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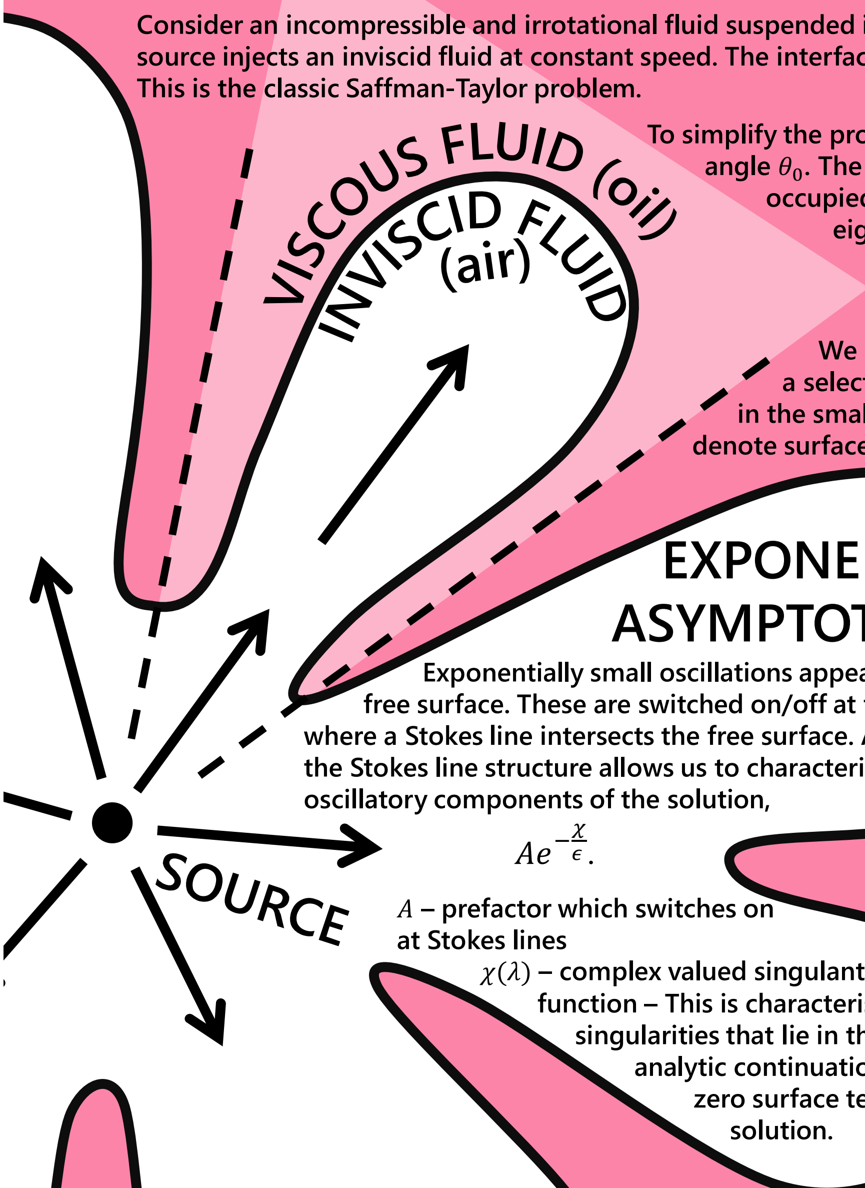
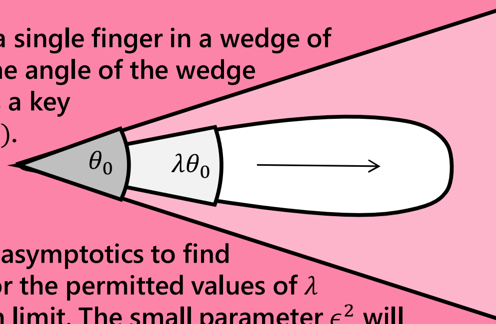
Exponential asymptotics for SAFFMAN-TAYLOR IN A WEDGE

Consider an incompressible and irrotational fluid suspended in a narrow space between two parallel glass plates. A source injects an inviscid fluid at constant speed. The interface develops fingers which split and spread over time. This is the classic Saffman-Taylor problem.



To simplify the problem, consider a single finger in a wedge of angle θ_0. The proportion of the angle of the wedge occupied by the finger is a key eigenvalue $\lambda \in (0,1)$.



We use exponential asymptotics to find a selection condition for the permitted values of λ in the small surface tension limit. The small parameter ϵ^2 will denote surface tension.

EXPONENTIAL ASYMPTOTICS

Exponentially small oscillations appear on the free surface. These are switched on/off at the places where a Stokes line intersects the free surface. Analysing the Stokes line structure allows us to characterise these oscillatory components of the solution,

$$Ae^{-\frac{\chi}{\epsilon}}.$$

A – prefactor which switches on at Stokes lines

$\chi(\lambda)$ – complex valued singulant function – This is characterised by singularities that lie in the analytic continuation of the zero surface tension solution.

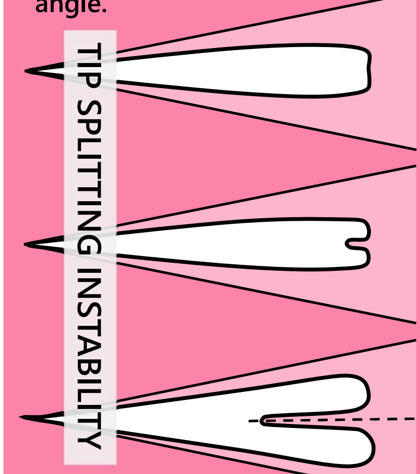
SELECTION CONDITION

At the corner of the wedge there are boundary conditions which require the interface to be oscillation-free. This means any oscillations present near the tip of the finger must be cancelled out by crossing Stokes lines before the end of the finger. Imposing this gives a selection condition which must be satisfied in order for ends of the finger to be oscillation-free, and thus for a solution to the model to exist:

$$\mathrm{F}(\lambda) = A_1 e^{-\frac{\chi_1}{\epsilon}} + A_2 e^{-\frac{\chi_2}{\epsilon}} + A_3 e^{-\frac{\chi_3}{\epsilon}} = 0$$

at ends of finger.

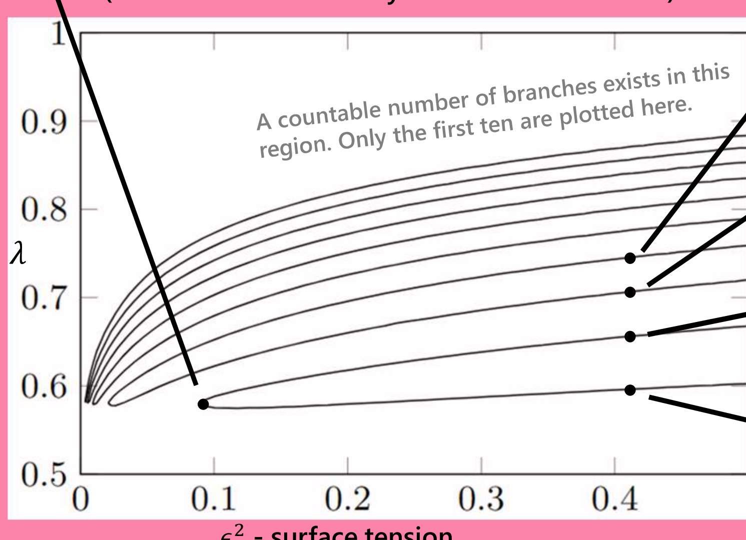
At this point solutions stop existing because a tip splitting instability occurs. The finger will split into multiple fingers where each can then be thought of as a single finger in a wedge of smaller angle.



There are now two fingers each with wedge angle $\theta_0 = 5^\circ$.

BIFURCATION DIAGRAM $\theta_0 = 10^\circ$

(values of λ which satisfy the selection criterion)



Exponentially small oscillations at tips of fingers.