Solutions: Week 4

Solution 11 Add slack variables to the first and second constraints and then add further artificial variables to the first and third constraints. Then apply the simplex algorithm (for example) as follows.

	$\mathbf{x_1}$	x_2	x_3	x_4	x_5	x_6	x_7	x_8	
$\mathbf{x_7}$	**1**	-1	-1	-2	-1	0	1	0	2
x_6	1	1	0	1	0	1	0	0	8
x_8	1	2	-1	0	0	0	0	1	4
	-2	-1	2	2	1	0	0	0	-6

	x_1	$\mathbf{x_2}$	x_3	x_4	x_5	x_6	x_7	x_8	
x_1	1	-1	-1	-2	-1	0	-	0	2
x_6	0	2	1	3	1	1	-	0	6
$\mathbf{x_8}$	0	**3**	0	2	1	0	-	1	2
	0	-3	0	-2	-1	0	-	0	-2

Last tableau in phase I, introduce extra row for coefficients of original objective written in terms of non-basic variables.

	x_1	x_2	x_3	$\mathbf{x_4}$	x_5	x_6	x_7	x_8	
x_1	1	0	-1	-4/3	-2/3	0	-	-	8/3
x_6	0	0	1	5/3	1/3	1	-	-	14/3
$\mathbf{x_2}$	0	1	0	**2/3**	1/3	0	-	-	2/3
	0	0	0	0	0	0	-	-	0
	0	0	1	-8/3	-1/3	0	-	-	10/3

	x_1	x_2	x_3	$\mathbf{x_4}$	x_5	x_6	
x_1	1	2	-1	0	0	0	4
x_6	0	-5/2	1	0	-1/2	1	3
x_4	0	3/2	0	1	1/2	0	1
	0	4	1	0	1	0	6

Solution $x_1 = 4, x_2 = x_3 = 0, x_4 = 1$, $(x_5 = 0, x_6 = 3)$ and $z = 6$.

Solution 12 Let (x_1, x_2, x_3), (x_4, x_5, x_6) and (x_7, x_8, x_9) be the quantities in kilos of seeds, rasins and nuts of the chewy, crunchy and nutty snacks respectively.

Considering for example chewy snacks we require that

$$x_2 \geq 0.6(x_1 + x_2 + x_3) \text{ and } x_3 \leq 0.25(x_1 + x_2 + x_3).$$

Writing out all constraints in simplified form we have the problem of:

maximimising

$$x_1 + 0.5x_2 + 1.2x_3 + 0.6x_4 + 0.1x_5 + 0.8x_6 + 0.2x_7 - 0.3x_8 + 0.4x_9$$

subjet to:

$$\begin{aligned}
0.6x_1 - 0.4x_2 + 0.6x_3 &\leq 0 \\
0.25x_1 + 0.25x_2 - 0.75x_3 &\geq 0 \\
-0.4x_4 + 0.6x_5 + 0.6x_6 &\leq 0 \\
-0.8x_7 + 0.2x_8 + 0.2x_9 &\geq 0 \\
0.6x_7 + 0.6x_8 - 0.4x_9 &\leq 0 \\
x_1 + x_4 + x_7 &\leq 100 \\
x_2 + x_5 + x_8 &\leq 80 \\
x_3 + x_6 + x_9 &\leq 60 \\
x_1, \cdots x_9 &\geq 0
\end{aligned}$$

Solution 13 Make x_1 kilos of super and x_2 kilos of delux. The problem becomes to

maximise $22x_1 + 30x_2$

subject to

$$\begin{aligned}
0.5x_1 + 0.25x_2 &\leq 120 \\
0.5x_1 + 0.75x_2 &\leq 160 \\
x_1, x_2 &\geq 0.
\end{aligned}$$

Adding slack variables and solving via the simplex algorithm:

	x_1	$\mathbf{x_2}$	x_3	x_4	
x_3	0.5	0.25	1	0	120
$\mathbf{x_4}$	0.5	**0.75**	0	1	160
	-22	-30	0	0	0

	$\mathbf{x_1}$	x_2	x_3	x_4	
$\mathbf{x_3}$	**1/3**	0	1	-1/3	200/3
x_2	2/3	1	0	4/3	640/3
	-2	0	0	40	6400

	x_1	x_2	x_3	x_4	
x_1	1	0	3	-1	200
x_2	0	1	-2	2	80
	0	0	6	38	6800

Solution is to make 200kg of super and 80kg of delux with profit 68 dollars.

Solution 14 Use $x_1, \cdots, x_4$ of the four ingredients for the high octane and use $x_5, \cdots, x_8$ of the four ingredients for the low octane. Then vapour pressuer constraint for high octane is

$$5x_1 + 6.5x_2 + 4x_3 + 18x_4 = 7(x_1 + x_2 + x_3 + x_4)$$

or otherwise said

$$-2x_1 - 0.5x_2 - 3x_3 + 11x_4 = 0.$$

However, in this case, because total production of high octane is fixed to be 1300, it is simpler to treat this constraint as

$$5x_1 + 6.5x_2 + 4x_3 + 18x_4 = 7 \times 1300.$$

Using this simplification and noting that the objective function coefficients are the difference between revenue of product and cost of ingredient, problem becomes:

maximise

$$-0.7x_1 + 2.15x_2 + 2.7x_3 + 2.2x_4 + 0.3x_5 + 3.15x_6 + 3.7x_7 + 3.2x_8$$

subject to

$$
\begin{aligned}
5x_1 + 6.5x_2 + 4x_3 + 18x_4 &= 9100 \\
5x_5 + 6.5x_6 + 4x_7 + 18x_8 &= 5600 \\
108x_1 + 94x_2 + 87x_3 + 108x_4 &= 130,000 \\
98x_5 + 87x_6 + 80x_7 + 100x_8 &= 72,000 \\
x_1 + x_2 + x_3 + x_4 &= 1300 \\
x_5 + x_6 + x_7 + x_8 &= 800 \\
x_1 + x_5 &\leq 700 \\
x_2 + x_6 &\leq 600 \\
x_3 + x_7 &\leq 900 \\
x_4 + x_8 &\leq 500 \\
x_i &\geq 0 \text{ for } i = 1, \cdots, 8.
\end{aligned}
$$