

Condensation phenomena in stochastic systems

Bath 4th-6th July 2016

| | Monday 4 | Tuesday 5 | Wednesday 6 |
|---------------|------------------------|----------------------|--------------------|
| 9:00 – 9:30 | Welcome | Amin Coja-Oghlan | Alessandra Bianchi |
| 9:30 – 10:00 | Gunter Schütz | | |
| 10:00 – 10:30 | | coffee | coffee |
| 10:30 – 11:00 | coffee | Steffen Dereich | Bati Şengül |
| 11:00 – 11:30 | Claude Godrèche | | |
| 11:30 – 12:00 | | Volker Betz | Paul Chleboun |
| 12:00 – 12:30 | Mim Jatuviriyapornchai | | |
| 12:30 – 14:00 | lunch | lunch | lunch |
| 14:00 – 14:30 | Michail Loulakis | Juraj Szavitz-Nossan | |
| 14:30 – 15:00 | | | |
| 15:00 – 15:30 | Inés Armendáriz | Ginestra Bianconi | |
| 15:30 – 16:00 | | | |
| 16:00 – 16:30 | coffee | coffee | |
| 16:30 – 17:00 | Daniel Ueltschi | Frank Redig | |
| 17:00 – 17:30 | | | |
| | reception | dinner | |

Inés Armendáriz (University of Buenos Aires)

– Condensing zero-range process.

Zero-range processes with decreasing jump rates are known to exhibit condensation, where a finite fraction of all particles concentrates on a single lattice site. We will consider two such models. In a first result, we study a zero-range process on a one-dimensional lattice with periodic boundary conditions in the thermodynamic limit with fixed, super-critical particle density. We show that the process exhibits metastability with respect to the condensate location, and the suitably accelerated process of the rescaled location converges to a limiting Markov process on the unit torus. In a second result we consider a zero-range process in the totally connected graph with L vertices and a number $N \sim \log(L)$ of particles that determine a vanishing density. We show that for this model the fluid configurations, where each site contains at most one particle are metastable as well. Moreover, for the right choice of parameters, the best way to perform a transition between condensate configurations is to go through the fluid, while each condensate well is deeper than the fluid set. Joint work with S. Grosskinsky, M. Loulakis, A de Masi and E. Presutti.

Volker Betz (University of Darmstadt)

– **Spatial random permutations**

Spatial random permutations are made so that they favor permutations that map points to nearby points. The strength of this effect is encoded in a parameter $\alpha > 0$, where larger alpha means stronger bias toward short jumps. I will introduce some variants of the model, and explain the connections to the theory of Bose-Einstein condensation. Then I will present a few older results, as well as very recent progress made jointly with Lorenzo Taggi (TU Darmstadt) for the regime of large alpha. Finally, I will discuss two conjectures suggested by numerical simulation: in two dimensions, the model appears to exhibit a Kosterlitz-Thouless phase transition, and there are reasons to believe that in the phase of algebraic decay of correlations, long cycles are Schramm-Löwner curves, with parameter between 4 and 8 depending on alpha.

Alessandra Bianchi (University of Padova)

– **Dynamics of the condensate in the reversible inclusion process on a finite set.**

The inclusion process is a stochastic lattice gas where particles perform random walks subjected to mutual attraction, thus providing the natural bosonic counterpart of the well-studied exclusion process. Due to attractive interaction between the particles, the inclusion process can exhibit a condensation transition, where a finite fraction of all particles concentrates on a single site. In this talk we characterize the dynamics of the condensate for the reversible inclusion process on a finite set S , in the limit of total number of particles going to infinity. By potential theoretic techniques, we determine the time-scales associated to the transitions of the condensate from one site to another, and we show that the limiting dynamics of the condensate is a suitable continuous time random walk on S . Joint work with S. Dommers and C. Giardinà.

Ginestra Bianconi (Queen Mary University of London)

– **Quantum statistics and condensation transitions in networks and simplicial complexes.**

In this talk I will provide an overview of the Bose-Einstein condensation in simple complex networks providing both analytical and extensive numerical results describing the detailed stochastic dynamics of the Bianconi-Barabási model above and below the condensation transition. These results indicate that in the network at there is a subextensive number of nodes that acquire a finite fraction of the links. I will discuss an interesting variation of the model to describe weighted complex networks where there is a condensation transition in which few links acquire a finite fraction of all the weight of the network. Finally I will show that the Bianconi-Barabási model is a subcase of a wider model called Network Geometry with Flavor describing simplicial complexes whose statistical properties depend on quantum statistics and I will describe the phase transition occurring in this model.

Paul Chleboun (University of Warwick)

– **Monotonicity and condensation in homogeneous stochastic particle systems.**

Stochastic monotonicity is an extremely useful property of many interacting particle systems, combined with coupling methods it is a key tool in determining the set of invariant measures for certain process and obtaining hydrodynamic limits. We study stochastic particle systems that conserve the particle density and exhibit a condensation transition due to the particle interaction. On a fixed lattice the condensation transition occurs as the total number of particles diverges, the system separates into homogeneous phase distributed at the maximal invariant measure, and a condensate where a diverging number of particles concentrates on a single lattice site. We study spatially homogeneous processes which exhibit this phenomena, and have product stationary measures. Such processes include zero-range process and more general misanthrope processes. All known examples are not stochastically monotone. We are able to

show that in fact all such processes, with finite critical density, are necessarily non-monotone. Condensation can occur on finite lattices even if the critical density is not finite, we discuss an example that appears to be both condensing and monotone. This is joint work with Thomas Rafferty and Stefan Grosskinsky.

Amin Coja-Oghlan (University of Frankfurt)

– **Condensation in random factor graph models.**

Factor graph models have been used to describe a wide range of objects in physics, combinatorics, information theory and computer science. In particular, this includes ‘mean-field’ models of disordered systems such as glasses. Physics methods predict that in many of these objects a condensation phase transition occurs, which is similar in nature to the Kauzmann phase transition in glasses. In this talk I will explore this phenomenon from the abstract perspective of limits of discrete measures.

Steffen Dereich (University of Münster)

– **Condensation in preferential attachment models with fitness.**

In 2001 Bianconi and Barabási observed an intriguing condensation phenomenon in preferential attachment models with fitness. In the condensation phase, in the limit, there is a comparably small set of vertices (the condensate) that attracts a constant fraction of new links established by new vertices. We provide new results on the formation of the condensate in the case where the fitness distribution is bounded and has polynomial tails.

Watthanan (Mim) Jatuviriyapornchai (University of Warwick)

– **Coarsening dynamics in condensing stochastic particle systems.**

Zero-range processes and inclusion processes are well known to exhibit a condensation transition under certain conditions on the jump rates, and the dynamics of this transition continues to be a subject of current research interest. Starting from homogeneous initial conditions, the time evolution of the condensed phase exhibits an interesting coarsening phenomenon of mass transport between cluster sites. The single site dynamics of the processes form a birth death chain describing the coarsening behaviour. We introduce a size-biased version of the single site process, which provides an effective tool to analyze the dynamics of the condensed phase without finite size effects.

Claude Godrèche (CEA Saclay)

– **Dynamics of condensation for a prototypical zero-range process.**

We study the coarsening dynamics of a prototypical model of condensing ZRP, with hopping rate $u_k = 1 + b/k^\sigma$, ($0 < \sigma < 1$).

Michail Loulakis (National TU of Athens)

– **Large deviations for subexponential sums and condensing zero-range processes.**

The conditional distribution of i.i.d. random variables with thin tails subject to a large deviation of their sum is described by Gibbs’ conditioning principle. We will present a version of Gibbs’ conditioning principle for subexponential random variables and show how this result explains condensation in the Evans model for Zero Range Processes. Further, we will derive how the condensate emerges around the critical density. Joint work with Inés Armendáriz and Stefan Grosskinsky.

Frank Redig (TU Delft)

– **Wealth distribution models with symmetries.**

We introduce a recent model from econo-physics, the so-called immediate exchange model. This

is a model with two-agent exchanges where the wealth of each agent is uniformly split in two parts, top-parts are exchanged, and the new parts are added up again. We show that this has a discrete dual which in turn is related to the symmetric inclusion process via thermalization. This immediately yields a generalization to a two parameter family of models (where the splitting is Beta distributed) having similar properties. This then further leads to a quite general class of models where a discrete or continuous quantity is split, exchanged and added up again. We discuss how derive to self-dualities for such models, via additive structure of symmetries. As a consequence, one can characterize stationary measures and ergodic behaviour. Based on joint work with Federico Sau (Delft).

Gunter Schütz (Jülich Research Centre)

– **The zero-range process conditioned on an atypical current.**

We study the asymmetric zero-range process (ZRP) with L sites and open boundaries, conditioned to carry an atypical current. Using a generalized Doob h -transform we compute explicitly the transition rates of an effective process for which the conditioned dynamics are typical. This effective process is a zero-range process with renormalized hopping rates, which are space dependent even when the original rates are constant. This leads to non-trivial density profiles in the steady state of the conditioned dynamics, and, under generic conditions on the jump rates of the unconditioned ZRP, to an intriguing supercritical bulk region where condensates can grow. These results provide a microscopic perspective on macroscopic fluctuation theory (MFT) for the weakly asymmetric case: It turns out that the predictions of MFT remain valid in the non-rigorous limit of finite asymmetry. In addition, the microscopic results yield the correct scaling factor for the asymmetry that MFT cannot predict.

Batı Şengül (University of Bath)

– **Existence of a phase transition of the interchange process on the Hamming graph.**

The interchange process on a finite graph is obtained by placing a particle on each vertex of the graph, then at rate 1, selecting an edge uniformly at random and swapping the two particles at either end of this edge. We consider this process on the 2-dimensional Hamming graph. The main result is a phase transition: in the subcritical phase, all of the cycles of the process have length $O(\log n)$, whereas in the supercritical phase a positive density of vertices lie in cycles of length at least $n^{2-\varepsilon}$. This is joint work with Piotr Miłoś.

Juraj Szavitz-Nossan (University of Edinburgh)

– **Conditioned random walks and spatially-extended condensation.**

We study a reflected random walk (random walk that stays non-negative), conditioned to atypical area and local time (the number of returns to the origin). For the double exponential (Laplace) jump distribution, we show that the process exhibits condensation phenomenon by which a single random walk excursion (path between two successive returns to the origin) takes a macroscopic fraction of the total area. Our results can be mapped to stochastic lattice gases with pair-factorised steady states, providing an alternative explanation for the spatially-extended (interaction-driven) condensation from the standpoint of the large deviation theory.

Daniel Ueltschi (University of Warwick)

– **The random interchange process on the hypercube.**

We study random permutations of the vertices of the hypercube. The permutations are given by products of (uniform, independent) random transpositions on edges. We establish the existence of a phase transition accompanied by cycles of diverging lengths. (Joint work with R. Kotecký and P. Miłoś.)