13, of which the first units appeared in March 1977. They were produced by practising teachers and tested in schools before publication in accordance with the standard practice of the SMP. A card system is used and, according to the publishers, 'the very able, average and less able children are well catered for'. A reviewer writes:

> The handbook states ... that there is not much new mathematics to be found in it. This disclaimer is somewhat coy as many topics are enhanced by the introduction of such ideas as sets, the laws of commutativity and distribution, Caley tables, equivalence, mapping, number line, ordering and bases other than denary. There is no doubt that the course emphasizes most of all the need for a firm foundation for work on number, and practice in computational skills, measurement and shape. Perhaps in the present educational climate, the course is being published at an opportune time.
>
> (Foster 1977)

A new USA programme, *Developing Mathematical Processes*, takes up a position closely similar to that of the SMP 7–13 course both in teaching approach and the introduction of modern terminology. (Moser 1970–76). The new version of the USA series, *Modern School Mathematics: Structure and Use*, (Duncan 1978), however, does not even use the word 'set', presumably reflecting the strength of the reaction of the elementary school teacher against the 'new math'.

In the SMP director's reports, it is stated (Thwaites 1972a, 115) that many of the new texts seem 'to have brought about, almost inevitably (as it now seems), a change of heart and of approach on the part of teachers who are now refashioning their teaching methods so as to sharpen their pupils' imagination, intuition and curiosity'. The mention of these possibilities serves as a timely reminder of the valuable intrinsic elements in mathematical education that transform the subject in its learning and teaching yet do not have obvious connection with the utilitarian ends for which the community is currently calling. 'To be educated is not to have arrived at a destination. It is to travel with a different point of view', (Peters 1963).

BRYAN THWAITES

SMP: A Response

Of all the critiques of the work of the SMP which have appeared from time to time, Wilfred Flemming's is the most sensitive and perceptive, and I greatly appreciate it. Certainly his analysis will help the SMP to examine its mechanisms and future objectives more effectively, and to identify its own strengths and weaknesses.[1] Given, then, that there is nothing in the account with which I take serious issue, all that I need do is to add one or two glosses which may further interest the reader.

Looking back over the last quarter-century, I am inclined to make the bold claim that it was the SMP which successfully established, for the first time, a definitive methodology of curriculum development. This methodology had five ingredients, all of them indispensable: authorship by experienced practising teachers, repeated pre-publication testing of draft materials in the classroom, GCE and CSE examinations to correspond to the courses, in-service training for teachers, and a continuing service for teachers. Permeating these ingredients is the additional idea of assessment of the pupil's materials; Wilfred Flemming interestingly observes that the SMP felt that subjective assessment of this kind is probably more valuable than the attempts at objective assessment which have been made elsewhere.

The SMPs methodology, then, set new standards for curriculum development and the reader may care to assess the projects described in this volume according to these standards, and to correlate his assessments with the quality of influence exerted by the projects.

Beyond that, and beyond its astonishing and sometimes slightly terrifying impact on mathematical education all over the world, what else might be deduced from the SMP's experience? Perhaps the deepest impression left on me by the recent period of extraordinary change is the long time-scale inherent in education in general and in curriculum development in particular. (The knowledge picked up by a young teacher in 1978 may be imparted at the end of his career and used around the year 2050!) In these circumstances, I dare say that the typical 'research project' funded for a short fixed term is virtually useless: there has to be almost limitless follow-up if a project seeks to have any but a trivial and ephemeral impact. It is for this reason that the affairs of the School Mathematics Project have been directed in such a way as to ensure that, for practical purposes, it has an indefinite life.

Finally, I cannot resist the temptation to offer a strictly personal view which, I know, is not universally shared. If only by virtue of the large numbers of academics who have become involved in curriculum development for schools, there has emerged the idea that mathematical education is a discipline in its own right which should carry with it all the conventional panoply of chairs, of journals, of international conferences and so on. I doubt whether this idea is either justified or valid. Education is ultimately manifested in the relationship between the individual teacher and the individual pupil, and I do not believe that the relationship is subject to the rigorous analysis which is a necessary characteristic of an academic discipline. Everything rests, in the end, on the personal qualities of the individual teacher and hardly anything else matters.

Notes

1. It may be noted that the SMP clearly carries the impress of Dr. Thwaites' personal views on education as they appear in his article 'Visions of greatness or the defence of values in education', *IMA Bulletin* 12, 10, 300–304 1976. From the start, the organisation was set on maintaining a position of independence, a 'position unencumbered by existing interests and independent of existing organisations'. At one stage, a grant of £30,000 from the Schools Council was refused. The position taken up by the SMP must have made for tension and conflict as may be gleaned from the Director's reports. Hidden in the process of curriculum development are personal, social, political and other beliefs and values whose effects do not lend themselves to ready assessment, either from within or without a project, to confound the theorist.

2. This was my reaction to an earlier version of Flemming's article which was considerably longer than that now published. The shortened version seems to lay greater emphasis on the original ideas of the SMP than is now relevant; from it, readers may obtain the impression that there is some fixed and unchanging view of school mathematics which essentially characterises the SMP. On the contrary, the SMP evolves inexorably and, in so doing, adjusts its attitudes to the current scene.