Writing mathematics

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But, isn't maths different?

No, and the basics can elude us

Yes, in style and precision

Coherence

Take pity on the reader!

Assist with navigation

Precise use of language and symbols

Notation, language and conventions

Example sources of ambiguity

Write, read, revise

Asking for help

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But, isn't maths different?

"For the most part, the writing of mathematics is like the writing of English prose. Indeed, it **is** *a part of the writing of English. [...] If you read your work aloud, then you should be reading complete sentences that flow from one to the next, just as they do in good prose." [1, page 23]*

"Mathematical expressions are part of the sentence and so should be punctuated." [2, page 24]

"The primary rule is that you write in simple, correctly punctuated sentences." [3, page 23]

"Use good English, writing carefully, clearly and correctly." [4, page 20]

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Some 'obvious' rules

- Write in sentences!
- Use punctuation.
- Obey the usual rules of English grammar.

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Consider this example. Is this a sentence, does it use punctuation and does it obey the rules of grammar?

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Example

$n \times n$ matrix multiplication $C = AB$ in terms of scalars:

$$c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj} \qquad 1 \leq i, j \leq n$$

At matrix-vector level:

$$C = [Ab_1, Ab_2, \ldots, Ab_n]$$

$B = [b_1, b_2, \ldots, b_n]$ a partition into columns

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For example, we can write the $n \times n$ matrix multiplication $C = AB$ in terms of scalars,

$$c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}, \qquad 1 \leq i, j \leq n,$$

or at the matrix-vector level,

$$C = \left[Ab_1, Ab_2, \ldots, Ab_n \right],$$

where $B = [b_1, b_2, \ldots, b_n]$ is a partition into columns.

Yes, in style and precision

There are some differences between mathematical writing and other scientific or formal writing you may encounter:

- We tend to write in the first person plural ("We" in the sense of "the reader and I").
- We tend to write in the active voice (prefer "X did Y" to "Y was done by X").
- We tend to write in the present tense ("Applying theorem 1.2 we find").

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We **must** maintain logical coherence and precise use of language and symbols to communicate effectively.

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"In mathematics, what is needed is a clear and orderly presentation of the ideas." [1, page 14]

"The ideas must be put in logical order and the chain of reasoning forged and fixed" [2, page 4]

"Readers are not psychic. It is crucial to explain what you are doing." [3, page 26]

"Keep in mind that you have to maintain the reader's interest at all times. An easy way to lose it is by stringing along a succession of statements with no discernible goal" [4, page 6]

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- Give an overview
- Keep the reader informed of progress
  - *Explain* the connection to what has come before and what will follow
- Use connecting phrases to communicate the logical connection e.g.
  - Deduction: hence, so, it follows, as a result, consequently, therefore, thus, accordingly, then...
  - Explanation: as, because, since, due to, in view of, owing to, from, using...
  - New ideas: let, suppose that, define, now...

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- When you read mathematics in your area pay attention to how the writer ensures their writing is coherent

Take pity on the reader!

"a good mathematics paper is **not** *necessarily written in strict logical order [as] this is not the way that we learn" [1, page 62]*

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- Don't expect a reader to wade through technical details without explanation or a framework to make sense of the new information
  - Get readers involved via summaries, clear definitions and statements of results
  - Keep readers involved by communicating the key steps and techniques in use
  - Let readers delay engaging with technical detail until they are sure it is worth the effort

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  - Provide comfortable "stopping off" places for the reader to pause and reflect
  - Interweave prose and computation
- Lighten the load on working memory
  - Use straightforward vocabulary and short sentences with a simple structure
  - Remind the reader of definitions and formulae you have not used in a while
  - Remind the reader of the plan and progress through it
  - Supply examples, special cases, visualisations when they will assist the reader to build understanding rather than distract from the current train of thought

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  - This includes sections, subsections, theorems, propositions, lemmas, algorithms, important formulae or groups of formulae, examples etc.
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Be **specific** and **precise** when referring a reader elsewhere in the document. Do not vaguely refer to the diagram above or the first formula of this form.

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Precise use of language and symbols

"Mathematics cannot tolerate imprecision. The nature of mathematical **notation** *is that it tends to rule out imprecision. But the* **words** *that connect our formulae are also important." [1, page 6]*

"Use symbols if the idea would be too cumbersome to express in words, or it if is important to make a precise mathematical statement." [2, page 20]

"the reader should not have to deduce what you mean from context; all the necessary information should be there. Nothing should be ambiguous." [3, page 28]

"Use standard or familiar or suggestive notation that your readers can assimilate easily, allowing them to devote their energies to the mathematics" [4, page 16]

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  - For most points x, $x \in S$.
  - We see that $x \in S$ for most points x.
- Take care with Latin abbreviations and the singular/plural versions of non-English words e.g. Datum/Data

Example sources of ambiguity

- Keep it simple: compound sentences should be avoided due to the risk of introducing ambiguity
- Many expressions are not equations! Call expressions by their correct name e.g. inequality
- Ensure subject and verb agree even when mathematical constructs are involved e.g.
  - The set of all morphisms are compact
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- Avoid *it*, *this*, *that*, *these*, *those* or *them* — replace with full word or phrase
  - The above words allow us to talk about things without saying what those things are which can introduce ambiguity or be used to disguise the writer's own confusion

Example sources of ambiguity

- If you use *if* then use *then* (except in the case *if and only if*) e.g.
  - if $a > 0$, $b > 0$, $a + b > 0$
  - if $a > 0$, then $b > 0$ and $a + b > 0$ (e.g. true if $b = 5a$, for instance)
  - if $a > 0$ and $b > 0$, then $a + b > 0$ (always true — recall truth tables)
- Take care with the word *denote*, it has a specific meaning: "Let X denote Y" means the specific item X stands for some other specific item Y e.g. compare
  - Let f be a continuous function
  - Let S denote the set of all continuous functions

Example sources of ambiguity

Take care using words interpreted as quantifiers:

- Avoid using *any* when *all*, *each* or *every* is intended
  - What does "Show that any continuous function f on the interval..." mean?
- It is easier to spot these and other issues if singular constructions ("function" rather than "functions") are used:
  - "All continuous functions have a maximum..." suggests that they all share the same maximum
  - "Each continuous function has a maximum..." is probably what was meant
- Beware changes in quantification order when rewriting sentences!

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"Reading your words aloud **forces** *you to make sense of what you have written and to deliver it as a coherent whole." [1, page 11]*

"Does the text flow? [...] Is there a good balance between equations and text? Does the paper look inviting? Do the key ideas stand out?" [2, page 94]

"Read your work slowly. [...] A useful proofreading method is to concentrate on one aspect of proofreading at a time." [3, page 32]

"When you get [your peers] comments, don't shout them down for being numbskulls: if they had trouble understanding you, others will too." [4, page 28]

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Writing is a loop: capture *some* version, read for various aspects and revise the text. What can you add? What should you take away? Is there a coherent and precise whole? Get feedback on early drafts.

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- Simple syntax and grammar
- Introduction and consistent use of appropriate notation and terminology
- Mathematical accuracy
- Appropriate levels of explanation and abstraction
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Reading aloud is extremely helpful!

Asking for help

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You can attend MASH dropins with questions where some fluency in reading mathematical notation seems to be required e.g. logical structure, incorporating expressions into sentences, proofreading strategies and LaTeX.

- During term time: attend a dropin in the first instance, times at `http://www.bath.ac.uk/study/mash`
- Outside of term time for PhD students only: email `mailto:mash@bath.ac.uk` to enquire

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