How to write maths

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Session 1 of 3
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These are the slides from the talk, annotated with some of the things I (Euan) said (which appear in boxes like this one).

Who are you?

- MMath student doing year-long/semester-long project
  - You'll do some maths....
    1. learn some new maths from a book/paper, or
    2. do some new research, or
    3. do a mixture of 1 and 2

    .....and write a report
- 1st-year PhD student
  - PhD = do research + write thesis
  - You'll write maths up as you go:
    - reports for supervisor
    - transfer report at either 12 or 18 months

Goal of these sessions:

Give you the fundamentals of writing maths well

Plan for these sessions:

- Session 1 (Euan): larger scale "how to do it" (level of whole document/sections)
- Session 2 (Emma): smaller scale aspects (level of paragraphs/sentences)
- Session 3: Interactive session:
  - Get feedback on maths you've written.

“Hold on a minute, what about typing maths?”

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- Before Session 3: complete Emma's "Getting started with LaTeX" mini-course online
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Brief comment on LaTeX:

You should separate the "understanding maths" from the "LaTeXing it"

Why?

1. LaTeX can give equations etc. an artificial "air of correctness" (so need to get maths straight beforehand).
2. Thinking about *both* maths *and* how to type it up in parallel less effective as in series

I claim the "real work" in writing maths should take place before you touch LaTeX.

“Ok, how do I do it?”

If you only remember one thing from this talk....

2 keys to writing maths (or indeed anything!):

1. have something to say (and know what this is)
2. know who the audience is

These 2 points sound a bit obvious, but I claim that you will find it very helpful to ask yourself these basic questions before starting to write something (and to return to them regularly throughout the writing process).

Who is my audience?

- MMath:
  - realistically, main audience is supervisor
  - should act as if your audience is *either* your past self *or* fellow maths student who didn't do this project
- PhD:
  - at first, audience will be supervisor; ultimately wider research community
  - should act as if your audience is *either* your past self *or* novice in the field

What is the "peak" you're trying to reach? How are you going to get there?

Broadly speaking, 2 aspects to the maths:

1. Individual results (theorems, lemmas, definitions, algorithms, examples, numerical results)

2. Flow of ideas (i.e. how the individual results fit together to get to the peak)

When you learn maths from a book/paper/lecture, you normally understand individual results first, and then understand the flow of ideas.

On the other hand, if you're doing research, at least some form of the flow of ideas has to come first (since this is what tells you what individual results to go for).

The key point from the point of view of writing maths is that once you decide the peak, the flow of ideas will dictate what individual results you need to get there (and in what order you need them).

Example: the fundamental theorem of calculus

Theorem

Suppose $f : (a, b) \to \mathbb{R}$ *is a continuous function and* $u \in (a, b)$. *If*

$$F(x) := \int_u^x f(t)\, dt,$$

then F *is differentiable on* (a, b) *and* $F'(x) = f(x)$ *for all* $x \in (a, b)$.

On the board I then went through the short proof of this result, and then drew a concept map to illustrate the flow of ideas (see `proof.pdf` and `concept.pdf`.)

If one were to write a piece of maths whose peak was the fundamental theorem of calculus, then the concept map would dictate the path to the peak.

Exercise:

For your favourite unit this semester, take one of the main results proved so far and draw a "concept map" for it.

Algorithm for writing maths

1a: understand individual results

1b: write out proofs of individual results (could LaTeX here)

2a: understand flow of ideas

2b: write down flow of ideas (e.g. as concept map)

(At this stage you have "proved" the result)

3: LaTeX any remaining results and proofs

“What about text?”

Writing text: what's the point?

- "Linking" text, i.e. text between individual results
  - Goal: "hold the reader's hand" from one result to another
- "Context" text, i.e. text in the introduction/discussion/conclusion
  - Goal: place the "peak" in context

How most people write text:

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"In mankind's quest for knowledge, Partial Differential Equations (PDEs) have proved to be an indispensable tool."

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4: write text by first bullet-pointing ideas (then sub-bullet-pointing), then writing sentences

A note on style...

Crustacean style

- structure on outside
- good for maths papers/notes/talks

Vertebrate style

- structure hidden inside
- good for detective stories

Common stumbling blocks for students

1. "Staring at a blank page" syndrome
2. Not wanting to LaTeX anything until the last minute
3. Expecting the first draft to be the final draft
4. Not getting feedback from your supervisor
5. When learning stuff from book/paper, question of how to take ownership of the material?

Regarding 1: the "peak and path" analogy and the idea that you should write text by first bullet-pointing ideas can be used to combat this.

Regarding 2: although we've emphasised here that one should separate the "understanding maths" from the "LaTeXing it", you don't want to leave it too late to LaTeX things. In particular, getting feedback from your supervisor will probably be easier if you give them a LaTeXed document (since it's usually easier to read than a hand-written one).

Regarding 3: if you follow the advice above, your first draft will probably be pretty good, but it will still need revising. Emma will talk more about how to do this next week (i.e. what to look out for when revising your drafts).

Regarding 4: people who hit the first two stumbling blocks usually get this fourth one for free. Needless to say, the more input your supervisor gives, the better your document will be.

Regarding 5: if you've been learning some maths from a book or paper, when you come to write it up, it's natural to think "How can I do anything different than just copying out the book/paper? (Surely the author is better at writing these results out than I am?)". Nevertheless, in the vast majority of cases the author has *either* a different audience to you, *or* a different peak to you (or they may have multiple peaks, only one of which is yours). These differences will naturally make your presentation of the material different to theirs.

Summary:

- 2 keys to writing maths (or indeed anything!):
  1. have something to say (and know what this is)
  2. know who the audience is
- What is the "peak" you're trying to reach? How are you going to get there?
- Follow the algorithm

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How do I get LaTeX on my machine? (MMath students only)

- Option 1: download LaTeX on own computer
- Option 2: use "Overleaf", access via internet
- Option 3: use MikTeX on Unidesk

For 1 and 2: advice in Emma's mini-course online

`www.bath.ac.uk/study/mash/resources/LaTeX/`

If you need help: talk to us or go to a MASH drop-in

LaTeX resources:

- "Detexify": symbols → LaTeX

`http://detexify.kirelabs.org/classify.html`

- Formulae → LaTeX

`http://webdemo.myscript.com/views/math.html`

- Google (e.g. "crossed-out text in latex?")

Task before 3rd session:

Pick your favourite equation and write it in LaTeX

References/acknowledgements

Two great books on how to write maths are:

1. "Handbook of Writing for the Mathematical Sciences" by Nicholas J. Higham, and
2. "A Primer of Mathematical Writing" by Steven G. Krantz.

These books have different styles, and complement each other very well.
Both are in the library (as 510.37(04) HIG and 808.0665 KRA respectively).

PhD students – I recommend you read both these books.

MMath students – having a look at these books will undoubtably improve your writing, but your time will probably be better spent on the maths in your project.

Acknowledgements: I learnt many of the principles in this talk from the two books above. Marj Batchelor (Cambridge) first taught me the "peak" analogy, and "crustacean" vs. "vertebrate" style.