Exercise 1.
(a) Repeat the mouse game of the first lecture with 4 mice starting at the node called "Start". Keep a count of how many steps you make. How many mice out of the 4 reach the "Cheese" node? How many steps the mice take altogether?
(b) For a single mouse moving at random, what are the probabilities of reaching "Cheese" and "Cat", respectively?
(c) What is the average number of moves for a single mouse?
(d) Carry out (a)–(c) starting from some other nodes.

Exercise 2. The probability of a possible path for the mouse is obtained by mutiplying the probabilities of each step along the path. For example, the probability that the mouse follows the path $0 \to 2 \to 3 \to$ Cheese is $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$. The probability of following the path $0 \to 1 \to 3 \to 4 \to$ Cheese is $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$.
(a) There are 4 possible paths for the mouse to reach "Cat" from "Start". Find all 4 of these paths, and their probabilities.
(b) Add the probabiltiies of all the paths leading from "Start" to "Cat". Does this match the result of Exercise 1(b)?
(c) Carry out (a)–(b) starting from some other nodes, and compare with the results you found in Exercise 1(d).
(d)*[If time permits]* The average number of moves can be found as follows. For all paths (regardless of whether it ends at "Cheese" or "Cat"), multiply the probability of the path by the number of steps and add. Check that this matches the answers in Exercise 1.

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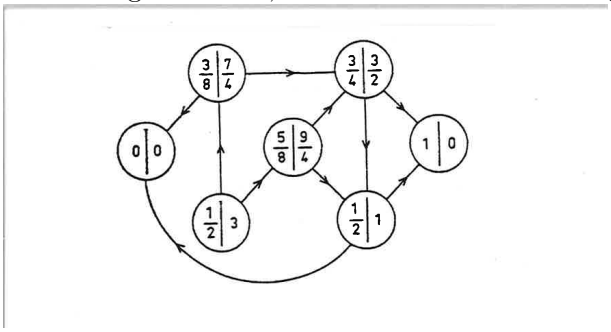
Answers:

Exercise 1(a): (a) 2 mice reach "Cheese" and 2 mice reach "Cat", and the 4 mice take 12 steps altogether.
Exercise 1(b): The probabilities are 2/4 and 2/4, respectively.
Exercise 1(c): The number of moves per mouse was $12/4 = 3$, so a single mouse will make 3 moves on average.
Exercise 1(d): The figure below shows the answers. The number on the left is the probability of reaching "Cheese", and the number on the right is the average number of steps taken.



Exercise 2(a): The 4 paths and their probabilities are:
$0 \to 1 \to$ Cat has probability 1/4;
$0 \to 1 \to 3 \to 4 \to$ Cat has probability 1/16;
$0 \to 2 \to 4 \to$ Cat has probability 1/8;
$0 \to 2 \to 3 \to 4 \to$ Cat has probability 1/16.
Exercise 2(b): We get $1/4 + 1/16 + 1/8 + 1/16 = 1/2$, and this matches the result of Exercise 1(b).
Exercise 2(c): Starting at Node 1:
$1 \to$ Cat has probability 1/2;
$1 \to 3 \to 4 \to$ Cat has probability 1/8;
The probability of reaching "Cat" is $1/2 + 1/8 = 5/8$.
Starting at Node 2:
$2 \to 4 \to$ Cat has probability 1/4;
$2 \to 3 \to 4 \to$ Cat has probability 1/8;
The probability of reaching "Cat" is $1/4 + 1/8 = 3/8$.
Starting at Node 3:
$3 \to 4 \to$ Cat has probability 1/4;
The probability of reaching "Cat" is 1/4.
Starting at Node 4:
$4 \to$ Cat has probability 1/2; The probability of reaching "Cat" is 1/2.
Exercise 2(d): For example, starting at Node 3:
$1 \times (1/2) + 2 \times (1/4) + 2 \times (1/4) = 3/2$.