University of Bath

DEPARTMENT OF MATHEMATICAL SCIENCES EXAMINATION

MA30087: OPTIMISATION METHODS OF OPERATIONAL RESEARCH

Wednesday 28th January 2004, 16.30–18.30

Candidates may use university-supplied calculators.

Full marks will be given for correct answers to THREE questions.
Only the best three answers will contribute towards the assessment.

Examiners will attach importance to the number of well-answered questions.

1. (a) Prove that if the canonical form linear programming problem has an optimal solution, there is a basic feasible solution which is optimal.

   (b) Consider the following programming problem. Show how it may be turned into a linear programming problem by substituting for x_1 by a function of two new variables, whose product is required to be zero. Explain why this is automatically satisfied at a basic solution.

$$\text{Maximise } c_1|x_1| + \textstyle\sum_{j=2}^{n} c_j x_j$$
$$\text{subject to } \textstyle\sum_{j=1}^{n} a_{ij} x_j = b_i \quad i = 1, ..., m.$$
$$x_j \geq 0,\ j = 2, ..., n,\quad x_1 \text{ not sign restricted.}$$

2. A manufacturer produces 3 products, each of which requires machining and hand finishing. The number of minutes of each process required for one unit of product is shown in the table below.

Required number of minutes per unit

	Machining	Hand finishing
Product 1	3	1
Product 2	2	2
Product 3	2	3

The unit profits on products 1, 2 and 3 are £8, £3 and £9 respectively. Each week the factory has available 2,000 minutes of machining time and 1,500 minutes of hand finishing time. The manufacturer has a contract which requires the combined number of units of products 1 and 2 to total at least 500.

(i) Formulate the problem of maximising weekly profit on these products as a linear programme and use the Two Phase Method of the Simplex algorithm to obtain a solution. This will probably take two iterations.

(ii) Write down the dual to the above problem (take care over signs) and demonstrate how, having solved the primal problem, you have also solved the dual. Use this to verify, by appealing to the Duality theorem, that your solutions are indeed optimal. Explain briefly how you would interpret the real world meaning of the dual variables.

(iii) Use sensitivity analysis to determine the effect of increasing the required combined number of products 1 and 2 to at least 600?

3. In a land levelling project, soil must be taken from four zones and taken to five other zones. The amounts (in '000 tons) to be removed from zones A,B,C and D are 25, 50, 30 and 40 respectively whilst the amounts needed at zones K,L,M,N and P are 30, 60, 20, 15 and 20. The average distances travelled (metres) in moving soil between the various zones are shown below.

	K	L	M	N	P
A	120	70	200	60	100
B	80	130	100	130	140
C	190	130	70	110	160
D	130	90	100	180	150

Formulate and solve the problem of moving the soil as required whilst minimising the effort required in terms of total tons times metres travelled.

Show how to modify the problem to allow for the following complications (separately). Obtain the new solution in each case

(a) At least 20,000 tons must be taken from A to K

(b) At most 20,000 tons can be taken from D to L

4. (a) Consider a capacitated network (N, γ) having a single source $s \in N$ and a single sink $d \in N$. Any flow f in the network satisfies the usual conditions

$$f(i,j) \leq \gamma(i,j) \quad \forall i,j \in N$$
$$f(i,j) = -f(j,i) \quad \forall i,j \in N$$
$$\textstyle\sum_{j \in N} f(i,j) = \sum_{j \in N} f(j,i) \quad \forall i \in N \setminus s, d$$

Define a **cut**, (A, B) and show that the total flow from the source equals the flow across any cut (A, B) i.e.

$$\textstyle\sum_{j \in N} f(s,j) = \sum_{i \in A} \sum_{j \in B} f(i,j)$$

(b) State without proof the maximal flow, minimal cut theorem

(c) Draw the 6 node network whose capacities are given in the following matrix and find the maximal flow. State the minimal cut. and verify that the theorem in (b) holds.

$$\begin{pmatrix} 0 & 10 & 14 & 0 & 0 & 0 \\ 0 & 0 & 2 & 12 & 0 & 0 \\ 0 & 2 & 0 & 0 & 4 & 6 \\ 0 & 12 & 0 & 0 & 6 & 6 \\ 0 & 0 & 4 & 6 & 0 & 10 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

MEB