$$\frac{F(x+h)-F(x)}{h} - f(x)$$

$$= \frac{1}{h}\left(\int_u^{x+h} f(t)\,dt - \int_u^{x} f(t)\,dt\right) - f(x)$$

$$= \frac{1}{h}\int_x^{x+h} f(t)\,dt \quad - f(x)$$

$$= \frac{1}{h}\int_x^{x+h} \big(f(t)-f(x)\big)\,dt \qquad \left(\text{since } \int_x^{x+h} dt = h\right)$$

since $|\text{integral}| \leq$ length $\times$ sup.

$$\left|\frac{F(x+h)-F(x)}{h} - f(x)\right| \leq \frac{1}{\cancel{h}}\cdot\cancel{h}\cdot \sup_{t\in[x,x+h]} |f(t)-f(x)|$$

$\longrightarrow 0$ as $h \longrightarrow 0$ by continuity of f.

$\square$