How to write maths (well)

Dr Euan Spence

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These are the slides from a talk I gave to the new first-year students at Bath, annotated with some of the things I said (which appear in boxes like this one).

Some of you may be thinking "What's writing got to do with maths? Is this some kind of joke? What's next week – how to dance maths?!?"

Well, although this talk was given on Friday afternoon, it was not a joke.

WHAT does writing maths well involve?

WHY write maths well?

HOW do you write maths well?

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At this point I asked the class to look at the two calculations on the handout (`example1.pdf`) and decide which one they find easier to understand.

A show of hands revealed that the majority of people found Calculation 2 easier to understand. This is the one that I find easier to understand, and I claim that the vast majority of mathematicians would also find this one easier to understand. The reason is that it keeps the reader informed about what's going on at each step of the calculation.

The point of this exercise was to demonstrate that the majority of you already have some sort of idea of what constitutes good mathematical writing.

Guiding principle of writing maths well

When you read the maths you've written it should make sense as a piece of English.

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At this point you might say to me "Ok, I can see that Calculation 2 makes sense when you read it aloud whereas Calculation 1 doesn't, and I can see that having these words in Calculation 2 makes it easier for the reader to follow the argument, but if the chips were down the reader would still be able to understand Calculation 1."

I can see where this (hypothetical) person is coming from. Indeed, the vast majority of the mathematical arguments that you've met so far (i.e. at school) are a bit like the one in Example 1 (calculating $\int \sin^4 x \, dx$): you can pretty much get away without writing any words.

However, the majority of mathematical arguments that you'll meet at university don't make sense without any words.

Why?

- The majority of mathematical arguments don't make sense without any words.
- In the future you will be in situations where you need to communicate mathematical arguments to other people:
  - short term: exams
  - longer term: in whatever you do after you graduate

Regarding the short term (i.e. exams): I marked $\sim$100 exam scripts last summer and, when looking at a new script, I could quickly make a good guess how well the student had done based on how many words there were on the page. Of course, this doesn't mean that writing down words just for the sake of it will get you a better mark, but it does mean that writing words is an inherent part of constructing correct mathematical arguments.

Regarding the longer term: even if you end up doing a job that doesn't involve any maths (as some of you will), the ability to construct valid, well-structured arguments will still be very useful.

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Example 2:

Prove that the sum of any two odd numbers is even.

When I was thinking about why the sum of any two odd numbers is even, I wrote down the following on the piece of paper in front of me:

$$2n + 1$$

$$\begin{aligned}(2n + 1) + (2m + 1) &= 2n + 1 + 2m + 1 \\ &= 2n + 2m + 2 \\ &= 2(n + m + 1)\end{aligned}$$

These few lines of maths are the essence of why the sum of any two odd numbers is even, but they're not an argument – when read aloud they don't make sense since they're just a collection of symbols and numbers.

Here is a proper argument that proves the result.

Proof.
Any odd number equals $2n+1$ for some integer n.
Therefore, given any two odd numbers p and q there exist integers n and m such that $p = 2n+1$ and $q = 2m+1$. Hence

$$p + q = (2n+1) + (2m+1).$$

Now

$$\begin{aligned}(2n+1) + (2m+1) &= 2n + 1 + 2m + 1 \\ &= 2n + 2m + 2 \\ &= 2(n+m+1).\end{aligned}$$

Since $2(n+m+1)$ is divisible by 2 it is even.
Therefore, $p+q$ is even.

□

Example 3:

Prove that the product of any two odd numbers is odd.

I gave the students in the class some time to have a go at this example themselves.

Similar to before, the lines of maths that I wrote down before I "got the right answer" (i.e. understood why the statement was true) are

$$2n + 1$$

$$\begin{aligned}(2n + 1)(2m + 1) &= 4nm + 2n + 2m + 1 \\ &= 2(2nm + n + m) + 1,\end{aligned}$$

however these don't constitute a mathematical argument.

Proof.

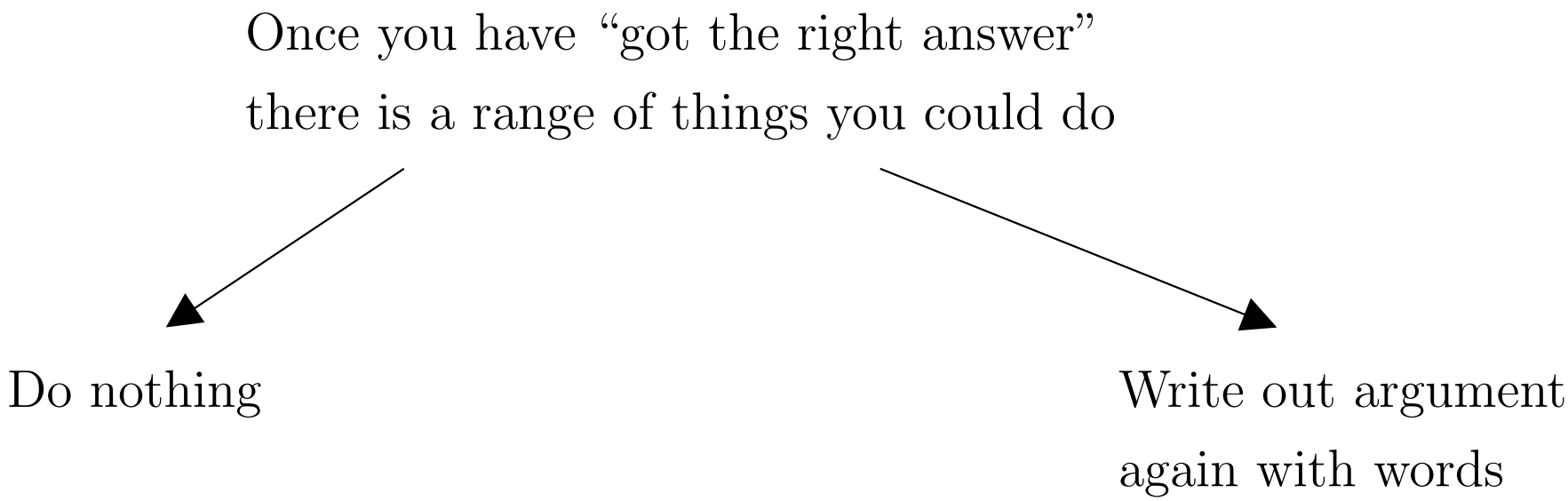
Any odd number equals $2n+1$ for some integer n.
Therefore, given any two odd numbers p and q there exist integers n and m such that $p = 2n+1$ and $q = 2m+1$. Hence

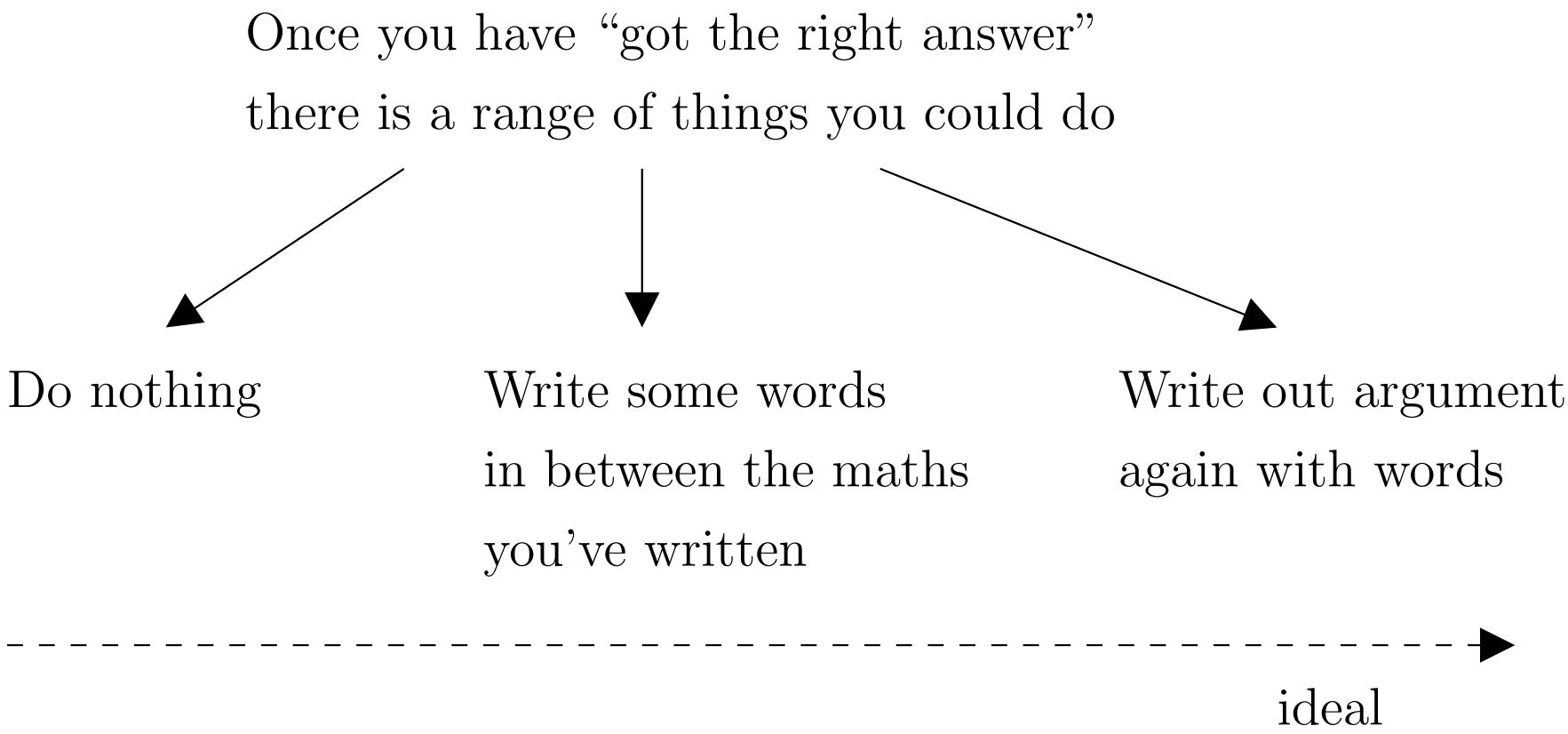
$$pq = (2n+1)(2m+1).$$

Now,

$$\begin{aligned}(2n+1)(2m+1) &= 4nm + 2n + 2m + 1 \\ &= 2(2nm + n + m) + 1.\end{aligned}$$

Therefore, $(2n+1)(2m+1) = 2r+1$ for $r = 2nm + n + m$ (which is an integer if n and m are integers).
Therefore, pq is odd. □


Once you have “got the right answer”
there is a range of things you could do
Do nothing
Write out argument
again with words


Once you have “got the right answer”
there is a range of things you could do
Do nothing
Write some words
in between the maths
you’ve written
Write out argument
again with words
ideal

Although writing out the argument again with words is what you should be aiming for, if you're short on time (e.g. in an exam) you could add some words in between the maths that you've written.

This is illustrated in `example1annotated.pdf`.

If you only remember one thing from this talk....

When you read the maths you've written it should make sense as a piece of English.

This is the beginning....

Learning to write maths well (i.e. learning to construct arguments that make sense when read aloud) will take some practice, but you will have lots of opportunities to do this through the problem sheets. You will also see many examples of mathematical arguments in lectures which (I hope!) will be written well.

Resources

- XX10190 book (Chapman and Epp) pages 154–158 (of Epp).
- Book: “How to study for a mathematics degree” by Lara Alcock, Chapter 8.

And finally, for those disappointed that there will be no talk on how to dance maths....

Beautiful Dance Moves

$\sin(x)$

$\cos(x)$

$\tan(x)$

$\cot(x)$

$|x|$

x

x^2

x^2+y^2

$\sqrt{x}$

$\sqrt{-x}$

$\frac{1}{x}$

crap.