Solutions: Week 8

Solution 25 In this question the idea is to split A into A_1 (the first six machines, more expensive) and A_2 (the last two machines, cheaper). However, as explained in the previous question, this splitting will not work on its own because the algorithm will always try to reduce the value of the objective. Thus, we need to make use of pricing out. Normally the algorithm would ship as much as possible from A_2 and then A_1 would have to send the machines to the dump. However, if we cost $A_1 \to$ dump at an arbitrarily large amount, M, then it turns out to be preferable to shipping from A_1 first is more preferable. Thus our cost matrix now takes the form:

	X	Y	Z	Dump
A_1	50	60	30	M
A_2	40	50	20	0
B	60	40	20	0
C	40	70	30	0

We proceed with the North West Corner method

	5	4	3	4	$u_i \downarrow$
6	$5_{[50]}$	$1^{-\eta}_{[60]}$	$0^{+\eta}_{[30]}$	$0_{[M]}$	0
2	$0_{[40]}$	$2_{[50]}$	$0_{[20]}$	$0_{[0]}$	-10
5	$0_{[60]}$	$1^{+\eta}_{[40]}$	$3^{-\eta}_{[20]}$	$1_{[0]}$	-20
3	$0_{[40]}$	$0_{[70]}$	$0_{[30]}$	$3_{[0]}$	-10
$v_j \rightarrow$	50	60	40	20	$\cdot$

Take $\eta = 1$.

	5	4	3	4	$u_i \downarrow$
6	$5^{-\eta}_{[50]}$	$0_{[60]}$	$1^{+\eta}_{[30]}$	$0_{[M]}$	0
2	$0^{+\eta}_{[40]}$	$2^{-\eta}_{[50]}$	$0_{[20]}$	$0_{[0]}$	0
5	$0_{[60]}$	$2^{+\eta}_{[40]}$	$2^{-\eta}_{[20]}$	$1_{[0]}$	-10
3	$0_{[40]}$	$0_{[70]}$	$0_{[30]}$	$3_{[0]}$	-10
$v_j \rightarrow$	50	50	30	10	$\cdot$

Note that with $\eta = 2$ this perturbation takes out one zero and introduces two zeros and hence the next solution will be degenerate.

	5	4	3	4	$u_i \downarrow$
6	3 [50]	0 [60]	$1^{+\eta}$ [30]	0 [M]	0
2	2 [40]	0 [50]	0 [20]	0 [0]	-10
5	0 [60]	4 [40]	0 [20]	1 [0]	-10
3	0 [40]	0 [70]	0 [30]	3 [0]	-10
$v_j \rightarrow$	50	50	30	10	$z = 480$

Despite the fact that we have a degenerate solution, it turns out that it is still optimal as $x_{ij} \geq 0$, $u_i + v_j \leq c_{ij}$ and $x_{ij}(u_i + v_j - c_{ij}) = 0$.

Now compare the above solution with the solution to Exercise 23 and note that the final objective of the latter is lower. Therefore, the solution to Exercise 23 is an optimal solution to the adapted problem. The reason for this is that the solution to Exercise 23 still respects the formulation of transportation costs in this question and yet has a cheaper overall cost. One might worry that there is still a better solution in which one ships only 5 from A, however the optimality of the solution to Exercise 23 shows that this scenario would not be cheaper. Moreover, the situation in which a minimal number of 6 are shipped from A is covered by the solution above which, as noted, is more expensive than the solution to Exercise 23.

Solution 26 First formulate the cost matrix to take account of storage and use pricing out to forbid shipping backwards in time. Note that the problem is not in balance as supply exceeds demand and hence it is necessary to introduce a dump. This yields the following.

	qtr1 750	qtr2 900	qtr3 1000	qtr4 850	dump 400
qtr1: 1000	110	125	140	M	0
qtr2: 1700	M	100	114	128	0
qtr3: 800	M	M	120	136	0
qtr4: 400	M	M	M	130	0

Be careful with the first row not to forget that goods cannot be stored for more than two extra quaters.

Now assigning with the matrix method and proceeding as usual we solve as follows (as usual $u_1 = 0$ always).

	750	900	1000	850	400	$u_i \downarrow$
1000	750 $_{[110]}$	0 $_{[125]}$	0 $_{[140]}$	0 $_{[M]}$	250 $_{[0]}$	0
1700	0 $_{[M]}$	900 $_{[100]}$	$800^{-\eta}$ $_{[114]}$	$0^{+\eta}$ $_{[128]}$	0 $_{[0]}$	-6
800	0 $_{[M]}$	0 $_{[M]}$	$200^{+\eta}$ $_{[120]}$	$450^{-\eta}$ $_{[136]}$	150 $_{[0]}$	0
400	0 $_{[M]}$	0 $_{[M]}$	0 $_{[M]}$	400 $_{[130]}$	0 $_{[0]}$	-6
$v_j \rightarrow$	110	106	120	136	0	.

Take $\eta = 450$

	750	900	1000	850	400	$u_i \downarrow$
1000	750 $_{[110]}$	0 $_{[125]}$	0 $_{[140]}$	0 $_{[M]}$	250 $_{[0]}$	0
1700	0 $_{[M]}$	900 $_{[100]}$	350 $_{[114]}$	450 $_{[128]}$	0 $_{[0]}$	-6
800	0 $_{[M]}$	0 $_{[M]}$	650 $_{[120]}$	0 $_{[136]}$	150 $_{[0]}$	0
400	0 $_{[M]}$	0 $_{[M]}$	0 $_{[M]}$	400 $_{[130]}$	0 $_{[0]}$	-4
$v_j \rightarrow$	110	106	120	134	0	.

In the new table we see that $x_{ij}(u_i + v_j - c_{ij}) = 0$ and $x_{ij} \geq 0$ and $u_i + v_j \leq c_{ij}$ for all i, j thus the conditions of the complementary slackness theorem are met and the solution is optimal with $z = 400,000$.

Solution 27 The first table from the North-West Corner method is the following

	5	10	10
10	5 $_{[4]}$	5 $_{[2]}$	0 $_{[3]}$
5	0 $_{[6]}$	5 $_{[5]}$	0 $_{[8]}$
10	0 $_{[1]}$	0 $_{[4]}$	10 $_{[3]}$

This solution turns out to be quite problematic to start with. One could use the method of choosing a zero to be part of the basic solution. Another trick with this question is to try and introduce two non-zero cells into the basis at the cost of knocking out one non-zero element. This moves the solution from degenerate basic to non-degenerate basic. Here is the necessary perturbation.

	5	10	10
10	$5^{-\eta}$ $_{[4]}$	5 $_{[2]}$	$0^{+\eta}$ $_{[3]}$
5	0 $_{[6]}$	5 $_{[5]}$	0 $_{[8]}$
10	$0^{+\eta}$ $_{[1]}$	0 $_{[4]}$	$10^{-\eta}$ $_{[3]}$

which yields the new table

	5	10	10	$u_i \downarrow$
10	0 [4]	5 [2]	5 [3]	0
5	0 [6]	5 [5]	0 [8]	3
10	5 [1]	0 [4]	5 [3]	0
$v_j \rightarrow$	1	2	3	

One may check using the usual criteria for the Complementary Slackness Theorem that the last table provides us with the optimal solution with $z = 70$.