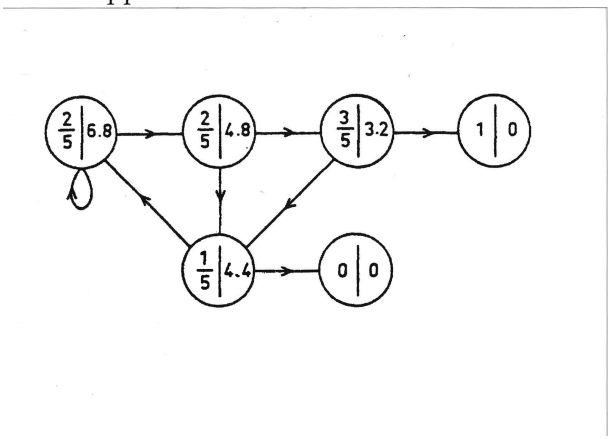
Exercise 1. Look at the graph of the game of Lecture 2. A coin with faces H and T is tossed repeatedly, until the either the pattern HHH or the pattern HTH has appeared. The progress made towards these patterns is represented by the Nodes of the graph. For example, if the tosses are $HHTTHHH$, we traverse the following Nodes:
Start $\to H \to HH \to HT \to$ Start $\to H \to HH \to HHH$.
(a) Find the path corresponding to the sequence of tosses $THTTTHTH$.
(b) What was the sequence of tosses, if the path traversed is:
Start $\to H \to HT \to$ Start $\to H \to HH \to HHH$?

Exercise 2. We look at the game on the accompanying graph.
(a) Begin the game at the "Start" Node, with no sticks anywehere. Play until you have 2 stick at "Start", some sticks at HHH and HTH, and no sticks anywhere else. How many sticks arrived in HHH and HTH, respectively?
(b) What is the probability that the pattern HHH appears before the pattern HTH?
(c) Begin the game with 2 sticks at Node HT and no sticks anywhere else. Perform a number of steps, adding more sticks at Node HT, when needed. Can you explain why, after the first step, there will always be at least one stick at the Node "Start"?
(d) Place now 1 stick at each of "Start", H, and HH (called *preloading*). Play the game beginning at the Node HT, until you arrive back at the preloaded configuration, that is, you have one stick at each of "Start", H, HH, and some sticks at HHH and HTH. How many sticks arrived in HHH and HTH, respectively?
(e) What is the probability that the pattern HHH appears before the pattern HTH, if you start with the sequence HT?
(f) Carry out (d)–(e) for the games beginning at some of the other Nodes. Always preload the other Nodes first. If you have time, compute the average number of moves as well.

Continued on page 2

Answers:

Exercise 1(a): Start $\to$ Start $\to H \to HT \to$ Start $\to$ Start $\to H \to HT \to HTH$.
Exercise 1(b): $HTTHHH$.
Exercise 2(a): After adding 7 sticks, 2 arrive in HHH, 3 arrive in HTH, and 2 arrive back at "Start".
Exercise 2(b): 2/5.
Exercise 2(c): Whenever sticks move at Node "Start", one of them stays there, so the number of saticks there can never go back to 0.
Exercise 2(d): After adding 5 sticks, 1 arrives at HHH, 4 arrive at HTH, and each of "Start", H and HH has 1 again.
Exercise 2(e): 1/5.
Exercise 2(f): The answers are shown below. The number on the left is the probability that HHH appears before HTH. The number on the right is the average number of steps taken.



Further reading: Today's activity was based on the following two papers (written by a mathematics teacher):
[1] Arthur Engel (1975): The probabilistic abacus. *Educational Studies in Mathematics*, **6**(1), 1–22.
[2] Arthur Engel (1976): Why does the probabilistic abacus work? *Educational Studies in Mathematics*, **7**(1), 59–69.