Better Contracting via Principal Agent Problems

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Current service contract model

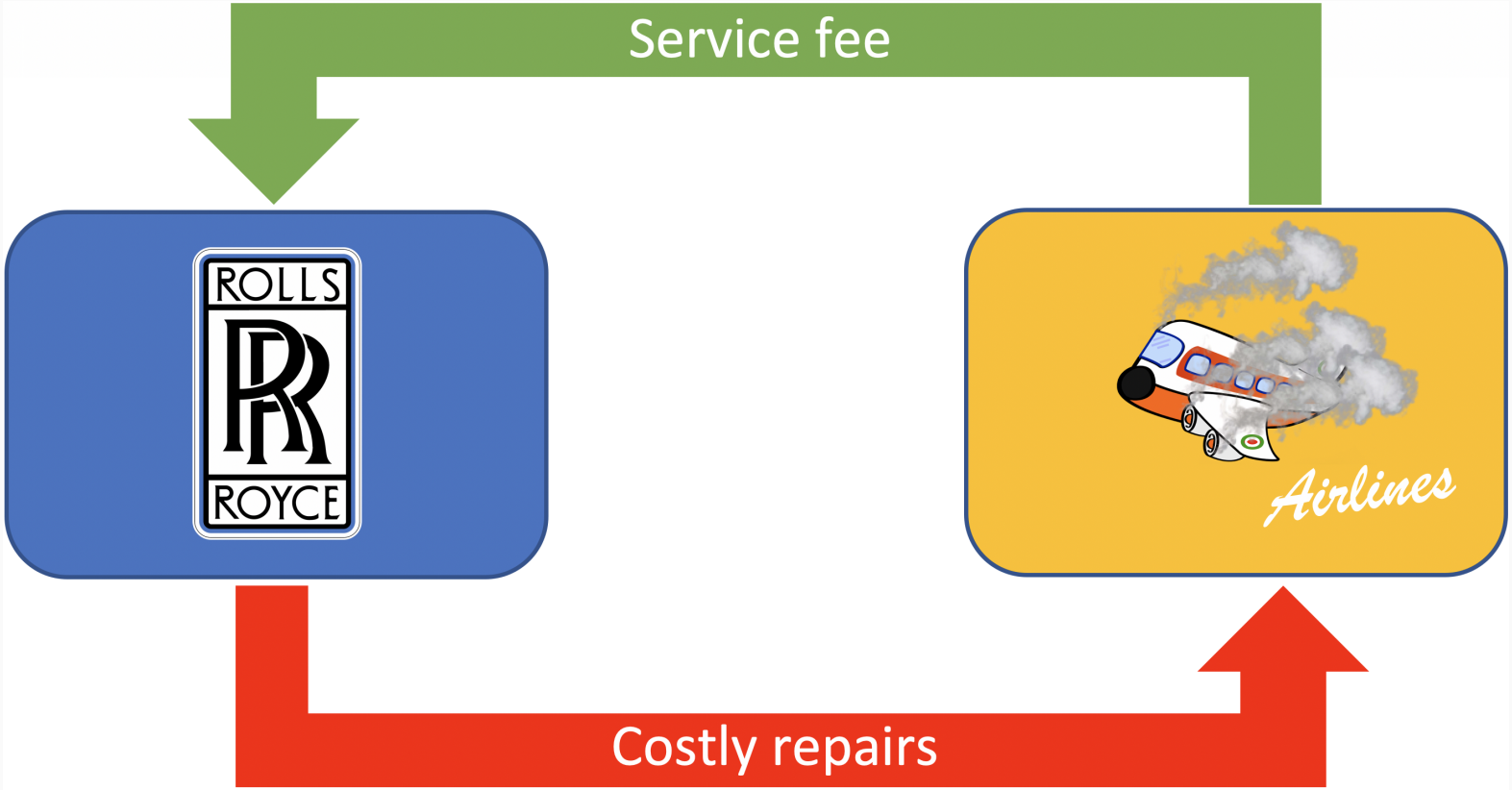

Figure 1: Airline pays a fixed service fee for Rolls-Royce to maintain engines

Principal Agent approach

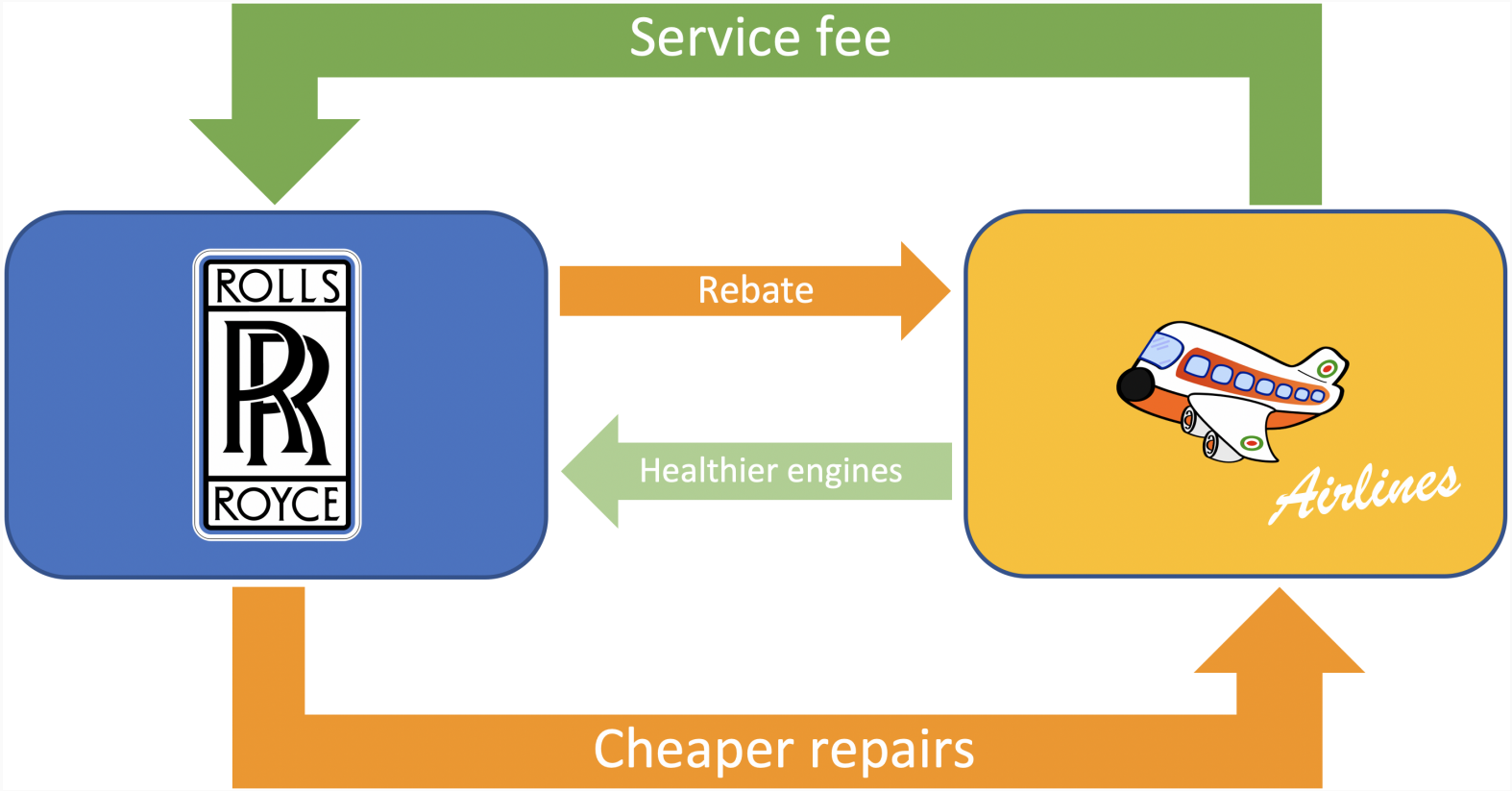

Figure 2: Rolls-Royce incentivises better behaviour through rebates

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- Deterioration of the engine depends on how it is used: many operational decisions by airlines will impact the general condition of the engine. Want to nudge the airline towards better behaviour.

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- Contract between a principal and an agent
- Principal wants to incentivise positive output from the agent
- The agent influences the output by his effort
- Agent dislikes effort and enjoys reward
- The principal sees only the output: a Brownian motion with drift that depends on the agent's effort
- **Goal**: Find the optimal contract that satisfies both parties

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$$dX_t = A_t dt + \sigma dZ_t$$

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- The agent exerts an effort, A, at a certain cost, $h(A)$, to themselves.
- The principal uses the observation of X to give the agent incentives to make effort

Principal Problem

Offer the agent a contract that maximizes the principal's profit

$$\mathbb{E}\left[r\int_0^\infty e^{-rt}\left(A_t - C_t\right)dt\right],$$

subject to providing the agent incentive enough to take up the contract,

$$\mathbb{E}\left[r\int_0^\infty e^{-rt}\left(u(C_t) - h(A_t)\right)dt\right] \geq \hat{W}.$$

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Principal problem

Assumption: the optimal contract can be written in terms of the agent's continuation value W_t

$$dW_t = r\left(W_t - u(c(W_t)) + h(a(W_t))\right)dt + rY(W_t)(dX_t - a(W_t)dt)$$

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Point: using W_t we can write the optimal contract by using just one variable.

Optimal contract

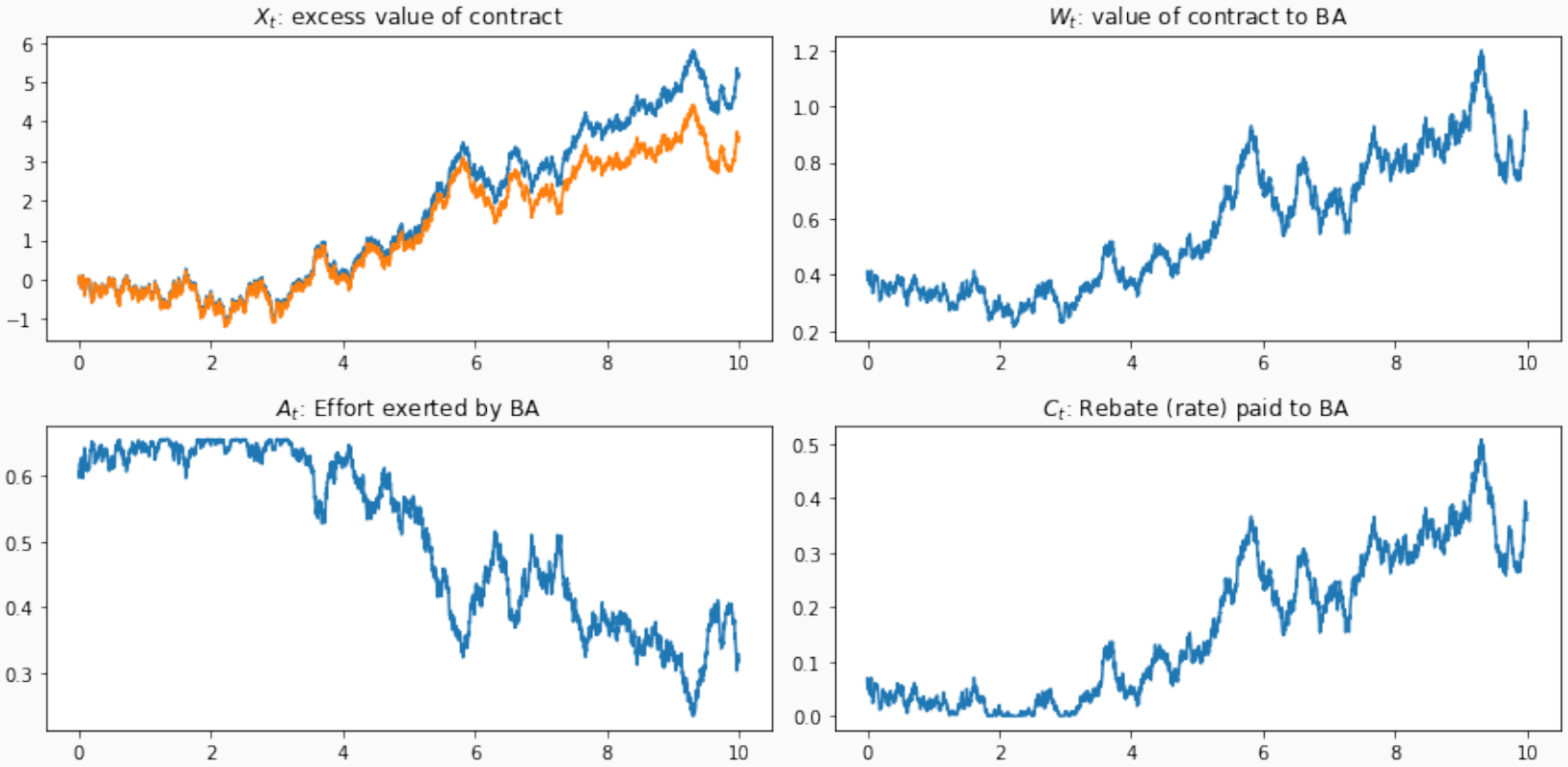

Figure 3: At the top left. Blue line: output process. Orange line: output minus cost

Other scenarios

The airline might decide to act suboptimally, i.e. not to follow the effort level suggested by Rolls-Royce.

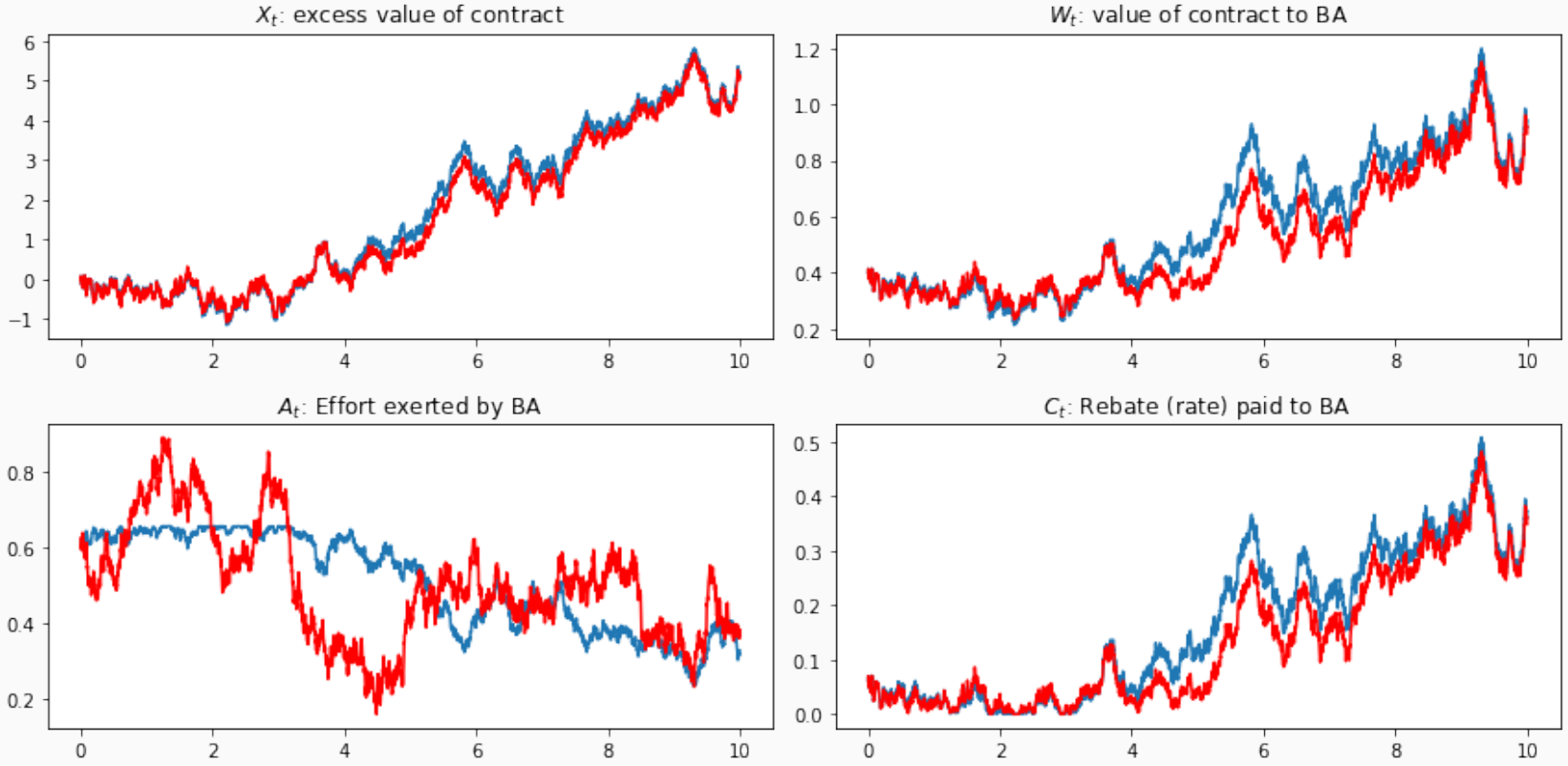

Figure 4: Blue line: optimal contract. Red line: suboptimal contract

The agent does nothing

Figure 5: Blue line: optimal contract. Red line: zero effort contract

Optimal effort and rebate

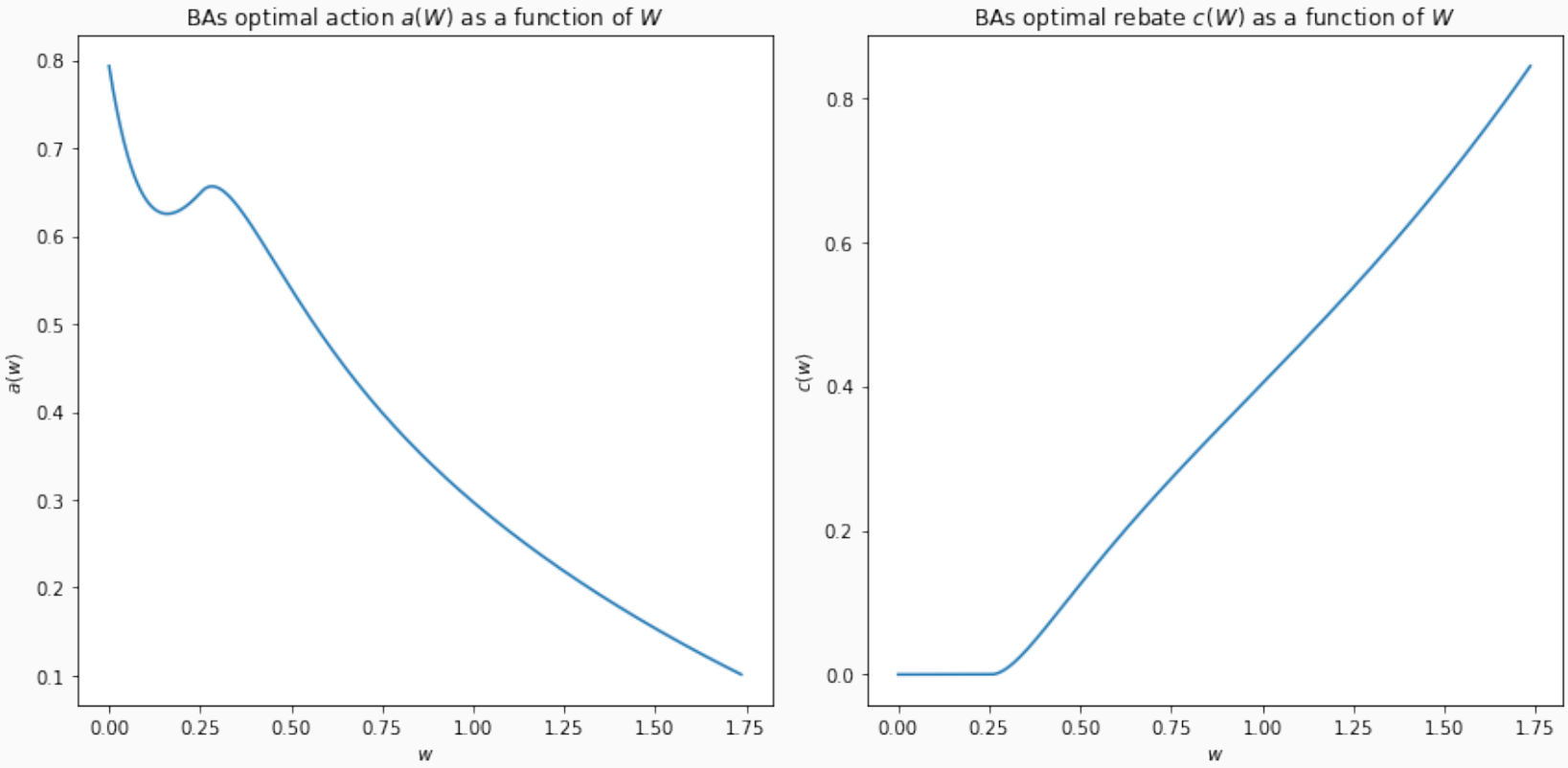

Figure 6: Left: optimal effort. Right: optimal consumption

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- Even if the agent acts suboptimally the contract tends to be beneficial to RR
- Agent is nudged towards better decisions which save money for RR and benefit the airline

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- What happens if the agent acts suboptimally? How is RRs profit affected?