Solutions: Week 1

Solution 1 (a). The problem is to maximise

(i) $z_1 = 15x_1 + 25x_2$ and

(ii) $z_2 = 15x_1 + 24x_2$ subject to

$$\begin{aligned} 5x_1 + 8x_2 &\leq 16,000 \\ 5x_1 + 4x_2 &\leq 14,000 \\ x_1 + 3x_2 &\leq 5,000 \\ x_1, x_2 &\geq 0. \end{aligned}$$

Note that the coefficients for z_2 are proportional to those of the coefficients in the first constraint! See Figure below for solution.

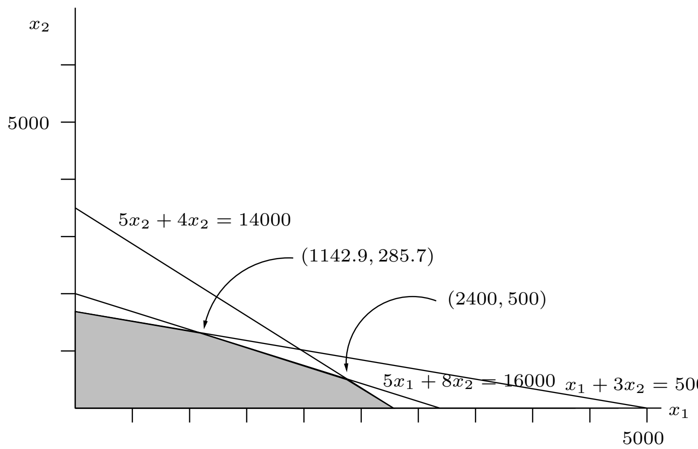

Figure 1: The grey area represents the feasible region. For solution to (i) the optimal solution occurs at $(x_1, x_2) = (1142.9, 1285.7)$ to one decimal place with optimal value $z_1 = 49258.7$. For solution to (ii) there are infinite optimal solutions which corresponds to the line segment between the solution to (i) and $(2400, 500)$ all with optimal value $z_2 = 48,000$.

(b) Let x_3 be the number of units of product 3 per week (casting at Betta) and x_4 be the number with casting subcontracted. Linear programming problem becomes to

Maximise $z = 15x_1 + 20x_2 + 18x_3 + 13x_4$

subject to

$$\begin{aligned} 5x_1 + 8x_2 + 8x_3 &\leq 16,000 \\ 5x_1 + 4x_2 + 3x_3 + 3x_4 &\leq 14,000 \\ x_1 + 3x_2 + 2x_3 + 2x_4 &\leq 5,000 \\ x_1, x_2, x_3, x_4 &\geq 0. \end{aligned}$$

Solution 2

(a)

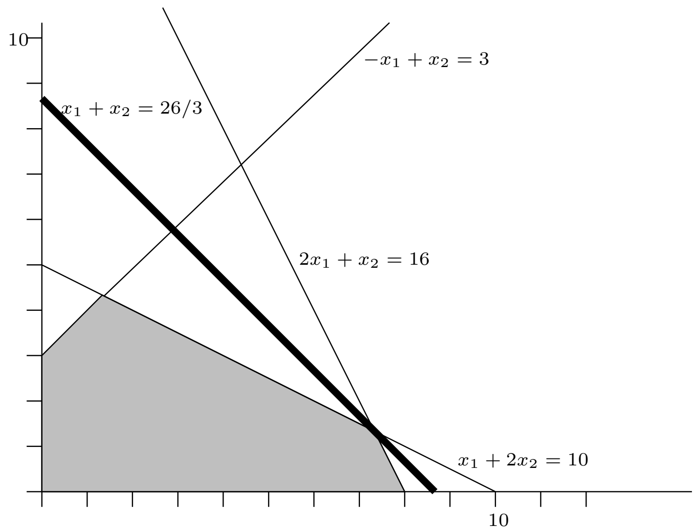

Figure 2: Grey area represents feasible region. The bold line is the line $z = x_1 + x_2$ such that z takes the maximal value for which there is an intersection with the feasible region. Optimal solution thus occurs when $(x_2, x_2) = (22/3, 4/3)$ and $z = 26/3$.

(b)&(c)

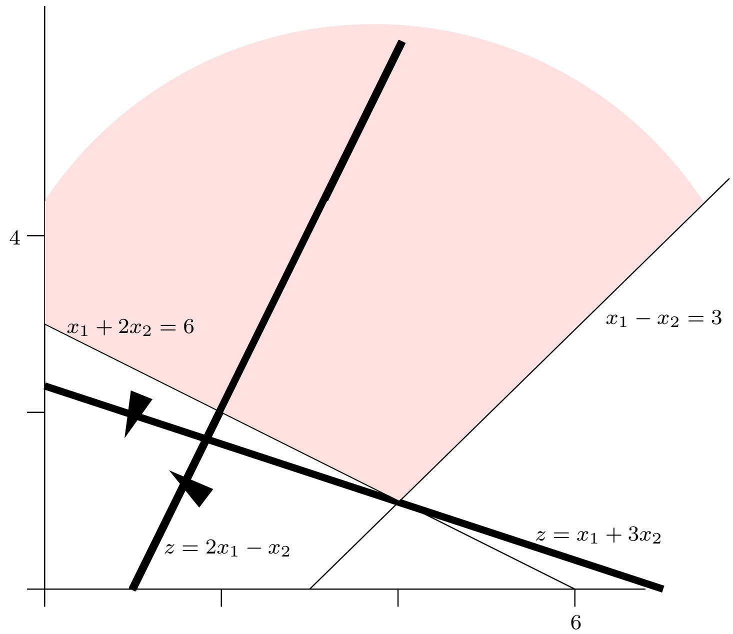

Figure 3: The grey area represents the feasible region. For (b) optimal solution occurs at $(x_1, x_2) = (4, 1)$ with optimal value $z = 7$. For (c) there is no finite optimal solution as one sees that moving the objective line to the left, it never exits the feasible region.

Solution 3 It is easy to see that teh optimal solution involves making no more of A, B and C than can be assembled into product. Make x_1 of product 1, x_2 of product 2. Hence one makes x_1 of A, $2x_2$ of B and $(x_1 + x_2)$ of C. The problem becomes to

maximise $z = 30x_1 + 45x_2$

subject to

$$\begin{aligned} 1.5x_1 + 2x_2 &\leq 300 \\ 0.25x_1 + 0.25x_2 &\leq 45 \\ x_1 &\leq 130 \\ x_2 &\leq 100 \\ x_1, x_2 &\geq 0. \end{aligned}$$

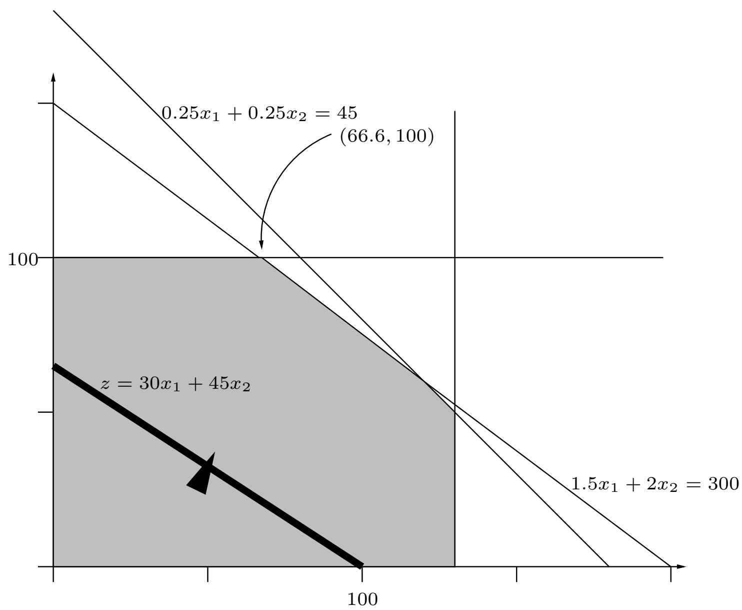

Figure 4: The grey area represents the feasible region. The optimal solution occurs at $(x_1, x_2) = (66\frac{2}{3}, 100)$ with optimal value $z = 6500$.

Solution 4 Say x_1 came on duty at 0.00, x_2 at 4.00 etc. The problem becomes to minimise $z = \sum_{i=1}^{6} x_i$ subject to

$$\begin{aligned} x_1 + x_6 &\geq 15 \\ x_1 + x_2 &\geq 35 \\ x_2 + x_3 &\geq 65 \\ x_3 + x_4 &\geq 80 \\ x_4 + x_5 &\geq 40 \\ x_5 + x_6 &\geq 25 \\ x_1, ..., x_6 &\geq 0. \end{aligned}$$