Learned regularisation with plug-and-play for ptychography

Diamond/Ada Lovelace Centre ITT 21 Challenge

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Motivation: mathematical forward model

Recover $\boldsymbol{x}$ from the **forward model** with **noise** $\boldsymbol{\epsilon}$:

$$\boldsymbol{y}_l^2 = |\mathcal{F}(A_l \boldsymbol{x})|^2 + \boldsymbol{\epsilon}_l, \qquad l \in \{1, ..., L\}$$

where l indexes the positions of the probe, $\boldsymbol{A}$ is a linear operator, $\boldsymbol{x}$ is the ground truth, $\boldsymbol{y}_l$ the data collected at position l and $\boldsymbol{\epsilon}_l$ represents noise.

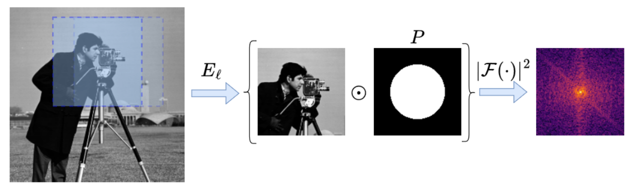

Figure 1: Illustration of Ptychography [Denker et al., 2024]

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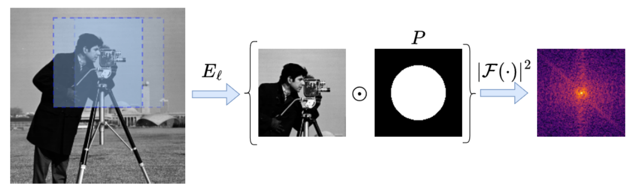

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- **Challenge:** Current algorithms take too long! :(

Ptychography imaging: variational form of inverse problem

The variational form of the inverse problem is given by

$$\operatorname*{argmin}_{\boldsymbol{x}} \left\{ \frac{1}{2} \sum_{l=1}^{L} \left\| y_l - \left| \mathcal{F}(A_l \boldsymbol{x}) \right| \right\|^2 + \lambda R(x) \right\}$$

where R is a chosen regulariser.

A **non-linear** problem of this form is solved iteratively through variable-splitting algorithms. Plug-and-play introduces a trained denoiser for optimising $R(x)$.

Half-quadratic splitting (HQS)

A standard variable-splitting algorithm with (x, z) for the problem:

$$\mathbf{z}_{k,\ell} = \arg\min_{\mathbf{z}} \|\mathbf{y}_\ell - |\mathcal{F}(\mathbf{z}_\ell)|\|^2 + \mu\|\mathbf{z}_\ell - A_\ell \mathbf{x}_{k-1}\|^2, \quad \ell = 1, ..., L \tag{6}$$

$$\mathbf{x}_k = \arg\min_{\mathbf{x}} \frac{\mu}{2\lambda} \sum_{\ell=1}^{L} \|A_\ell \mathbf{x} - \mathbf{z}_{k,\ell}\|^2 + R(\mathbf{x}). \tag{7}$$

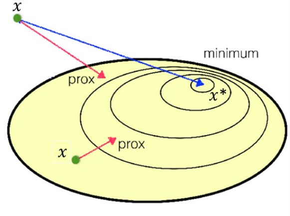
where this is solved iteratively between the data fidelity and regulariser terms [Denker et al., 2024].

Plug-and-Play adaptation (PnP)

- In the HQS algorithm

$$\mathbf{x}_k = \arg\min_{\mathbf{x}} \frac{\mu}{2\lambda}\|D\mathbf{x} - D\tilde{\mathbf{z}}_k\|^2 + R(\mathbf{x}) = D^{-1}\text{prox}_{\frac{\lambda}{\mu}R\circ D^{-1}}(D\tilde{\mathbf{z}}_k).$$

- This is the Gaussian maximum a-posteriori estimate
- Replace proximal operator with any Gaussian denoiser, $\mathcal{D}_\sigma$ [Venkatakrishnan et al., 2013]:

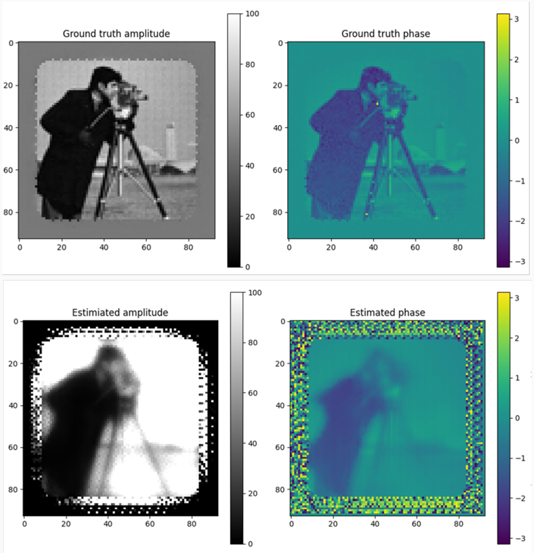


$$\textbf{Prox}_{\frac{\lambda}{\mu}R} \longrightarrow \mathcal{D}_\sigma$$

The denoisers considered by [Denker et al., 2024] are:

1. The handcrafted regulariser total variation (**TV**) - $R(\boldsymbol{x}) = ||\nabla \boldsymbol{x}||_1$
2. Pre-trained denoiser neural networks **DRUNet** and **WCRR**

Very preliminary results... (Unregularised)



Algorithm

Algorithm 1 Half-quadratic splitting algorithm for Ptychography

Input: Measurements $(\mathbf{y}_\ell)_{\ell=1}^{L}$, denoising parameters $(\tau_k)_{k\in\mathbb{N}}$, regularisation strength λ, initialization $\mathbf{x}_0$

for $k = 1, 2, \ldots$ **do**

$\mu_k = \lambda/\tau_k^2$

$c_k = \frac{n}{n+\mu_k}$

for $\ell = 1, \ldots, L$ **do**

$\hat{\mathbf{x}}_{k-1,\ell} = \mathcal{F}(A_\ell \mathbf{x}_{k-1})$

$\mathbf{z}_{k,\ell} = \mathcal{F}^{-1}(\hat{\mathbf{z}}_{k,\ell})$ with $\hat{\mathbf{z}}_{k,\ell} = (c_k \mathbf{y}_\ell + (1 - c_k)|\hat{\mathbf{x}}_{k-1,\ell}|) \exp(\mathrm{i}\,\mathrm{phase}(\hat{\mathbf{x}}_{k-1,\ell}))$

end for

$\tilde{\mathbf{z}}_k = \frac{\sum_{\ell=1}^{L} A_\ell^* \mathbf{z}_{k,\ell}}{\sum_{\ell=1}^{L} |A_\ell|^2}$

$\mathbf{x}_k = D^{-1}\mathrm{prox}_{\tau_k R \circ D^{-1}}(D\tilde{\mathbf{z}}_k)$

end for

- **Main problem:** sparse matrices and hocus pocus denoising parameters

Results from the paper

- Comparisons with **SequentialPIE** and **SimultaneousPIE**
- Tested on **natural images** dataset and a higher-res. brain phantom
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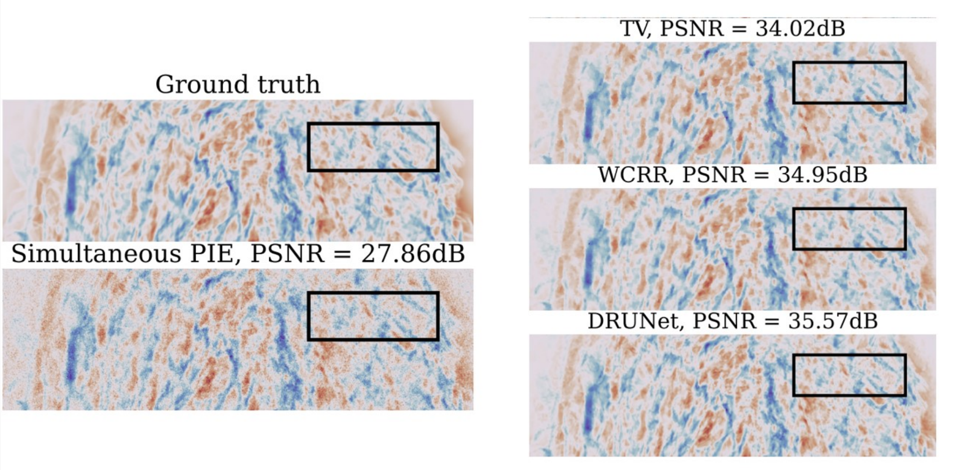
- **SeqPIE**, 225 positions: 21.91 ± 1.17 (PSNR mean $\pm$ standard dev.)
- **SimPIE**, 225 positions: 31.40 ± 0.99

- Best **PnP**, 49 positions: 27.03 ± 2.90
- Best **PnP**, 81 positions: 30.24 ± 2.07

Brain Phantom Example



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- Identified the limitations: **assumed** known probe, **unknown** computational cost of regularising

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Next steps:

- Timed experiments against SimPIE, SeqPIE
- Updating the probe and object together
- Combining with other algorithms
- Verify faster convergence with loss plots
- Possibly train the denoisers on **Ptychography images**


“That's all Folks!”

References

Denker, A., Hertrich, J., Kereta, Z., Cipiccia, S., Erin, E., and Arridge, S. (2024). **Plug-and-play half-quadratic splitting for ptychography.** *arXiv preprint arXiv:2412.02548*.

Venkatakrishnan, S. V., Bouman, C. A., and Wohlberg, B. (2013). **Plug-and-Play priors for model based reconstruction.** In *2013 IEEE Global Conference on Signal and Information Processing*, pages 945–948, Austin, TX, USA. IEEE.

Extra detail for HQS

- Full algorithm