The ant in the labyrinth. Day 2.

Manuel Cabezas. Universidad Católica de Chile

BUC-Chile Probability meeting

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- The branches $(\mathcal{B}_k)_{k\in\mathbb{N}}$ are i.i.d. and distributed as critical percolation trees.
- Those Branches act like traps for the ant.

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- The topological space $\mathfrak{T}_f := [0,1]/\sim$ is arc-connected and has no subspace which is homeomorphic to the circle

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- The CRT is the limit, as $K \to \infty$ of $\mathfrak{T}^{(K)}$.

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- Convergence of trees: Convergence of spanned subtrees + leaf tightness.

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- Existence: Krebs (CRT), Kigami, Croydon (General).

Theorem

Croydon 08

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Percolation

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  - van der Hofstadt, Jarai, rigorous definition of the IIC.

The Alexander-Orbach conjecture in high dimension

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- Kozma and Nachmias verified those conditions for the IIC.

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- Consider the random graph $\omega_n \subseteq \mathbb{Z}^d$ obtained as the image of $\mathcal{T}_n$.

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Scaling limit for the Ant in Critical Branching Random Walks

X^{ω_n}: Random walk on ω_n.

Theorem. Ben Arous, C., Fribergh

Let $d \geq 14$,

$$(n^{-\alpha} X^{\omega_n}_{n^{\beta} t})_{t \geq 0} \to (B^{ISE}_t)_{t \geq 0}$$

- What is B^{ISE}?
- What is α?, β?.