

DECOUPLING NORMALISATION MECHANISMS
WITH AN EYE TOWARDS CONCURRENCY

ALESSIO GUGLIELMI (BATH)

TORINO 14/2/17

LYON 18/4/17 - PALAISEAU 21/4/17 (with additions)

DECOUPLING NORMALISATION MECHANISMS WITH AN EYE TOWARDS CONCURRENCY

ALESSIO GUGLIELMI (BATH)

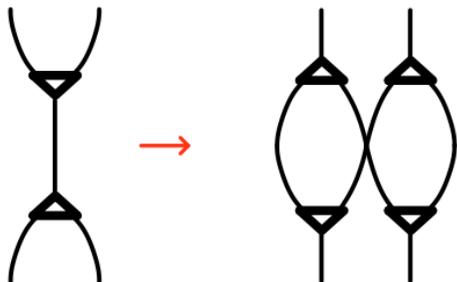
TORINO 14/2/17

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Designing proof systems for concurrency starting from the desired normalisation theory

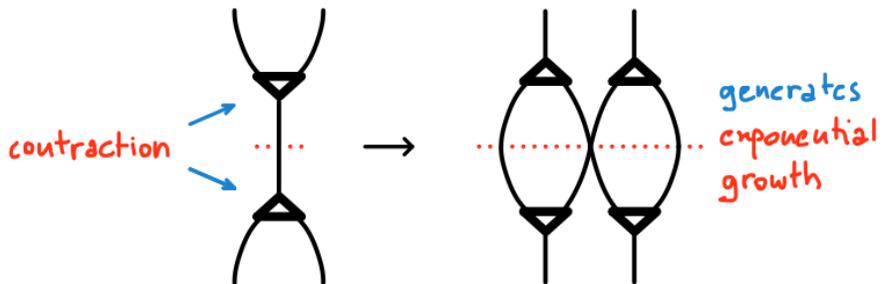
GRAPHICAL NORMALISATION MECHANISMS

Computations, for example



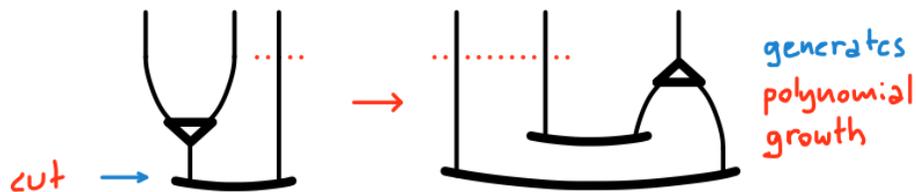
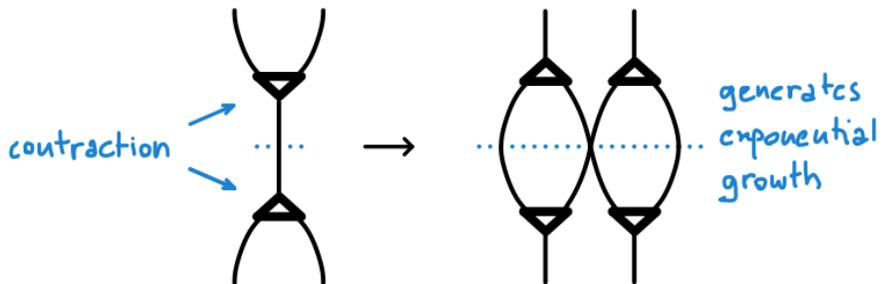
GRAPHICAL NORMALISATION MECHANISMS

Computations that increase complexity



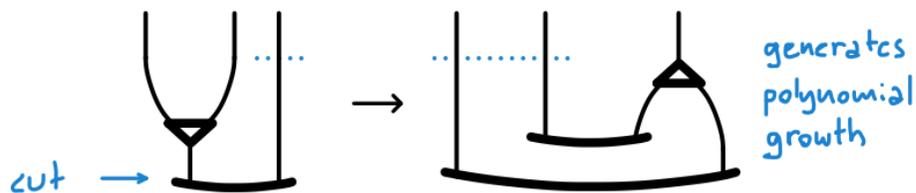
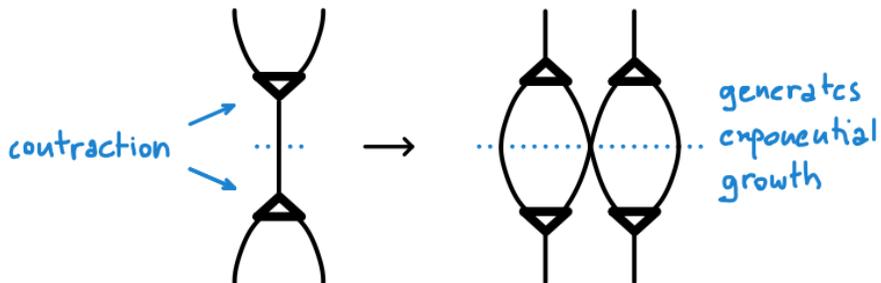
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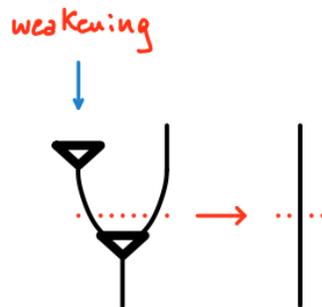


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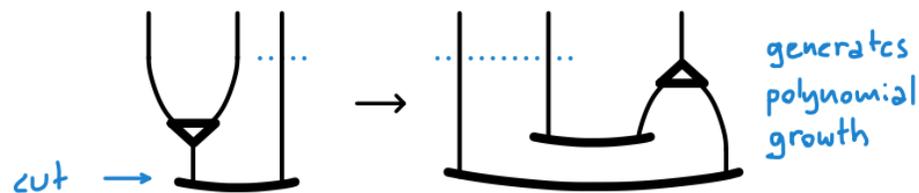
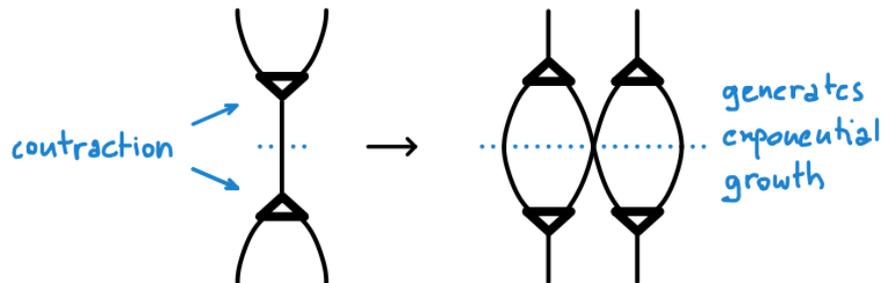


Computations that decrease complexity

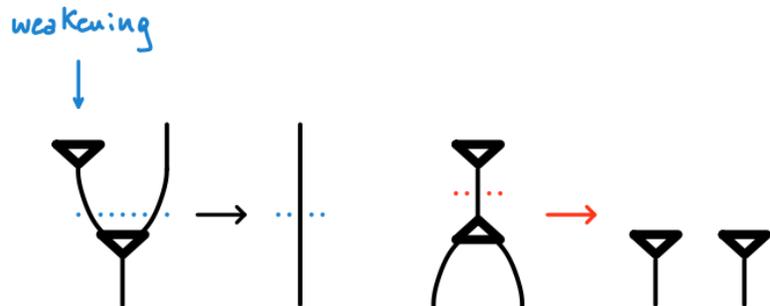


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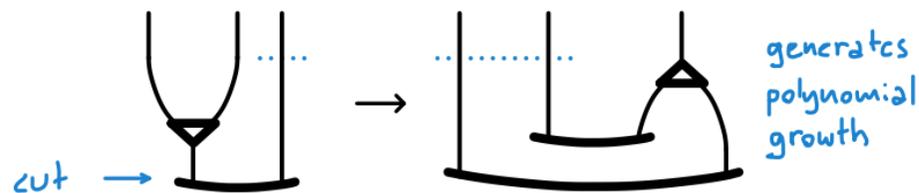
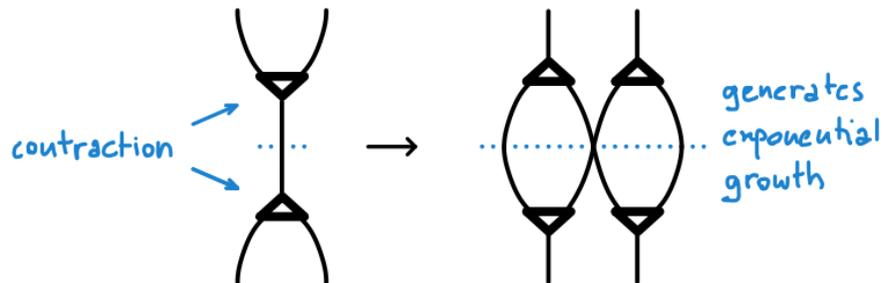


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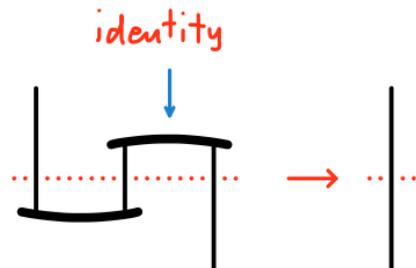
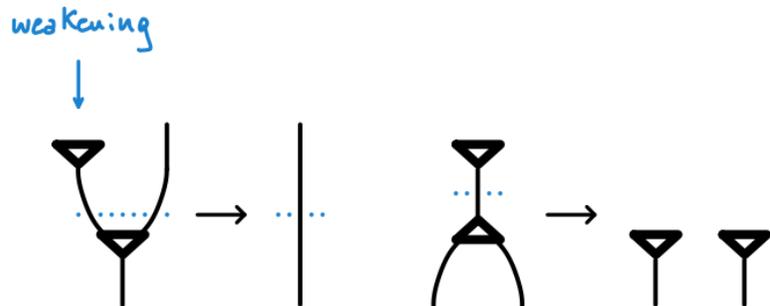


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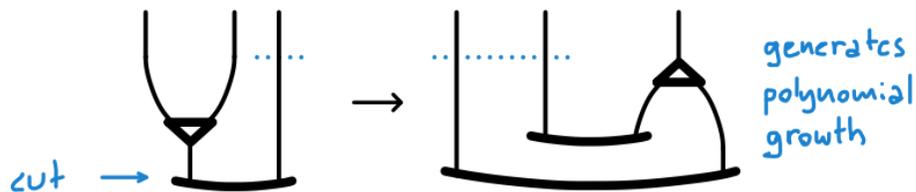
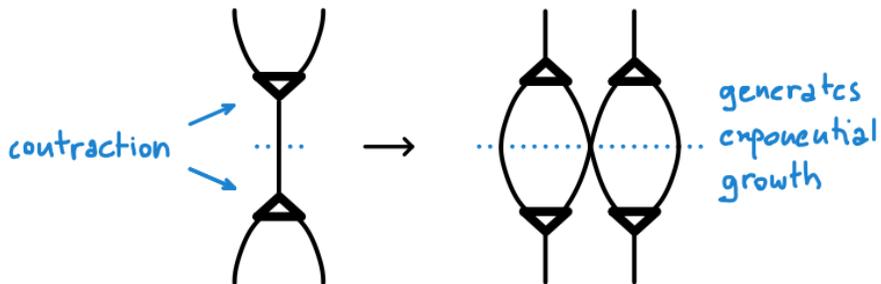


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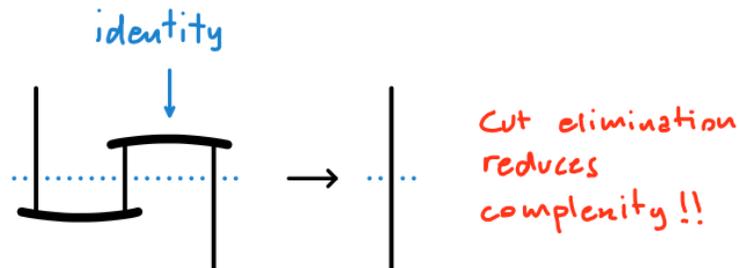
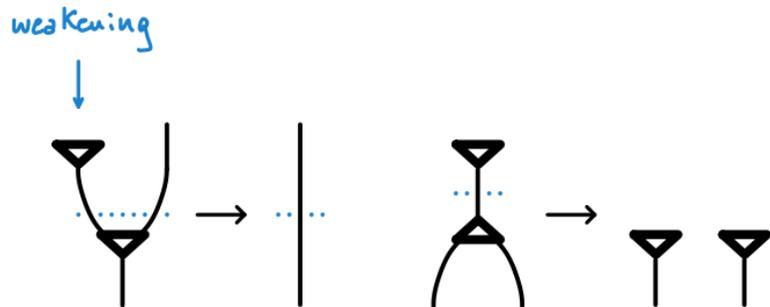


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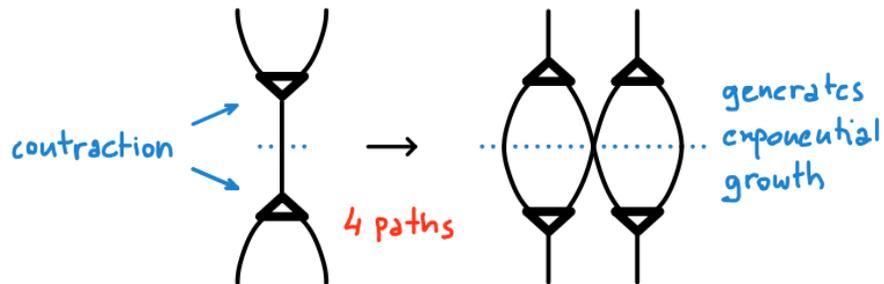


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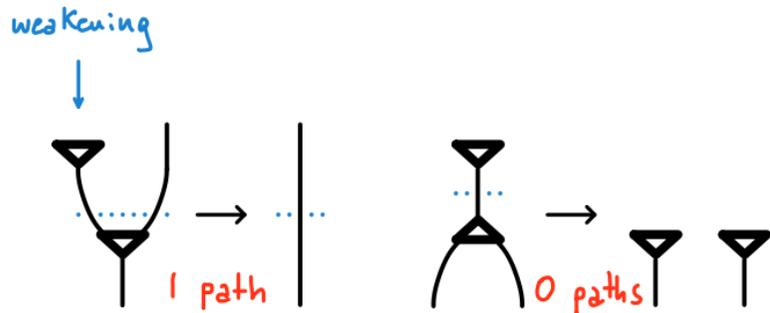


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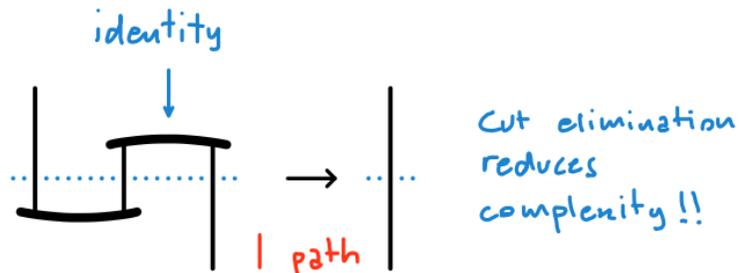
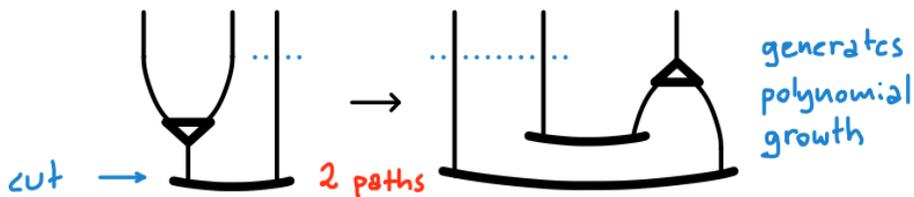
Computations that increase complexity



Computations that decrease complexity



Invariant number of paths



GRAPHICAL NORMALISATION MECHANISMS



Same as Lafont interaction combinators

Lamping showing graphs
cellular automata

Therefore they can be a universal model for concurrent computation

GRAPHICAL NORMALISATION MECHANISMS

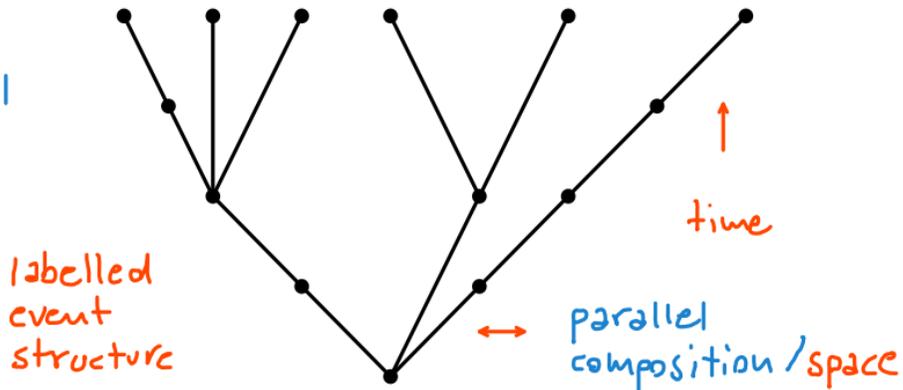


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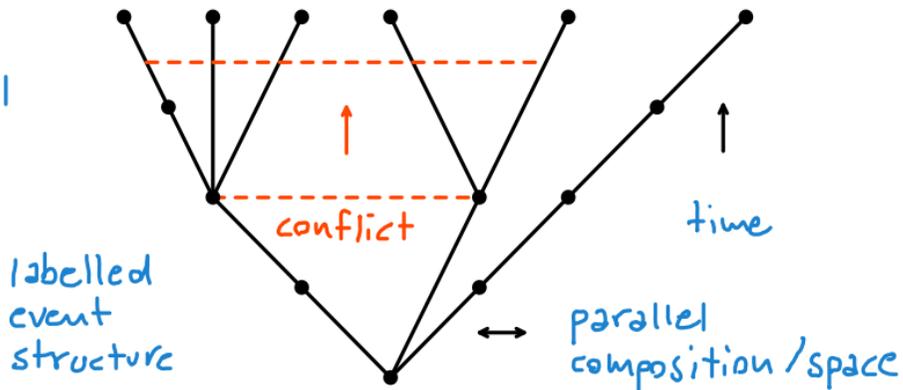


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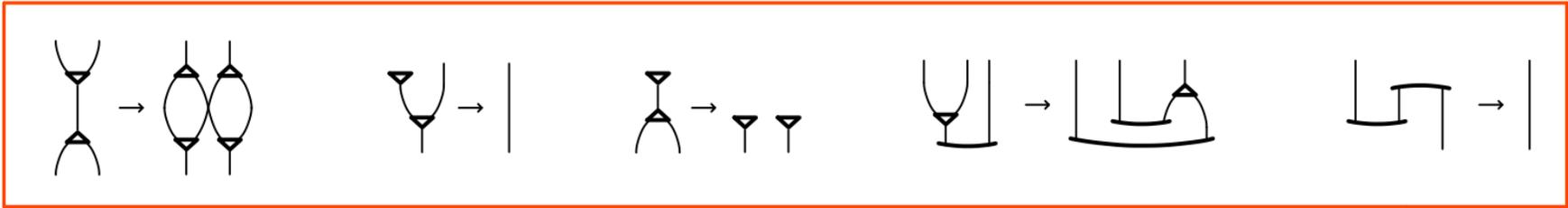
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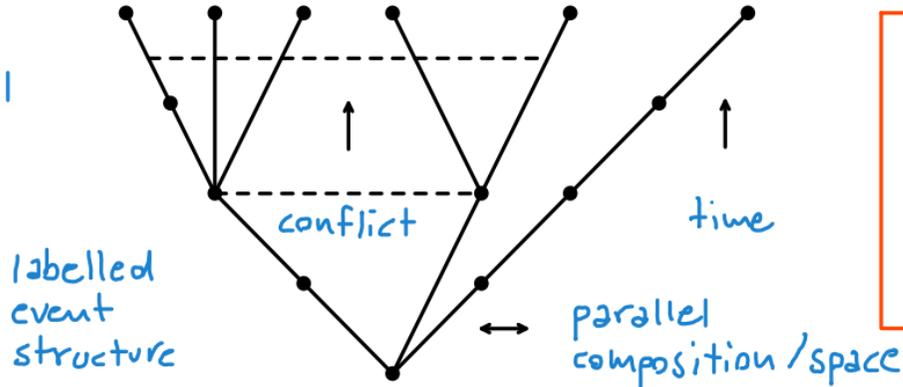
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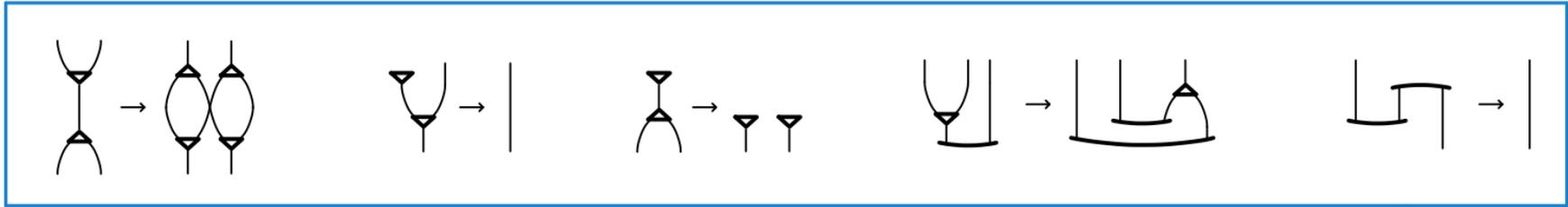
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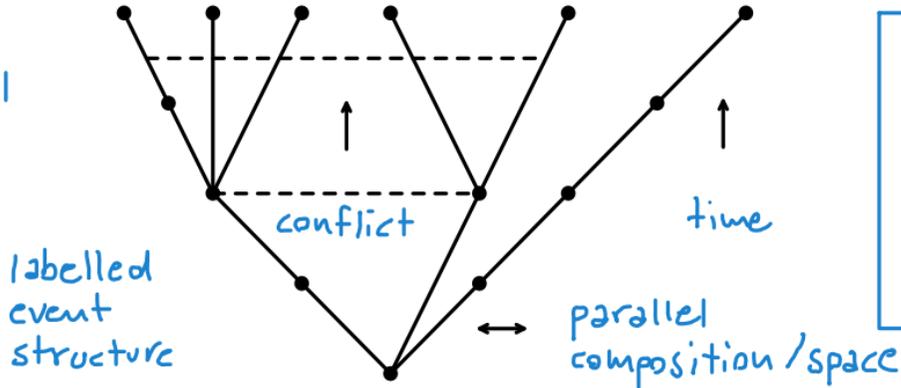
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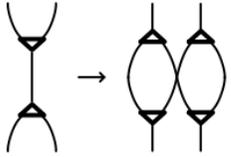
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We can see time
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 Conflict needs trees
 (or boxes), i.e., terms

GRAPHICAL NORMALISATION MECHANISMS

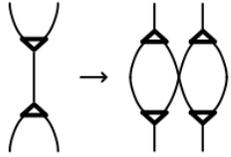


increases complexity



decreases complexity

TERN-BASED NORMALISATION MECHANISMS



increases complexity



elimination ↑

$$\text{mix } \frac{\vdash \Gamma, mA \quad \vdash n\bar{A}, \Delta}{\vdash \Gamma, \Delta}$$



decreases complexity

two opposite forces conflated

TERN-BASED NORMALISATION MECHANISMS

Problems in Gentzen
theory

poor control of
complexity

$$\text{mix} \frac{\vdash \Gamma, mA \quad \vdash n\bar{A}, \Delta}{\vdash \Gamma, \Delta}$$

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non-elementary bloating of predicate calculus analytic proofs for trivial reasons (Aguilera & Bazz 2016)

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poor modelling of concurrency

inability to model $a.b \mid \bar{a}.b \rightarrow 0$

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... but we need trees, scopes, abstractions, substitutions, ...

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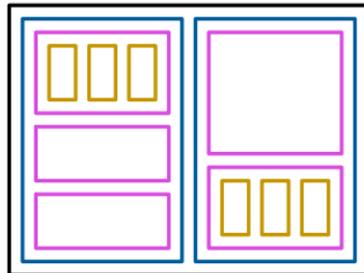
poor complexity

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... but we need trees, scopes, abstractions, substitutions, ...

deep inference

↕ inference
↔ any connective!!



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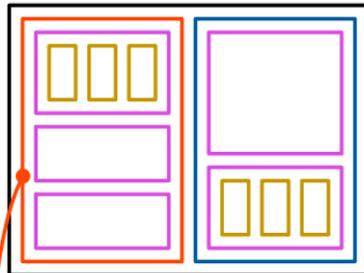
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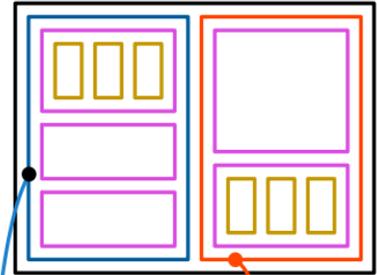
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this branch is not OK in Gentzen

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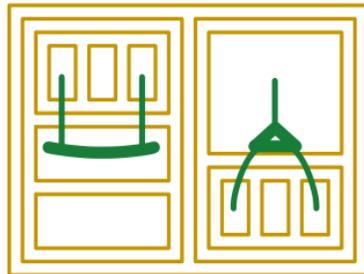
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the term-based and graphical mechanisms become compatible

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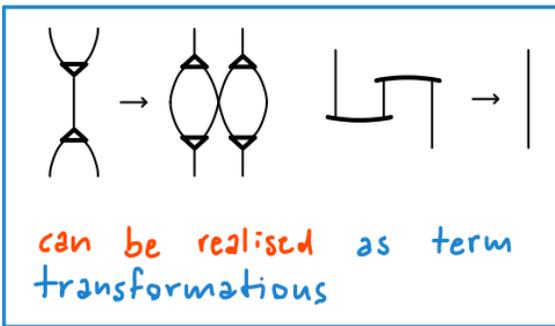
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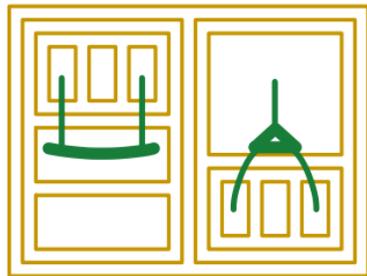
Solutions in deep inference



nonelementary speed-up thanks to in-loco creation of witnesses (Bruscoli & G 2009 + w.i.p. with Ralph)

system BV (this talk)

↕ inference
↔ any connective



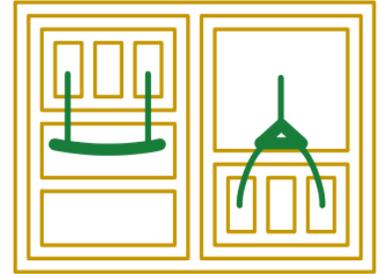
the term-based and graphical mechanisms become compatible

COMPATIBILITY BETWEEN THE TERM-BASED AND THE GRAPHICAL VIEW

What does 'compatible' mean?

Is there a reason for this 'compatibility'?

↕ inference
↔ any connective



the term-based and graphical mechanisms become compatible

COMPATIBILITY BETWEEN THE TERM-BASED AND THE GRAPHICAL VIEW

What does 'compatible' mean?

Is there a reason for this 'compatibility'?

Term-based inference rules deal with connectives:

$$\frac{(A \wedge C) \vee (B \wedge D)}{(A \vee B) \wedge (C \vee D)} \quad ||$$

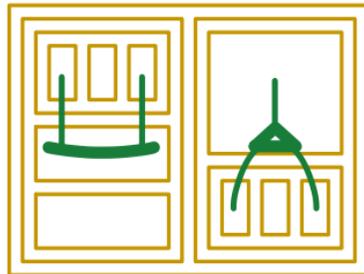
medial
(classical logic)

$$\frac{(A \wp C) \triangleleft (B \wp D)}{(A \triangleleft B) \wp (C \triangleleft D)} \quad ||$$

seq
(BV)

...

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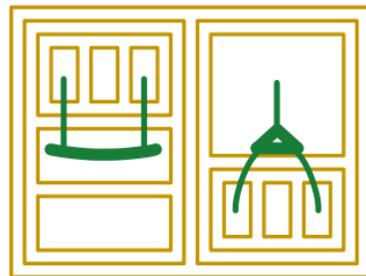
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← any connective



Graphical inference rules deal with atoms (structural rules):

$$\frac{1}{a \vee \bar{a}}$$

identity

$$\frac{a \wedge \bar{a}}{0}$$

cut

$$\frac{a \vee a}{a}$$

contraction

$$\frac{0}{a}$$

weakening

the term-based and graphical mechanisms become compatible

COMPATIBILITY BETWEEN THE TERM-BASED AND THE GRAPHICAL VIEW

What does 'compatible' mean?

Is there a reason for this 'compatibility'? Because atoms are connectives and all the rules come from one matrix!!

Term-based inference rules deal with connectives:

(see Aler Tubella PhD 2016):

$$\frac{(A \wedge C) \vee (B \wedge D)}{(A \vee B) \wedge (C \vee D)} \quad \parallel \quad \frac{(A \multimap C) \triangleleft (B \multimap D)}{(A \triangleleft B) \multimap (C \triangleleft D)} \quad \parallel \quad \dots \quad \frac{(A \alpha C) \beta (B \hat{\alpha} D)}{(A \beta B) \alpha (C \hat{\beta} D)}$$

medial
(classical logic)

seq
(BV)

obvious for these...
... what about these?

Graphical inference rules deal with atoms (structural rules):

$$\frac{I}{a \vee \bar{a}} \quad \frac{a \wedge \bar{a}}{0} \quad \frac{a \vee a}{a} \quad \frac{0}{a} \quad \dots$$

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seq
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Let's see cut and contraction

Graphical inference rules deal with atoms (structural rules):

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$$\frac{a \vee a}{a}$$

contraction

$$a \leftarrow (0 a 1)$$

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Graphical inference rules deal with atoms (structural rules):

$$\frac{(0 \wp 1) \wedge \bar{a}}{0}$$

$$\frac{(0 \wp 1) \vee (0 \wp 1)}{(0 \wp 1)}$$

$$\frac{a \leftarrow (0 \wp 1)}{\bar{a} \leftarrow (1 \wp 0)}$$

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Graphical inference rules deal with atoms (structural rules):

$$\frac{(0 \triangleright 1) \wedge (1 \triangleright 0)}{0}$$

$$\frac{(0 \triangleright 1) \vee (0 \triangleright 1)}{(0 \triangleright 1)}$$

$$\frac{\begin{matrix} \triangleright \leftarrow (0 \triangleright 1) \\ \triangleright \leftarrow (1 \triangleright 0) \end{matrix}}{}$$

use the matrix knowing that $0 \triangleright 0 = 0$
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Graphical inference rules deal with atoms (structural rules):

$$\frac{(0 \wp 1) \wedge (1 \wp 0)}{(0 \wedge 1) \wp (1 \wedge 0)}$$

cut!

$$\frac{(0 \wp 1) \vee (0 \wp 1)}{(0 \vee 0) \wp (1 \vee 1)}$$

contraction!

$$\begin{matrix} \wp \leftarrow (0 \wp 1) \\ \wp \leftarrow (1 \wp 0) \end{matrix}$$

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contraction



Same shape!

What about their normalisation behaviour?

COMPATIBILITY BETWEEN THE TERM-BASED AND THE GRAPHICAL VIEW

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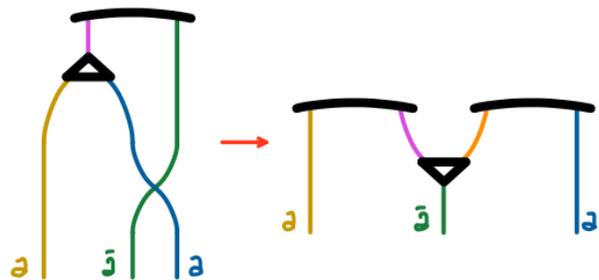
Does normalisation work at the subatomic level?

COMPATIBILITY BETWEEN THE TERM-BASED AND THE GRAPHICAL VIEW

What does 'compatible' mean?

Does normalisation work at the subatomic level?

$$\frac{\frac{1}{a} \vee \bar{a}}{a \wedge a} \rightarrow \frac{\frac{1}{a \vee \bar{a}} \wedge \frac{1}{a \vee \bar{a}}}{(a \wedge a) \vee \frac{\bar{a} \vee a}{a}}$$



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Does normalisation work at the subatomic level?

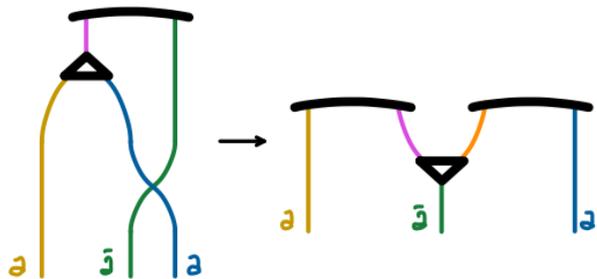
$$\frac{\frac{1}{a \wedge \bar{a}} \vee \bar{a}}{a \wedge \bar{a}} \rightarrow \frac{\frac{1}{a \vee \bar{a}} \wedge \frac{1}{a \vee \bar{a}}}{(a \wedge \bar{a}) \vee \frac{\bar{a} \vee \bar{a}}{\bar{a}}}$$

$$\frac{((0 \wedge 0) \vee 1) \bar{a} ((1 \wedge 1) \vee 0)}{(0 \wedge 0) \bar{a} (1 \wedge 1)} \vee (1 \bar{a} 0)$$

$$(0 \bar{a} 1) \wedge (0 \bar{a} 1)$$

$$\frac{((0 \vee 1) \wedge (0 \vee 1)) \bar{a} ((1 \vee 0) \wedge (1 \vee 0))}{(0 \bar{a} 1) \vee (1 \bar{a} 0) \wedge (0 \bar{a} 1) \vee (1 \bar{a} 0)}$$

$$\rightarrow \frac{(0 \bar{a} 1) \wedge (0 \bar{a} 1) \vee \frac{(1 \bar{a} 0) \vee (1 \bar{a} 0)}{(1 \vee 1) \bar{a} (0 \vee 0)}}$$

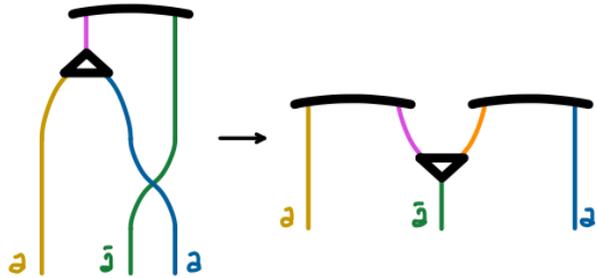


COMPATIBILITY BETWEEN THE TERP-BASED AND THE GRAPHICAL VIEW

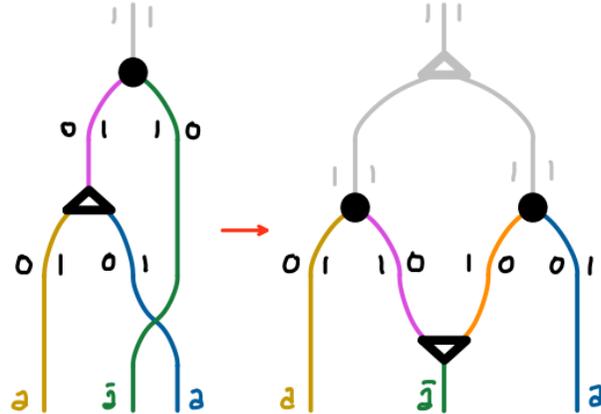
What does 'compatible' mean?

Does normalisation work at the subatomic level?

$$\frac{\frac{1}{a \wedge \bar{a}} \vee \bar{a}}{a \wedge \bar{a}} \rightarrow \frac{\frac{1}{a \vee \bar{a}} \wedge \frac{1}{a \vee \bar{a}}}{(a \wedge \bar{a}) \vee \frac{\bar{a} \vee \bar{a}}{\bar{a}}}$$



$$\frac{\frac{((0 \wedge 0) \vee 1) \wedge ((1 \wedge 1) \vee 0)}{(0 \wedge 0) \wedge (1 \wedge 1)} \vee (1 \wedge 0)}{(0 \wedge 1) \wedge (0 \wedge 1)} \rightarrow \frac{\frac{(0 \vee 1) \wedge (0 \vee 1)}{(0 \wedge 1) \vee (1 \wedge 0)} \wedge \frac{(0 \vee 1) \wedge (1 \vee 0)}{(0 \wedge 1) \vee (1 \wedge 0)}}{((0 \wedge 1) \wedge (0 \wedge 1)) \vee \frac{(1 \wedge 0) \vee (1 \wedge 0)}{(1 \vee 1) \wedge (0 \vee 0)}}$$

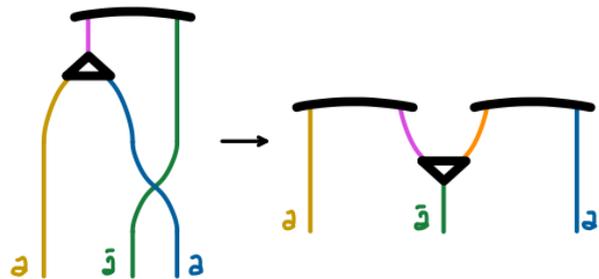


COMPATIBILITY BETWEEN THE TERP-BASED AND THE GRAPHICAL VIEW

What does 'compatible' mean?

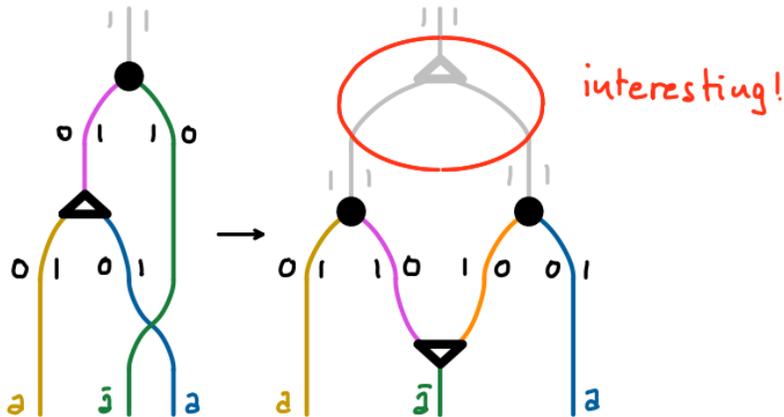
Does normalisation work at the subatomic level? **Yes**

$$\frac{\frac{1}{a \wedge \bar{a}} \vee \bar{a}}{a \wedge \bar{a}} \rightarrow \frac{\frac{1}{a \vee \bar{a}} \wedge \frac{1}{a \vee \bar{a}}}{(a \wedge \bar{a}) \vee \frac{\bar{a} \vee \bar{a}}{\bar{a}}}$$



$$\frac{\frac{((0 \wedge 0) \vee 1) \wedge ((1 \wedge 1) \vee 0)}{(0 \wedge 0) \wedge (1 \wedge 1)} \vee (1 \wedge 0)}{(0 \wedge 1) \wedge (0 \wedge 1)} \rightarrow \frac{((0 \vee 1) \wedge (0 \vee 1)) \wedge ((1 \vee 0) \wedge (1 \vee 0))}{(0 \wedge 1) \vee (1 \wedge 0) \wedge \frac{(1 \wedge 0) \vee (1 \wedge 0)}{(1 \vee 1) \wedge (0 \vee 0)}}$$

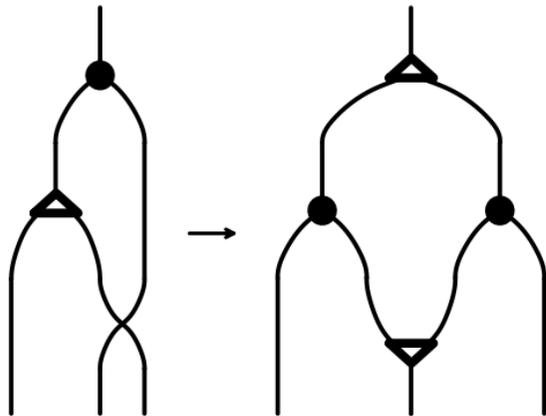
$$\frac{((0 \vee 1) \wedge (0 \vee 1)) \wedge ((1 \vee 0) \wedge (1 \vee 0))}{(0 \wedge 1) \vee (1 \wedge 0) \wedge \frac{(1 \wedge 0) \vee (1 \wedge 0)}{(1 \vee 1) \wedge (0 \vee 0)}}$$



COMPATIBILITY BETWEEN THE TERM-BASED AND THE GRAPHICAL VIEW

What does 'compatible' mean?

Does normalisation work at the subatomic level? Yes, and we get a **general reduction shape**. Let's use it to design a proof system for concurrency.



COMPATIBILITY BETWEEN THE TERM-BASED AND THE GRAPHICAL VIEW

What does 'compatible' mean?

Let us say that we want:

- parallelism \bowtie
- sequentiality \triangleleft (self-dual)
- 'a' Kleene star s.t. $\star A = \star A \triangleleft A$

$$\star A \triangleleft A$$

$$\hline \star A$$

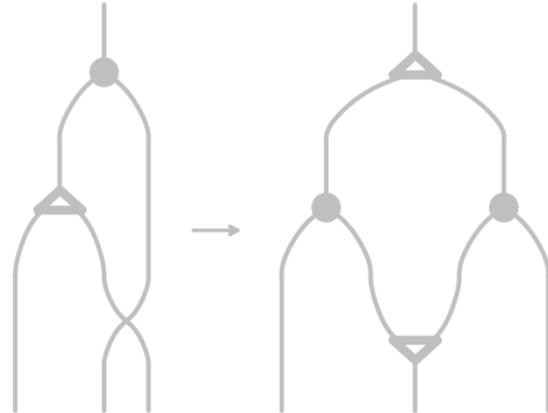
$$\star A$$

$$\hline \star A \triangleleft A$$

Because of self-duality:

$$\star(A \bowtie B)$$

$$\hline \star A \bowtie \star B$$



COMPATIBILITY BETWEEN THE TERM-BASED AND THE GRAPHICAL VIEW

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$$\star A \triangleleft A$$

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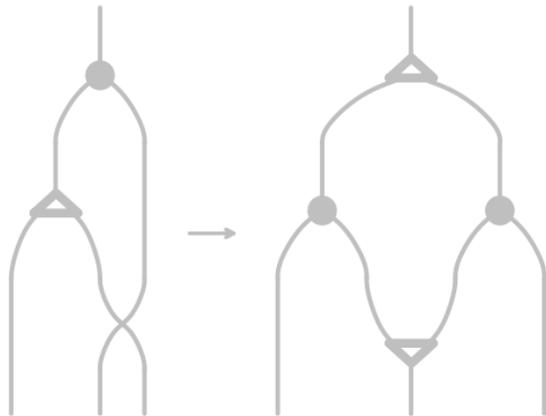
$$\star A \triangleleft A$$

Because of self-duality:

$$\star(A \bowtie B)$$

$$\star A \bowtie \star B$$

'term-based' and
'graphical' structures
together!



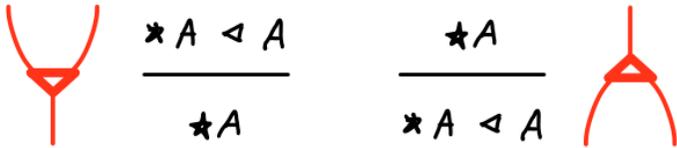
Can normalisation work?

COMPATIBILITY BETWEEN THE TERM-BASED AND THE GRAPHICAL VIEW

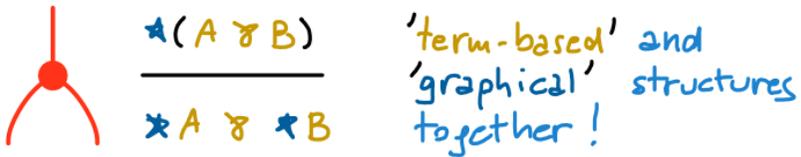
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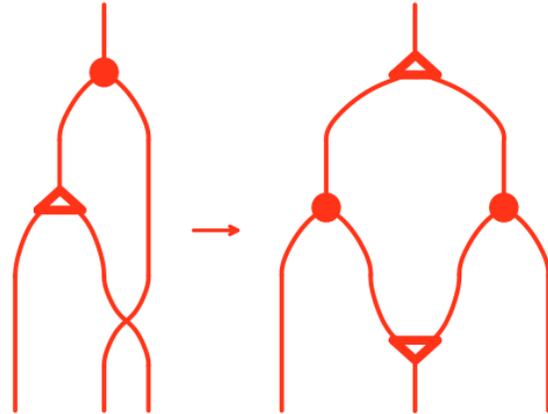
- parallelism \wp
- sequentiality \triangleleft (self-dual)
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Because of self-duality:



Can normalisation work? Use the shape!

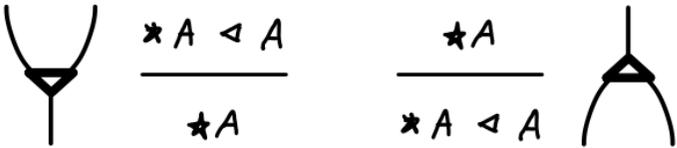


COMPATIBILITY BETWEEN THE TERM-BASED AND THE GRAPHICAL VIEW

What does 'compatible' mean?

Let us say that we want:

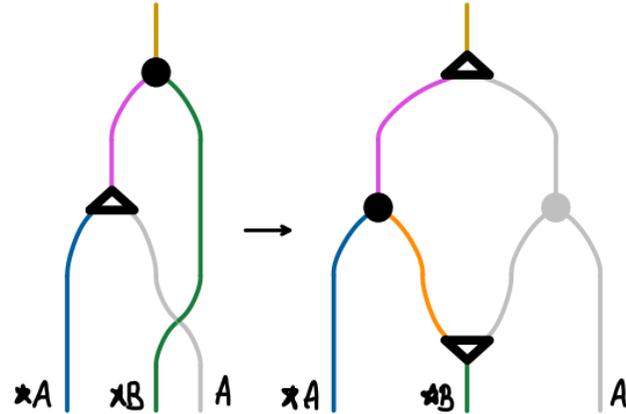
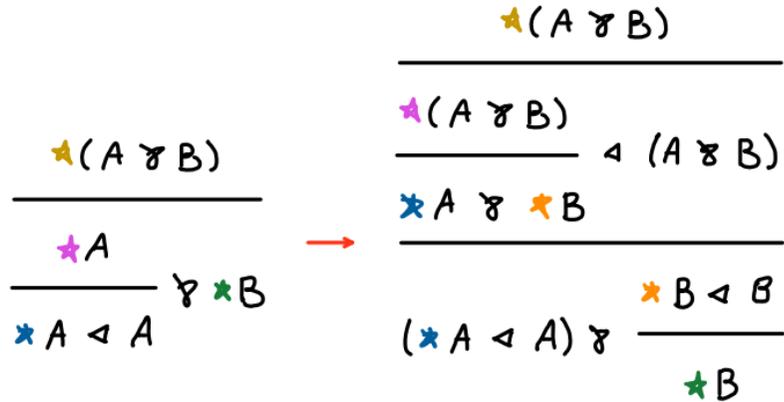
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Because of self-duality:



Can normalisation work? Use the shape.

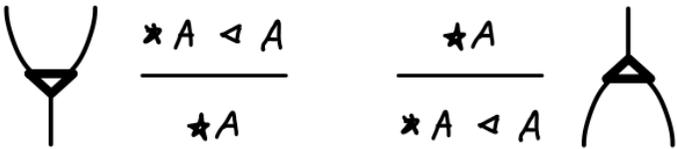


COMPATIBILITY BETWEEN THE TERM-BASED AND THE GRAPHICAL VIEW

What does 'compatible' mean?

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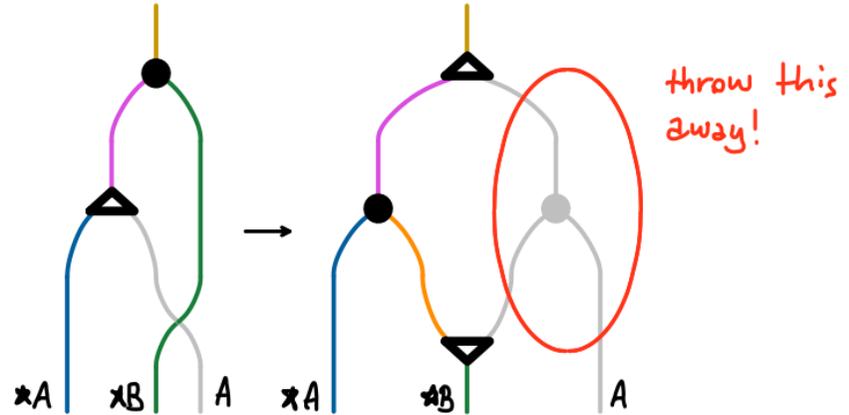
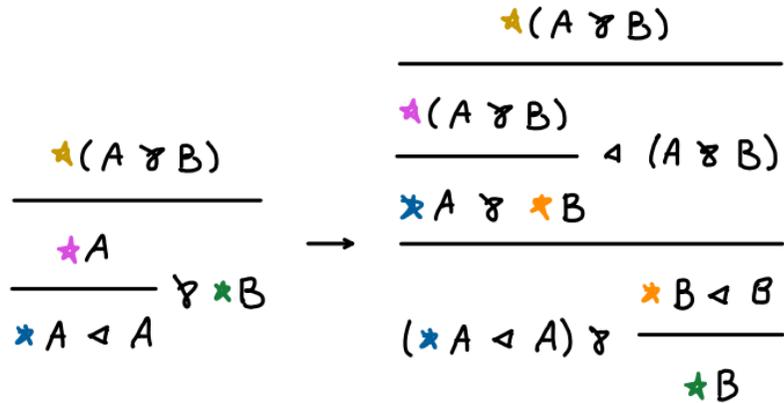
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Because of self-duality:



Can normalisation work? Use the shape.

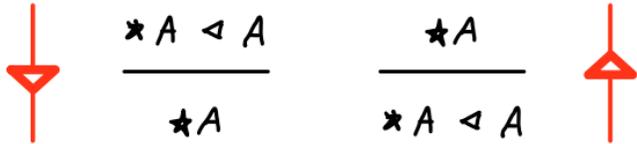


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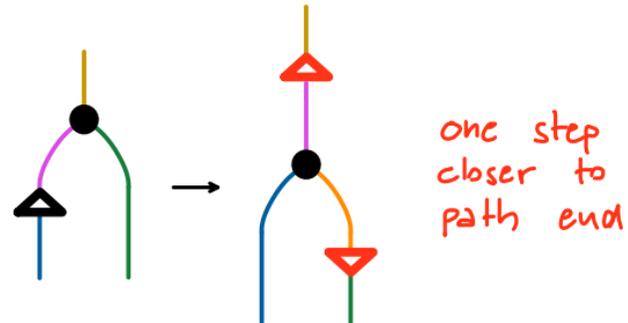
$$\frac{\star(A \bowtie B)}{\star A} \bowtie \star B$$

$$\frac{\star A \triangleleft A}{\star A \triangleleft A} \bowtie \star B$$

$$\frac{\star(A \bowtie B)}{\star(A \bowtie B) \triangleleft (A \bowtie B)}$$

$$\frac{\star A \bowtie \star B}{(\star A \triangleleft A) \bowtie \frac{\star B \triangleleft B}{\star B}}$$

Because of self-duality:



Can normalisation work? It works and it gives us an induction measure.

COMPATIBILITY BETWEEN THE TERM-BASED AND THE GRAPHICAL VIEW

What does 'compatible' mean?

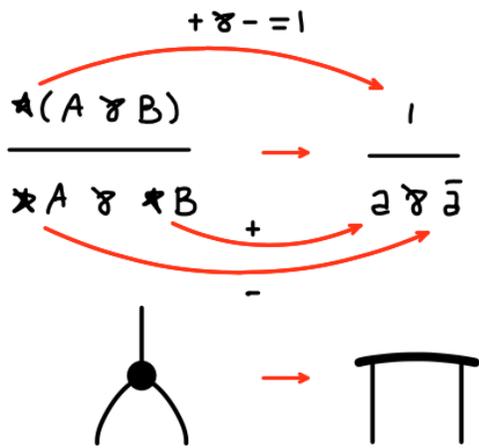
They can be used together to design proof systems.

COMPATIBILITY BETWEEN THE TERM-BASED AND THE GRAPHICAL VIEW

What does 'compatible' mean?

They can be used together to design proof systems.

There's more

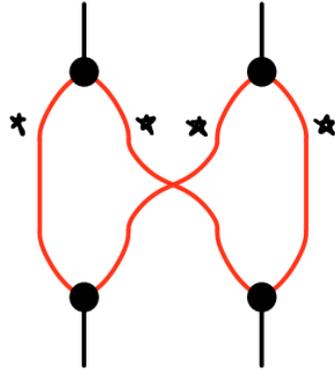
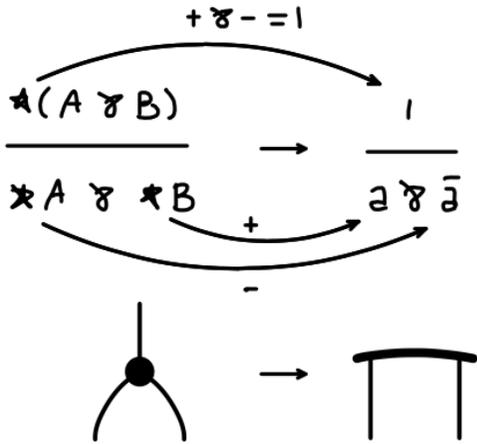


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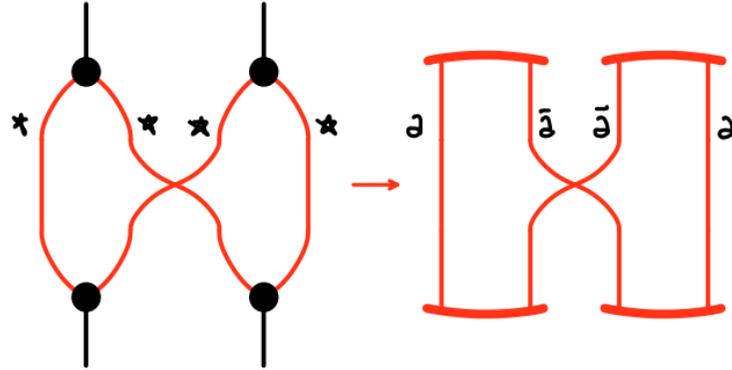
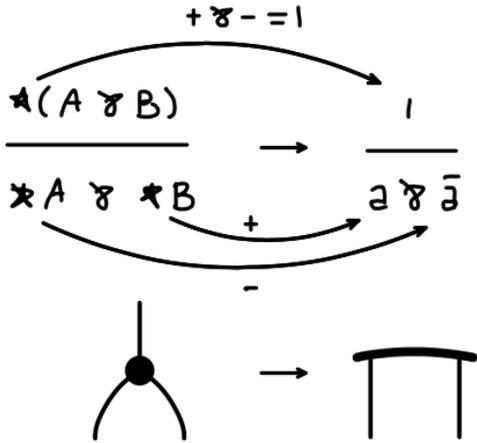
proof with a cycle of \star 's

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proof with a cycle of \star 's



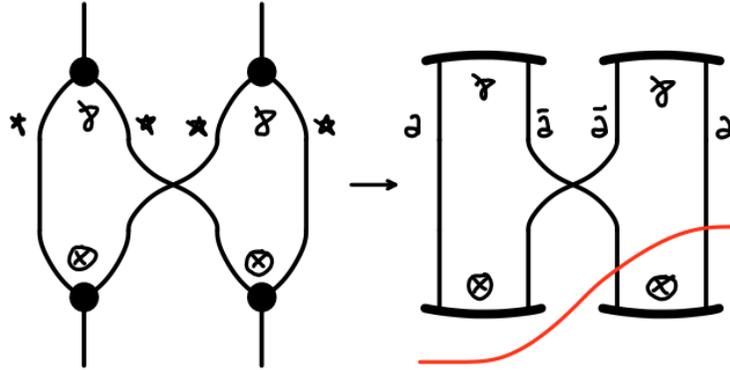
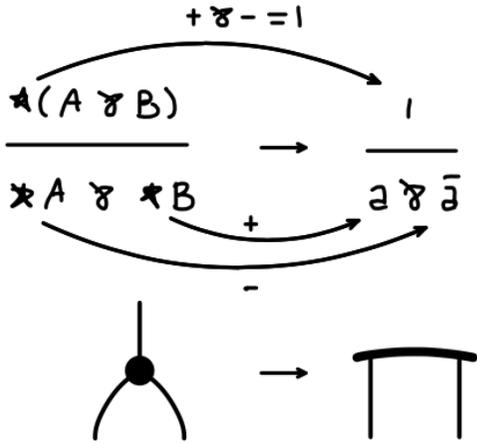
proof with a cycle of a 's in a linear system

COMPATIBILITY BETWEEN THE TERM-BASED AND THE GRAPHICAL VIEW

What does 'compatible' mean?

They can be used together to design proof systems.

There's more



proof with a cycle of \star 's



proof with a cycle of atoms in a linear system

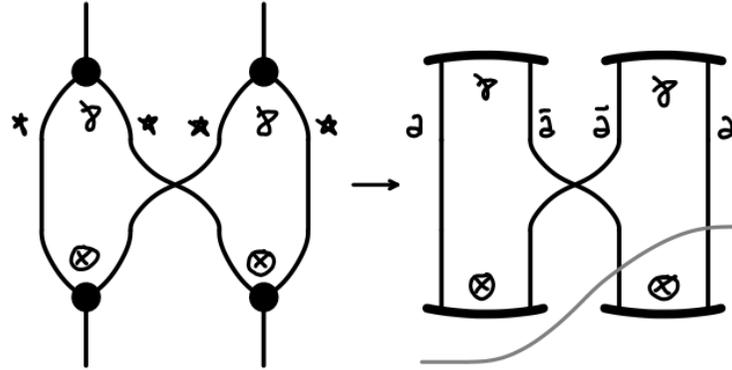
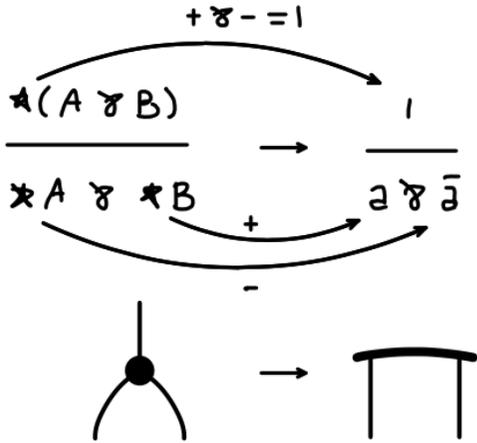
if we break the proof here, we prove $\bar{a} \otimes a$

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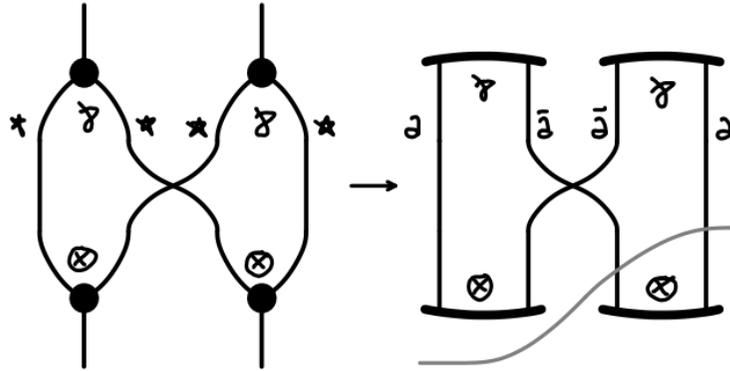
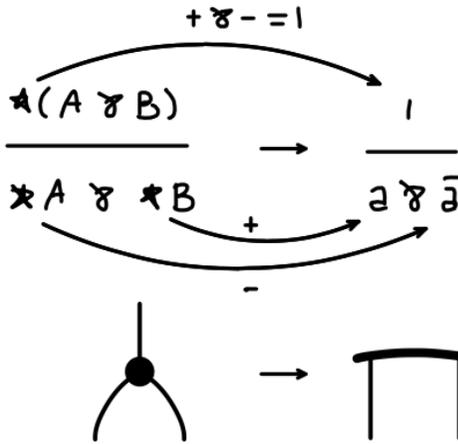
excluded by cut elimination (splitting theorem)

COMPATIBILITY BETWEEN THE TERM-BASED AND THE GRAPHICAL VIEW

What does 'compatible' mean?

They can be used together to design proof systems.

There's more: **cut elimination** in the linear fragment can be used to prove **topological** properties.



proof with a cycle of $*$'s

proof with a cycle of atoms in a linear system

if we break the proof here, we prove $\bar{a} \otimes a$

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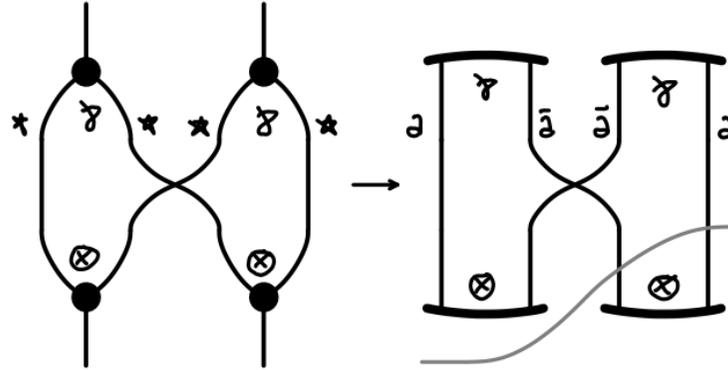
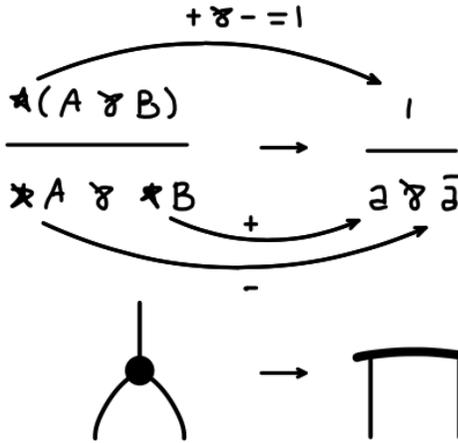
$*$'s don't cycle

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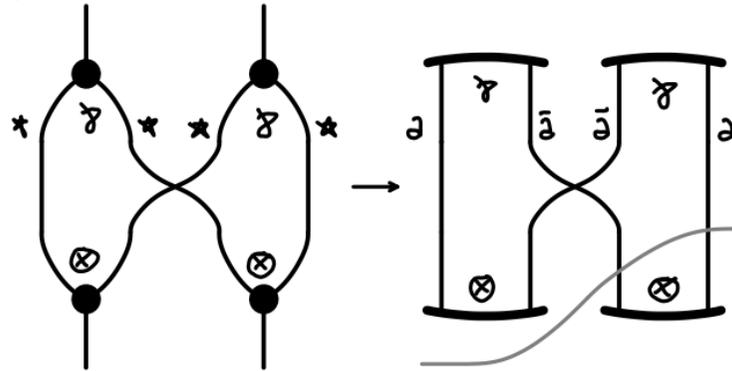
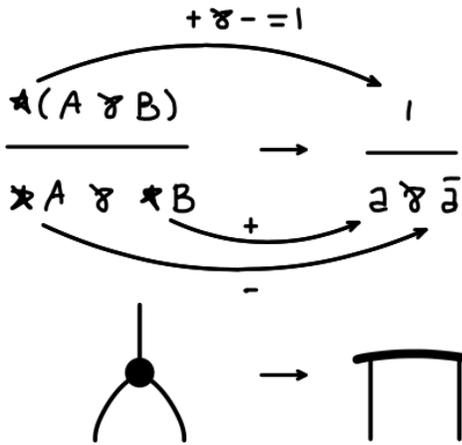
the Δ induction measure works \leftarrow \star 's don't cycle

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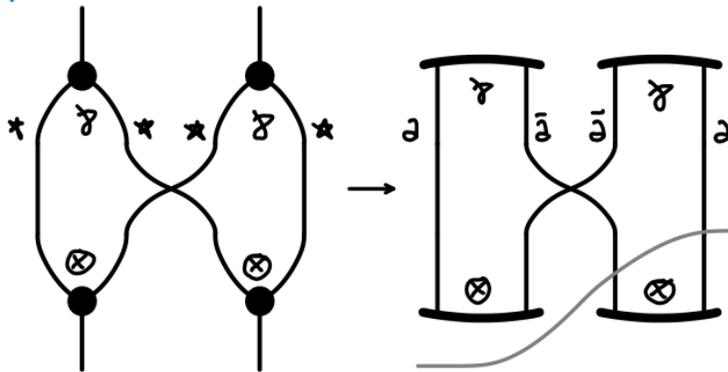
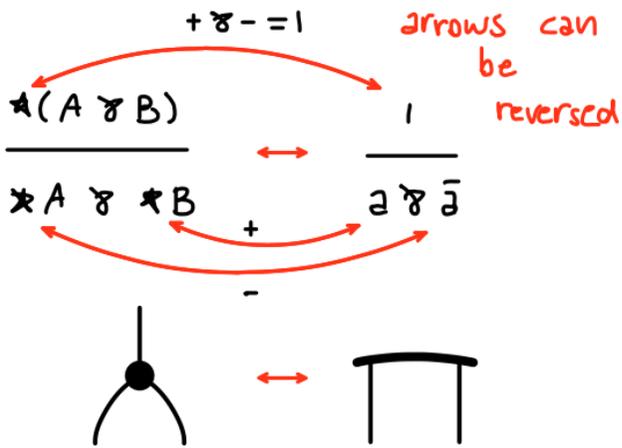
the \star system has cut elimination \leftarrow the \triangle induction measure works \leftarrow \star 's don't cycle

COMPATIBILITY BETWEEN THE TERM-BASED AND THE GRAPHICAL VIEW

What does 'compatible' mean?

There's more: cut elimination in the linear fragment can be used to prove topological properties.

1. They can be used together to design proof systems.
2. There is a symmetry between the two views.



proof with a cycle of $*$'s

proof with a cycle of atoms in a linear system

if we break the proof here, we prove $\bar{a} \otimes a$

excluded by cut elimination (splitting theorem)

the $*$ system has cut elimination ← the \triangle induction measure works ← $*$'s don't cycle

OBTAINING A CORE PROOF SYSTEM FOR CONCURRENCY

Term-based and graphical views: 1. they can be used together to design proof systems.
2. there is a symmetry between the two views.

previously studied system BV

+ rules obtained as described

$\frac{0}{\exists \bar{\exists}}$	$\frac{(A \bar{\exists} C) \otimes (B \bar{\exists} D)}{(A \otimes B) \bar{\exists} (C \bar{\exists} D)}$	$\frac{(A \bar{\exists} C) \triangleleft (B \bar{\exists} D)}{(A \triangleleft B) \bar{\exists} (C \triangleleft D)}$	$\frac{\star(A \bar{\exists} B)}{\star A \bar{\exists} \star B}$	$\frac{\star A \triangleleft A}{\star A}$
identity	switch	seq	promotion	contraction

+ equations: associativity, commutativity of $\bar{\exists}$ and \otimes , unit 0 and $\star 0 = 0$

OBTAINING A CORE PROOF SYSTEM FOR CONCURRENCY

We close by symmetry:

SBV			SKV	
$\frac{\exists \otimes \bar{\exists}}{\circ}$ <p>cut</p>	$\frac{(A \wp B) \otimes (C \otimes D)}{(A \otimes C) \wp (B \otimes D)}$ <p>switch</p>	$\frac{(A \triangleleft B) \otimes (C \triangleleft D)}{(A \otimes C) \triangleleft (B \otimes D)}$ <p>coseq</p>	$\frac{\star(A \otimes B)}{\star A \otimes \star B}$ <p>copromotion</p>	$\frac{\star A}{\star A \triangleleft A}$ <p>cocontraction</p>
$\frac{\circ}{\exists \wp \bar{\exists}}$ <p>identity</p>	$\frac{(A \wp C) \otimes (B \wp D)}{(A \otimes B) \wp (C \otimes D)}$ <p>switch</p>	$\frac{(A \wp C) \triangleleft (B \wp D)}{(A \triangleleft B) \wp (C \triangleleft D)}$ <p>seq</p>	$\frac{\star(A \wp B)}{\star A \wp \star B}$ <p>promotion</p>	$\frac{\star A \triangleleft A}{\star A}$ <p>contraction</p>

+ equations: associativity, commutativity of \wp and \otimes , unit \circ and $\star \circ = \circ$

OBTAINING A CORE PROOF SYSTEM FOR CONCURRENCY

We collapse the two switches (they turn out to be equivalent)

SBV		SKV	
$\frac{a \otimes \bar{a}}{\circ}$ <p>cut</p>	$\frac{(A \wp C) \otimes B}{(A \otimes B) \wp C}$ <p>switch</p>	$\frac{(A \triangleleft B) \otimes (C \triangleleft D)}{(A \otimes C) \triangleleft (B \otimes D)}$ <p>coseq</p>	$\frac{\star(A \otimes B)}{\star A \otimes \star B}$ <p>copromotion</p>
$\frac{\circ}{a \wp \bar{a}}$ <p>identity</p>	$\frac{(A \otimes B) \wp C}{(A \wp C) \triangleleft (B \wp D)}$ <p>seq</p>	$\frac{\star(A \wp B)}{\star A \wp \star B}$ <p>promotion</p>	$\frac{\star A}{\star A \triangleleft A}$ <p>cocontraction</p>
		$\frac{\star A \triangleleft A}{\star A}$ <p>contraction</p>	

+ equations: associativity, commutativity of \wp and \otimes , unit \circ and $\star \circ = \circ$

OBTAINING A CORE PROOF SYSTEM FOR CONCURRENCY

What about normalisation for this system?

SBV		SKV	
$\frac{a \otimes \bar{a}}{\quad}$ <p style="text-align: center;">o cut</p>	$\frac{(A \wp C) \otimes B}{\quad}$ <p style="text-align: center;">(A ⊗ B) ⋈ C switch</p>	$\frac{(A \triangleleft B) \otimes (C \triangleleft D)}{\quad}$ <p style="text-align: center;">(A ⊗ C) △ (B ⊗ D) coseq</p>	$\frac{\star(A \otimes B)}{\quad}$ <p style="text-align: center;">⋈ A ⊗ ⋈ B copromotion</p>
$\frac{o}{\quad}$ <p style="text-align: center;">a ⋈ ā identity</p>	$\frac{(A \wp C) \triangleleft (B \wp D)}{\quad}$ <p style="text-align: center;">(A △ B) ⋈ (C △ D) seq</p>	$\frac{\star(A \wp B)}{\quad}$ <p style="text-align: center;">⋈ A ⋈ ⋈ B promotion</p>	$\frac{\star A}{\quad}$ <p style="text-align: center;">⋈ A △ A cocontraction</p>
		$\frac{\star A \triangleleft A}{\quad}$ <p style="text-align: center;">⋈ A contraction</p>	

+ equations: associativity, commutativity of \wp and \otimes , unit o and $\star o = o$

NORMALISATION OF CONTRACTIONS — DECOMPOSITION

We push (ω) contractions along their natural paths.

$$\frac{*A \triangleleft A}{*A} \downarrow \text{contraction}$$
$$\frac{*A}{*A \triangleleft A} \uparrow \text{coccontraction}$$

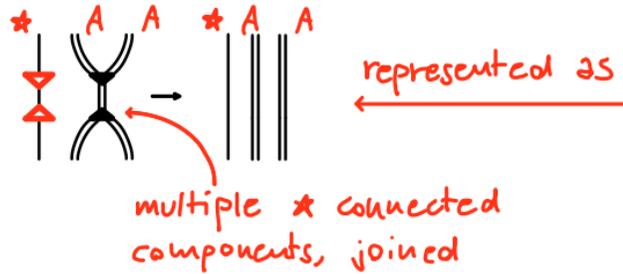
NORMALISATION OF CONTRACTIONS — DECOMPOSITION

We push (ω) contractions along their natural paths.

$$\frac{\star A \triangleleft A}{\star A} \rightarrow \star A \triangleleft A$$
$$\star A \triangleleft A$$

NORMALISATION OF CONTRACTIONS — DECOMPOSITION

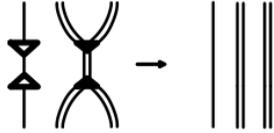
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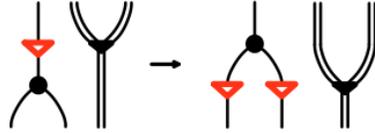
$$\frac{\star A \triangleleft A}{\star A} \rightarrow \star A \triangleleft A$$
$$\star A \triangleleft A$$

NORMALISATION OF CONTRACTIONS — DECOMPOSITION

vs. ccontraction



vs. promotion



↑
represented
as

$$\frac{* (A \wp B) \triangleleft (A \wp B)}{\quad}$$

$$\frac{* (A \wp B)}{\quad}$$

$$* A \wp * B$$

→

$$\frac{* (A \wp B)}{* A \wp * B} \triangleleft (A \wp B)$$

$$\frac{* A \triangleleft A \quad * B \triangleleft B}{* A \wp * B}$$

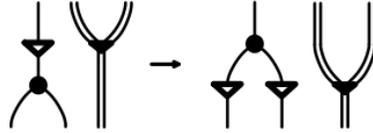
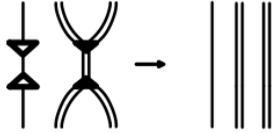
$$\frac{* A \triangleleft A}{* A} \wp \frac{* B \triangleleft B}{* B}$$

NORMALISATION OF CONTRACTIONS — DECOMPOSITION

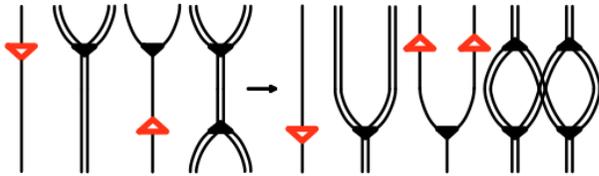
vs. contraction

vs. promotion

direct



external



representational
as

$$*K\{ *A \} \triangleleft K\{ *A \}$$

$$*K\left\{ \frac{*A}{*A \triangleleft A} \right\}$$

→

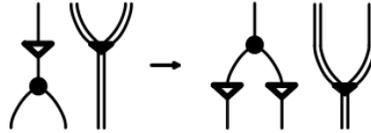
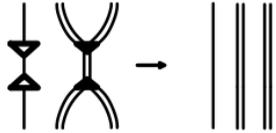
$$\frac{*K\left\{ \frac{*A}{*A \triangleleft A} \right\} \triangleleft K\left\{ \frac{*A}{*A \triangleleft A} \right\}}{*K\{ *A \triangleleft A \}}$$

NORMALISATION OF CONTRACTIONS — DECOMPOSITION

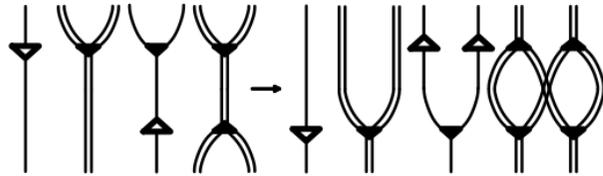
vs. contraction

vs. promotion

direct



external



Induction measure

$\langle E, H \rangle$

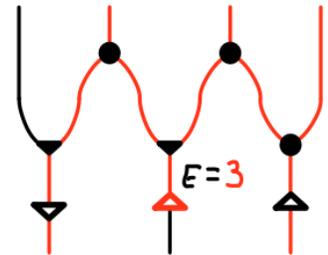
$E =$

$H =$

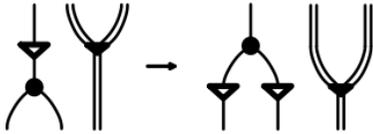
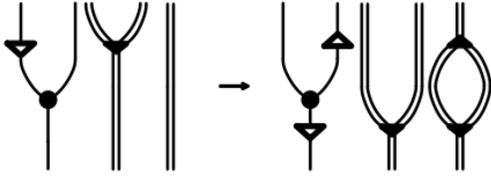
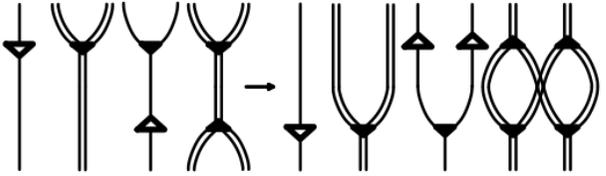
energy
contraction-height

energy =
number of ● in the
forward path of

$\triangleleft \blacktriangle$



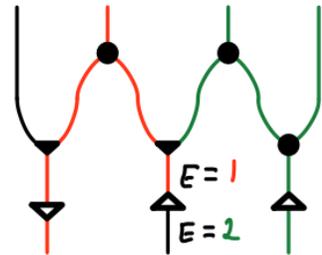
NORMALISATION OF CONTRACTIONS — DECOMPOSITION

vs. contraction	vs. promotion	vs. copromotion
direct		
external	<p>problem: creates paths — the measure in the rest of the flow is adversely affected</p>	
		

Induction measure $\langle E, H \rangle$, lexicographic

multiset ordering $\begin{cases} E = \{c_i\}_i^+; \text{ clone energies} \\ c_i = \{e_i\}_i^+; \text{ energies} \\ H = \{h_i\}_i^+; \text{ contraction-heights} \end{cases}$

solution:
energy = number of \bullet in the forward path of \triangle per clone



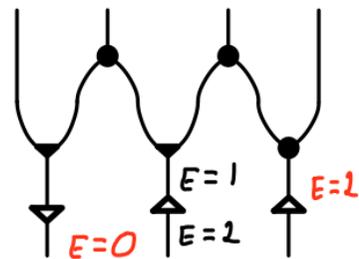
NORMALISATION OF CONTRACTIONS — DECOMPOSITION

	vs. contraction	vs. promotion	vs. copromotion
direct			
external			

+ internal (trivial) + dual

Induction measure $\langle E, H \rangle$, lexicographic

multiset ordering $\left\{ \begin{array}{l} E = \{c_i\}_i^+, \text{ clone energies} \\ c_i = \{e_i\}_i^+, \text{ energies} \\ H = \{h_i\}_i^+, \text{ contraction-heights} \end{array} \right.$



NORMALISATION OF CONTRACTIONS — DECOMPOSITION

	vs. contraction	vs. promotion	vs. copromotion
direct			
external			

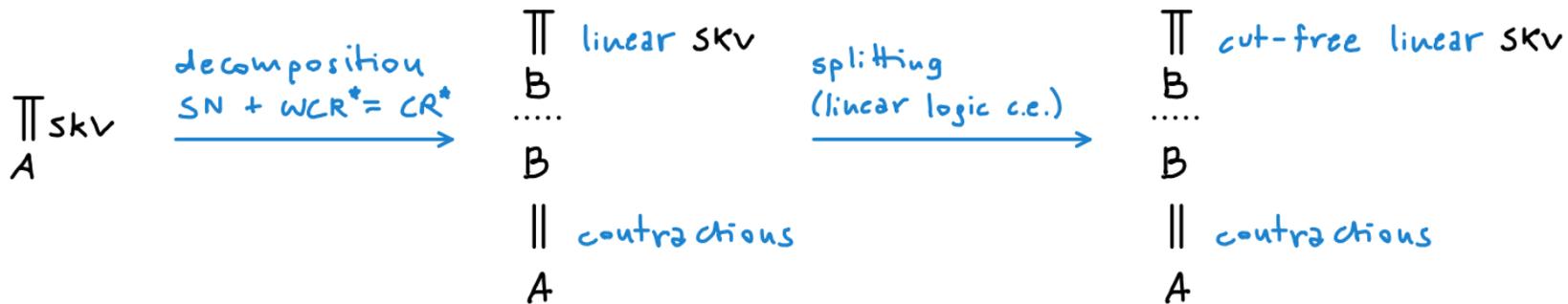
+ internal (trivial) + dual

Induction measure $\langle E, H \rangle$, lexicographic

multiset ordering $\left\{ \begin{array}{l} E = \{c_i\}_i^+, \text{ clone energies} \\ c_i = \{e_i\}_i^+, \text{ energies} \\ H = \{h_i\}_i^+, \text{ contraction-heights} \end{array} \right.$

This works!

NORMALISATION



* modulo permutations of (co)contractions.

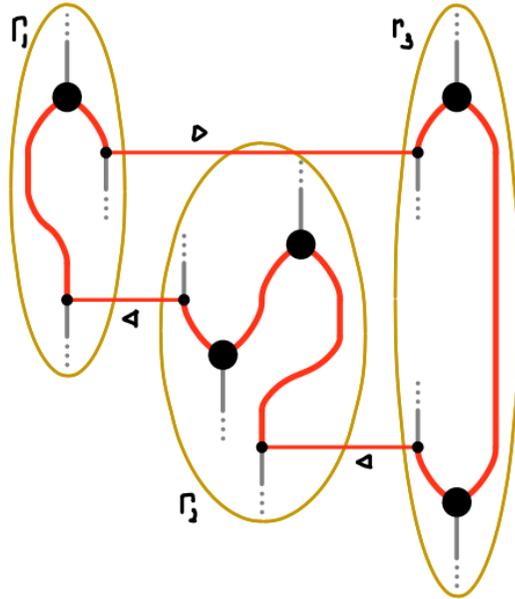
NO CYCLES AMONG \star CONNECTED COMPONENTS

Do the \star connected components form a well-order for \triangleleft ?

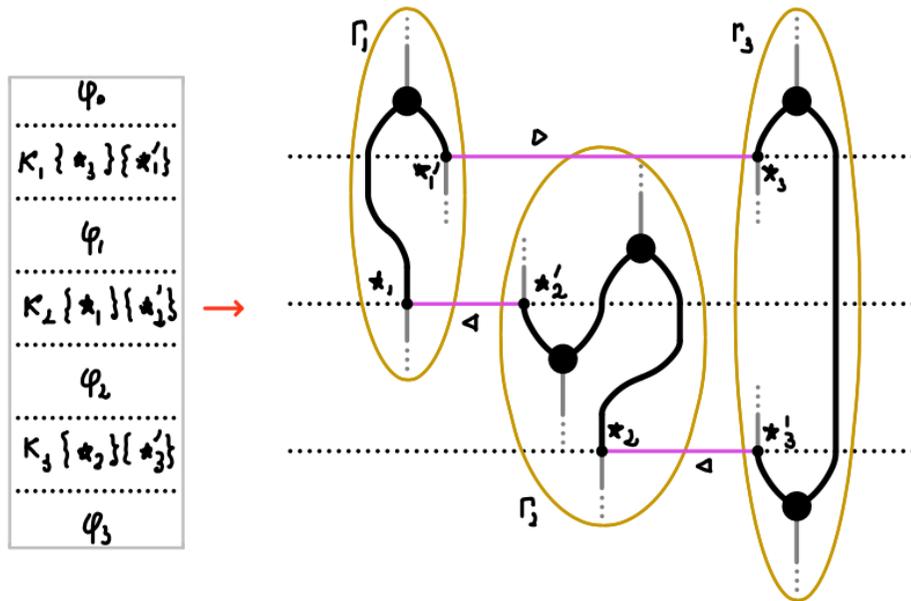
Another proof that exploits cut elimination in the linear fragment...

NO CYCLES AMONG \star CONNECTED COMPONENTS

Assume there are
 \star connected
components $\Gamma_1, \Gamma_2, \Gamma_3$
such that
they form a
 \triangleleft cycle

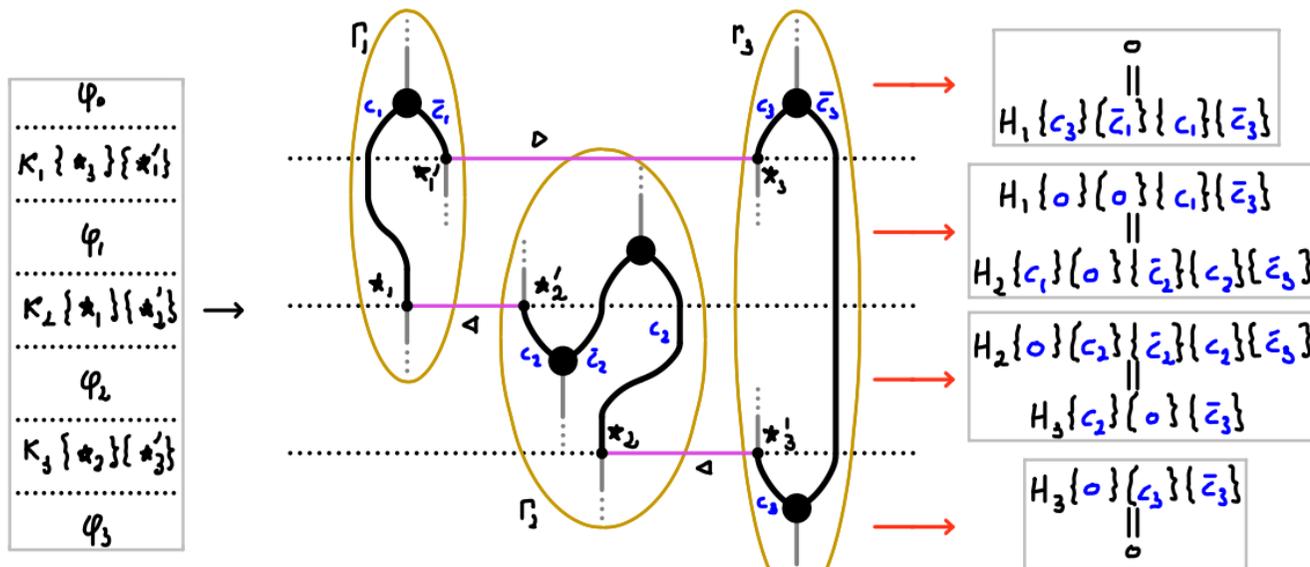


NO CYCLES AMONG \star CONNECTED COMPONENTS



This is the proof corresponding to the diagram

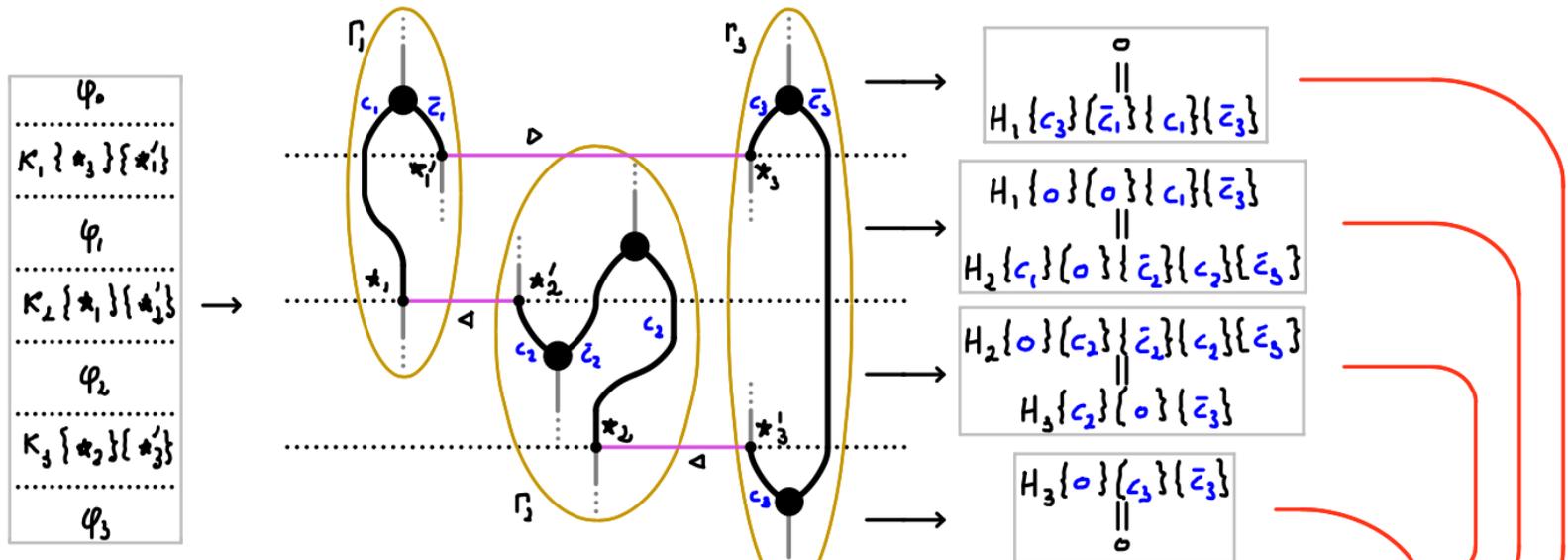
NO CYCLES AMONG \star CONNECTED COMPONENTS



We project \star 's into atoms in the linear fragment, separately for $\varphi_0, \varphi_1, \varphi_2$ and φ_3 .

The idea is to expose the \triangleleft relations.

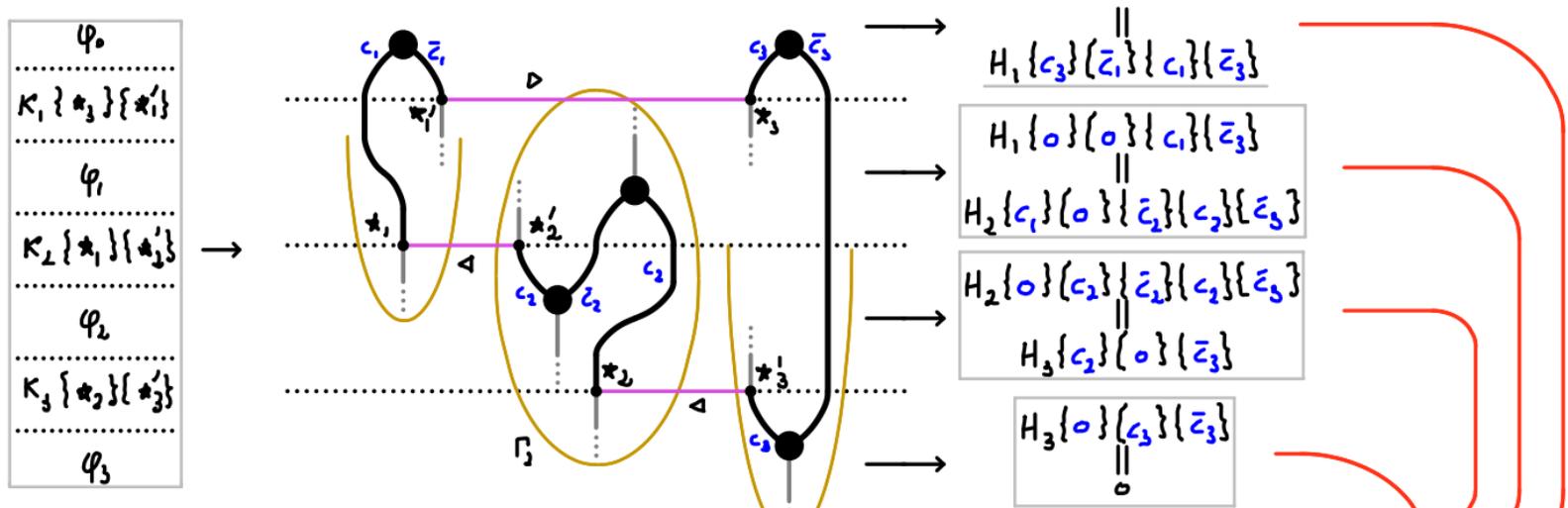
NO CYCLES AMONG \star CONNECTED COMPONENTS



We apply the deduction theorem.

$$\begin{array}{c}
 \Pi \\
 \circ \neq H_1\{c_3\}\{z_1\}\{c_1\}\{z_3\} \otimes \\
 \Pi \\
 \bar{H}_1\{0\}\{0\}\{z_1\}\{c_3\} \neq H_2\{c_1\}\{0\}\{z_2\}\{c_2\}\{z_3\} \otimes \\
 \Pi \\
 \bar{H}_2\{0\}\{z_2\}\{c_2\}\{z_2\}\{c_3\} \neq H_3\{c_2\}\{0\}\{z_3\} \otimes \\
 \Pi \\
 \bar{H}_3\{0\}\{z_3\}\{c_3\} \neq \circ
 \end{array}$$

NO CYCLES AMONG \star CONNECTED COMPONENTS



We apply the deduction theorem.

$$\prod \circ \nabla H_1 \{ c_3 \} \{ \bar{c}_1 \} \{ c_1 \} \{ \bar{c}_3 \}$$

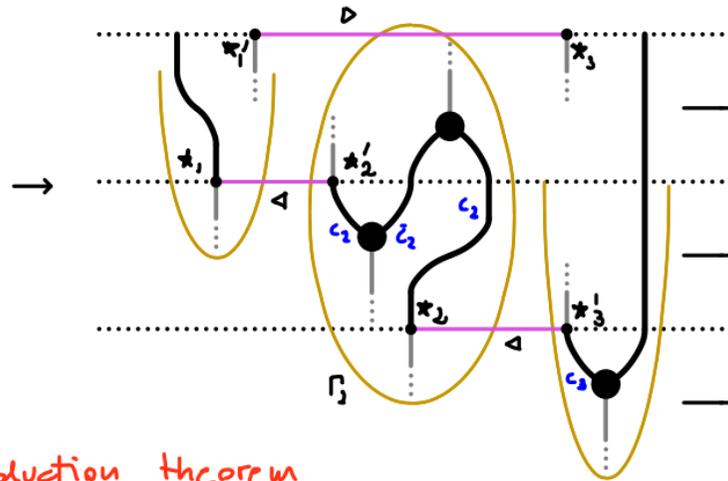
$$\prod \bar{H}_1 \{ 0 \} \{ 0 \} \{ \bar{c}_1 \} \{ c_3 \} \nabla H_2 \{ c_1 \} \{ 0 \} \{ \bar{c}_2 \} \{ c_2 \} \{ \bar{c}_3 \}$$

$$\prod \bar{H}_2 \{ 0 \} \{ \bar{c}_2 \} \{ c_2 \} \{ \bar{c}_2 \} \{ c_3 \} \nabla H_3 \{ c_2 \} \{ 0 \} \{ \bar{c}_3 \}$$

$$\prod \bar{H}_3 \{ 0 \} \{ \bar{c}_3 \} \{ c_3 \} \nabla 0$$

NO CYCLES AMONG \star CONNECTED COMPONENTS

$\dots\dots\dots$
 $K_1 \{ \star_3 \} \{ \star'_1 \}$
 φ_1
 $\dots\dots\dots$
 $K_2 \{ \star_1 \} \{ \star'_3 \}$
 φ_2
 $\dots\dots\dots$
 $K_3 \{ \star_2 \} \{ \star'_2 \}$
 φ_3
 $\dots\dots\dots$



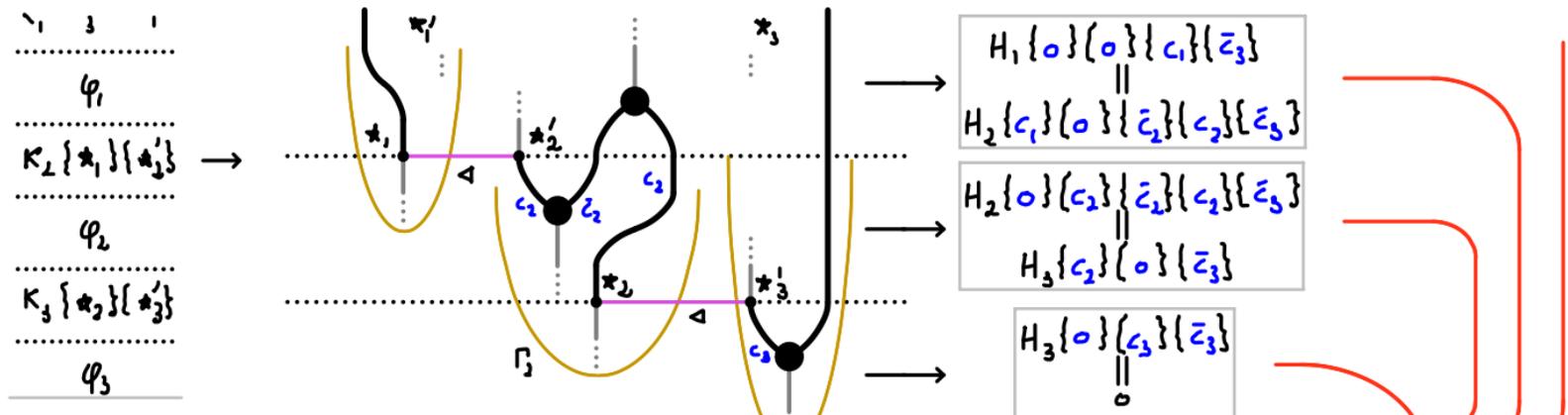
$\overline{c_3} \quad c_1 \quad c_1 \quad c_3$

$H_1 \{ 0 \} \{ 0 \} \{ c_1 \} \{ \bar{c}_3 \}$
$H_2 \{ c_1 \} \{ 0 \} \{ \bar{c}_2 \} \{ c_2 \} \{ \bar{c}_3 \}$
$H_2 \{ 0 \} \{ c_2 \} \{ \bar{c}_2 \} \{ c_2 \} \{ \bar{c}_3 \}$
$H_3 \{ c_2 \} \{ 0 \} \{ \bar{c}_3 \}$
$H_3 \{ 0 \} \{ c_3 \} \{ \bar{c}_3 \}$

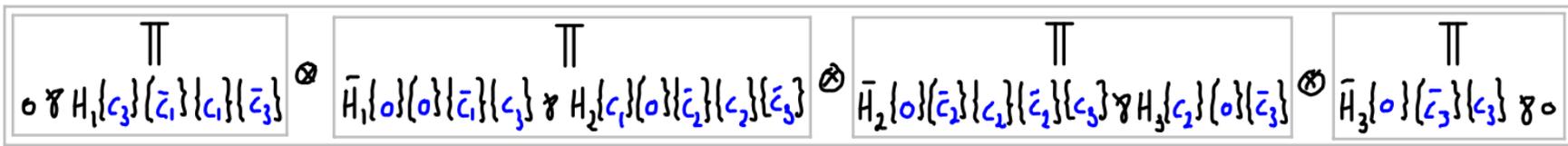
We apply the deduction theorem.

Π $\circ \nabla H_1 \{ c_3 \} \{ \bar{c}_1 \} \{ c_1 \} \{ \bar{c}_3 \}$	\otimes	Π $\bar{H}_1 \{ 0 \} \{ 0 \} \{ \bar{c}_1 \} \{ c_3 \} \nabla H_2 \{ c_1 \} \{ 0 \} \{ \bar{c}_2 \} \{ c_2 \} \{ \bar{c}_3 \}$	\otimes	Π $\bar{H}_2 \{ 0 \} \{ \bar{c}_2 \} \{ c_2 \} \{ \bar{c}_2 \} \{ c_3 \} \nabla H_3 \{ c_2 \} \{ 0 \} \{ \bar{c}_3 \}$	\otimes	Π $\bar{H}_3 \{ 0 \} \{ \bar{c}_3 \} \{ c_3 \} \nabla \circ$
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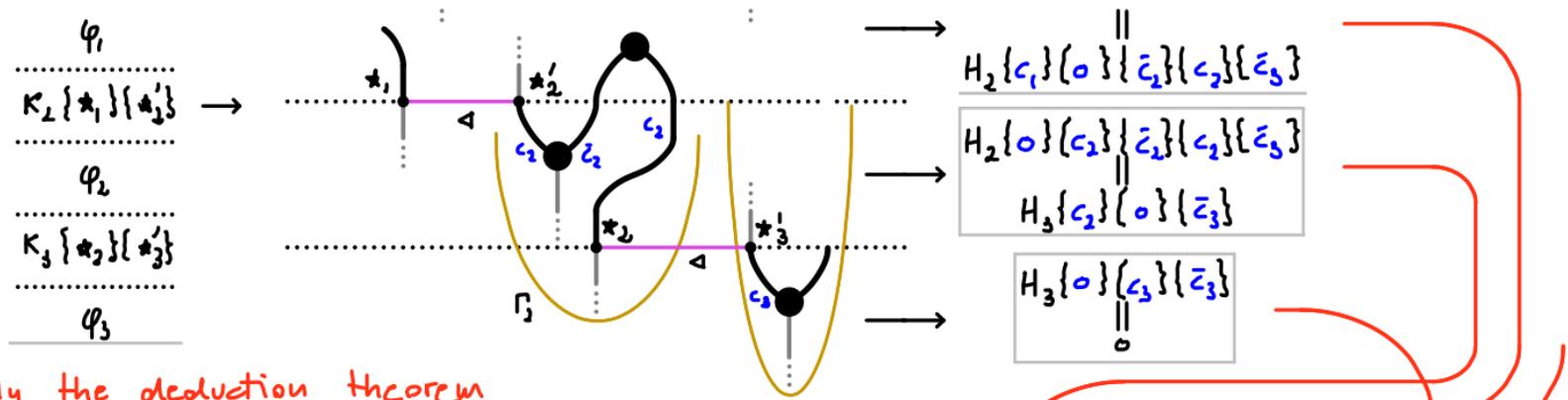
NO CYCLES AMONG * CONNECTED COMPONENTS



We apply the deduction theorem.



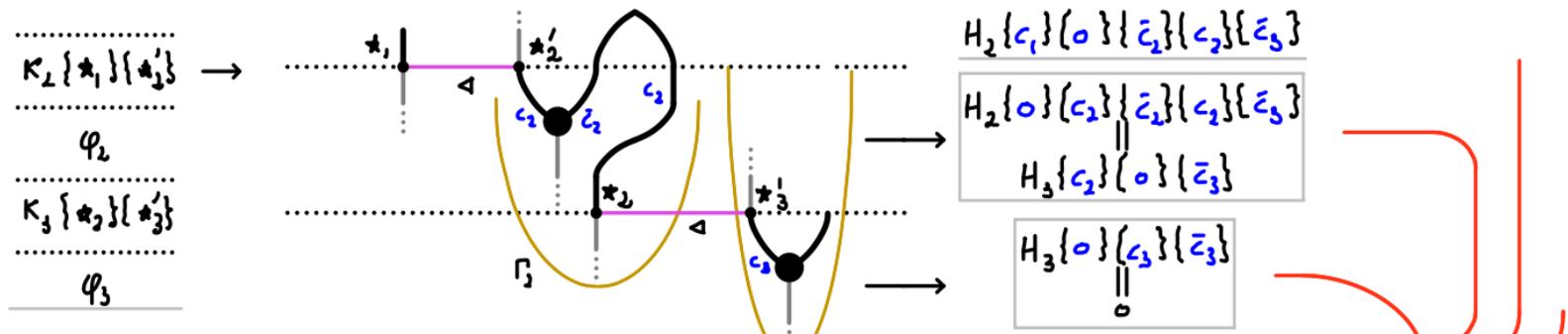
NO CYCLES AMONG \star CONNECTED COMPONENTS



We apply the deduction theorem.

$$\prod_{\circ} \bar{H}_1\{c_3\}\{\bar{c}_1\}\{c_1\}\{\bar{c}_3\} \otimes \prod_{\bar{H}_1\{0\}\{0\}\{\bar{c}_1\}\{c_3\}} \bar{H}_2\{c_1\}\{0\}\{\bar{c}_2\}\{c_2\}\{\bar{c}_3\} \otimes \prod_{\bar{H}_2\{0\}\{\bar{c}_2\}\{c_2\}\{\bar{c}_3\}} \bar{H}_3\{c_2\}\{0\}\{\bar{c}_3\} \otimes \prod_{\bar{H}_3\{0\}\{\bar{c}_3\}\{c_3\}} \circ$$

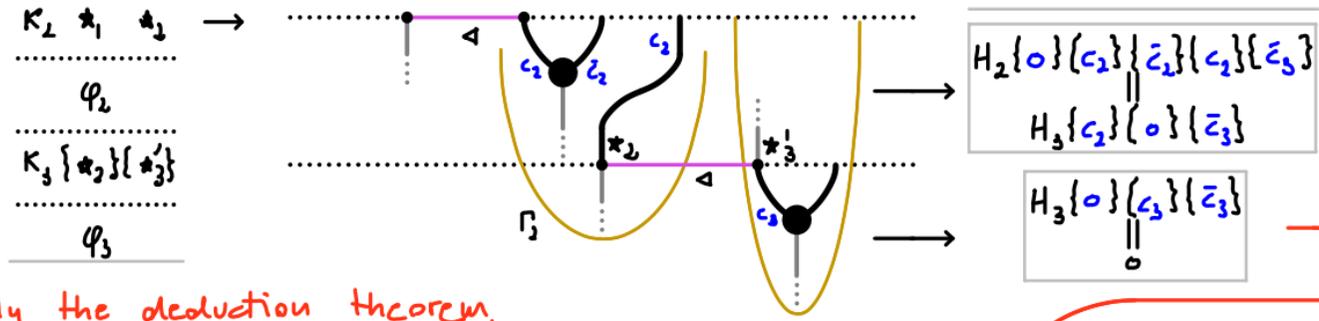
NO CYCLES AMONG \star CONNECTED COMPONENTS



We apply the deduction theorem.

$$\begin{array}{|c|} \hline \Pi \\ \hline \circ \nabla H_1 \{ c_3 \} \{ \bar{c}_1 \} \{ c_1 \} \{ \bar{c}_3 \} \\ \hline \end{array} \otimes \begin{array}{|c|} \hline \Pi \\ \hline \bar{H}_1 \{ 0 \} \{ 0 \} \{ \bar{c}_1 \} \{ c_3 \} \nabla H_2 \{ c_1 \} \{ 0 \} \{ \bar{c}_2 \} \{ c_2 \} \{ \bar{c}_3 \} \\ \hline \end{array} \otimes \begin{array}{|c|} \hline \Pi \\ \hline \bar{H}_2 \{ 0 \} \{ \bar{c}_2 \} \{ c_2 \} \{ \bar{c}_2 \} \{ c_3 \} \nabla H_3 \{ c_2 \} \{ 0 \} \{ \bar{c}_3 \} \\ \hline \end{array} \otimes \begin{array}{|c|} \hline \Pi \\ \hline \bar{H}_3 \{ 0 \} \{ \bar{c}_3 \} \{ c_3 \} \nabla \circ \\ \hline \end{array}$$

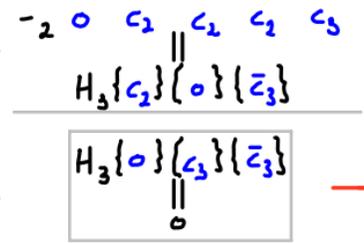
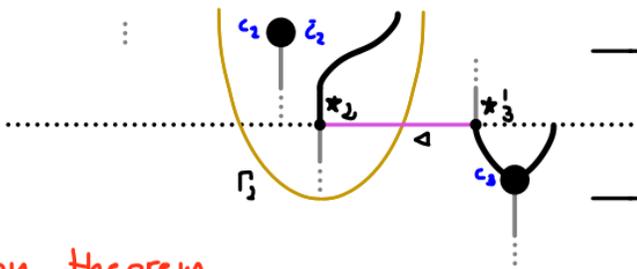
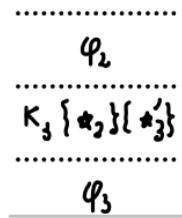
No Cycles Among \star Connected Components



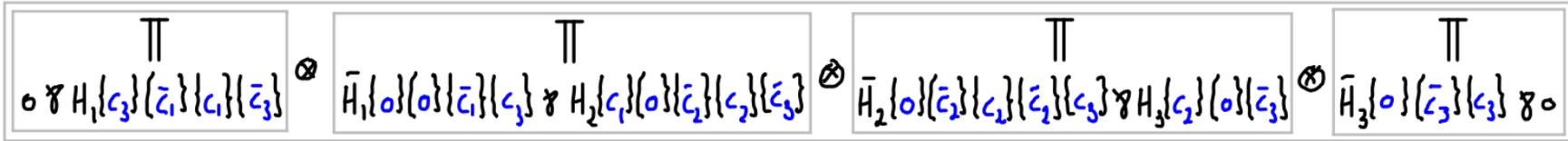
We apply the deduction theorem.

$$\begin{array}{|c|} \hline \Pi \\ \hline \circ \nabla H_1\{c_3\}\{\bar{c}_1\}\{c_1\}\{\bar{c}_3\} \\ \hline \end{array} \otimes \begin{array}{|c|} \hline \Pi \\ \hline \bar{H}_1\{0\}\{0\}\{\bar{c}_1\}\{c_3\} \nabla H_2\{c_1\}\{0\}\{\bar{c}_2\}\{c_2\}\{\bar{c}_3\} \\ \hline \end{array} \otimes \begin{array}{|c|} \hline \Pi \\ \hline \bar{H}_2\{0\}\{\bar{c}_2\}\{c_2\}\{\bar{c}_2\}\{c_3\} \nabla H_3\{c_2\}\{0\}\{\bar{c}_3\} \\ \hline \end{array} \otimes \begin{array}{|c|} \hline \Pi \\ \hline \bar{H}_3\{0\}\{\bar{c}_3\}\{c_3\} \nabla \circ \\ \hline \end{array}$$

NO CYCLES AMONG * CONNECTED COMPONENTS

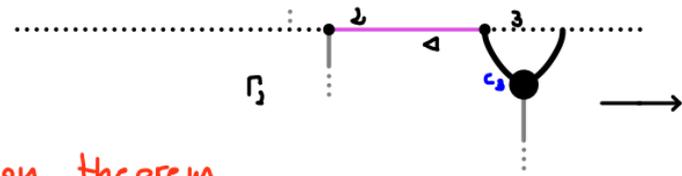


We apply the deduction theorem.



No Cycles Among * Connected Components

$$\frac{K_3 \{ *_2 \} \{ *'_3 \}}{\varphi_3}$$



$$\frac{H_3 \{ 0 \} \{ c_3 \} \{ \bar{c}_3 \}}{0}$$

We apply the deduction theorem.

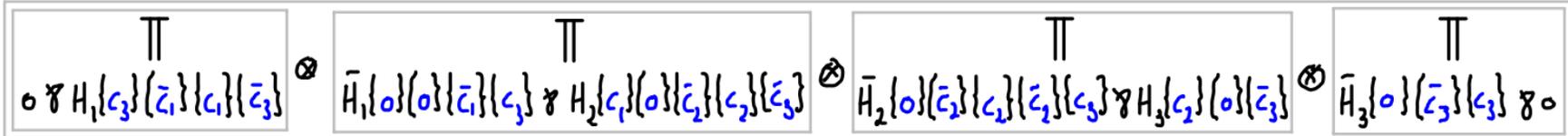
Four boxes representing logical derivations, connected by red arrows from the diagram above:

- Box 1: $\frac{\Pi}{0 \wp H_1 \{ c_3 \} \{ \bar{c}_1 \} \{ c_1 \} \{ \bar{c}_3 \}} \otimes$
- Box 2: $\frac{\Pi}{\bar{H}_1 \{ 0 \} \{ 0 \} \{ \bar{c}_1 \} \{ c_3 \} \wp H_2 \{ c_1 \} \{ 0 \} \{ \bar{c}_2 \} \{ c_2 \} \{ \bar{c}_3 \}} \otimes$
- Box 3: $\frac{\Pi}{\bar{H}_2 \{ 0 \} \{ \bar{c}_2 \} \{ c_2 \} \{ \bar{c}_2 \} \{ c_3 \} \wp H_3 \{ c_2 \} \{ 0 \} \{ \bar{c}_3 \}} \otimes$
- Box 4: $\frac{\Pi}{\bar{H}_3 \{ 0 \} \{ \bar{c}_3 \} \{ c_3 \} \wp 0}$

NO CYCLES AMONG * CONNECTED COMPONENTS



We apply the deduction theorem.



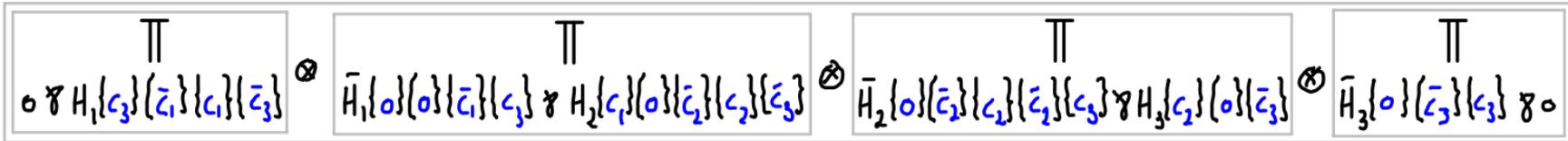
No Cycles Among \star Connected Components

_____ \downarrow

We apply the deduction theorem.

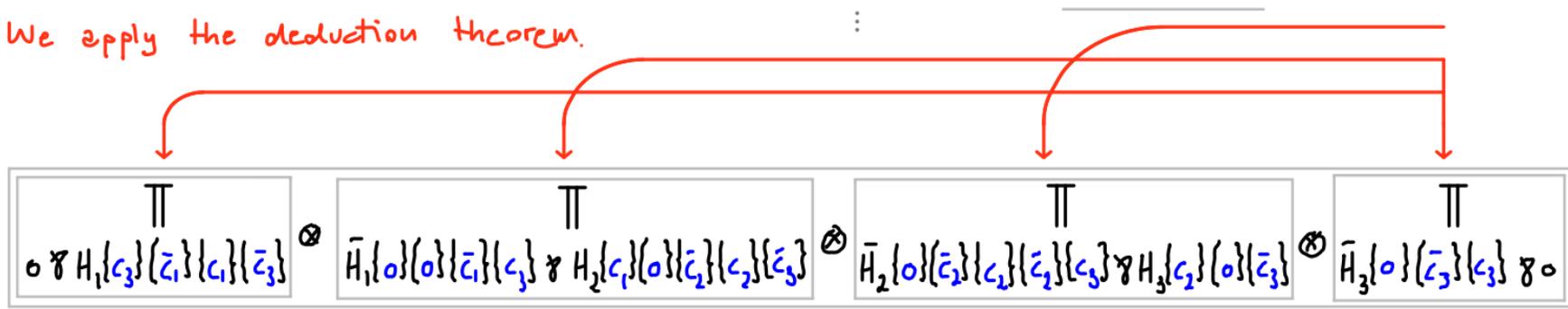
\vdots

_____ \circ

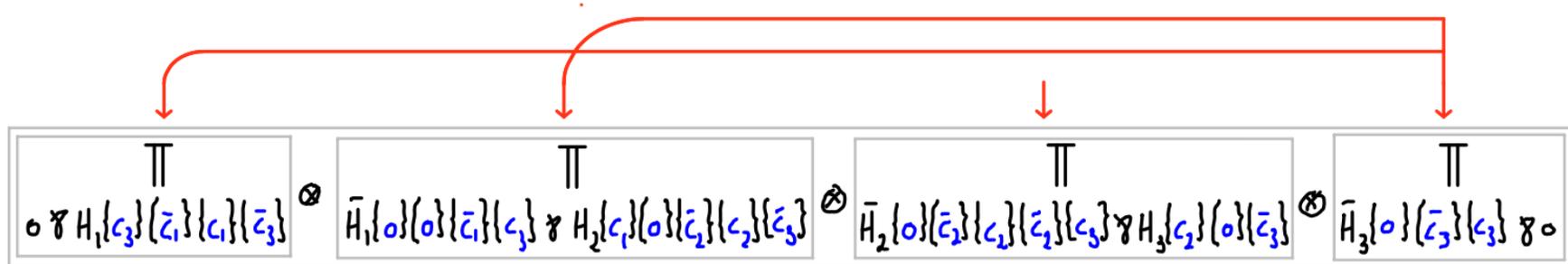


No Cycles Among \star Connected Components

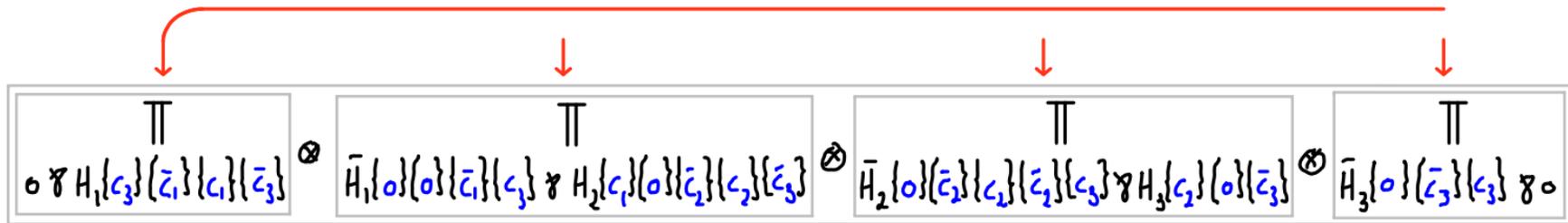
We apply the deduction theorem.



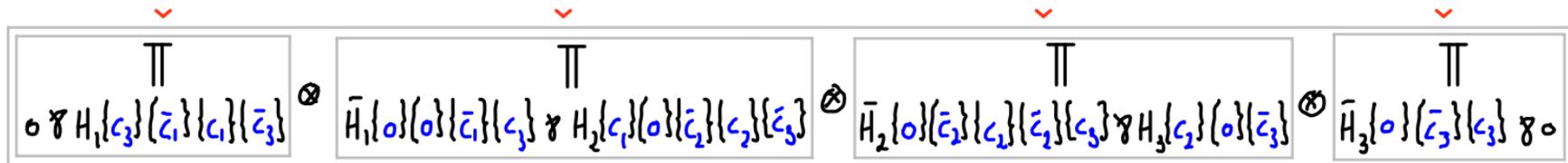
No Cycles Among \star Connected Components



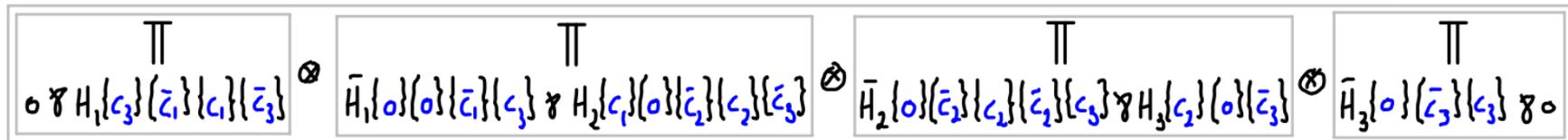
No Cycles Among \star Connected Components



No Cycles Among \star Connected Components



No Cycles Among \star Connected Components



NO CYCLES AMONG \star CONNECTED COMPONENTS

$$\begin{array}{|c|} \hline \prod \\ \hline \circ \not\star H_1\{c_3\}\{\bar{c}_1\}\{c_1\}\{\bar{c}_3\} \otimes \bar{H}_1\{o\}\{o\}\{\bar{c}_1\}\{c_3\} \not\star H_2\{c_1\}\{o\}\{\bar{c}_2\}\{c_2\}\{\bar{c}_3\} \otimes \bar{H}_2\{o\}\{\bar{c}_2\}\{c_2\}\{\bar{c}_1\}\{c_3\} \not\star H_3\{c_2\}\{o\}\{\bar{c}_3\} \otimes \bar{H}_3\{o\}\{\bar{c}_3\}\{c_3\} \not\star \circ \\ \hline \end{array}$$

||

$H_1\{c_3\}\{\bar{c}_1\}\{c_1\}\{\bar{c}_3\} \otimes \bar{H}_1\{o\}\{o\}\{\bar{c}_1\}\{c_3\}$ <p style="text-align: center;"> </p> <div style="border: 1px solid black; padding: 5px; margin: 5px auto; width: 80%;"> $H'_1\{c_3\}\{\bar{c}_1\} \left\{ \frac{c_1 \otimes \bar{c}_1}{o} \right\} \left\{ \frac{\bar{c}_3 \otimes c_3}{o} \right\}$ </div> <p style="text-align: center; color: red;">$c_3 \triangleleft \bar{c}_1$</p>	$H_2\{c_1\}\{o\}\{\bar{c}_2\}\{c_2\}\{\bar{c}_3\} \otimes \bar{H}_2\{o\}\{\bar{c}_2\}\{c_2\}\{\bar{c}_1\}\{c_3\}$ <p style="text-align: center;"> </p> <div style="border: 1px solid black; padding: 5px; margin: 5px auto; width: 80%;"> $H'_2\{c_1\}\{\bar{c}_2\} \left\{ \frac{\bar{c}_2 \otimes c_2}{o} \right\} \left\{ \frac{c_2 \otimes \bar{c}_2}{o} \right\} \left\{ \frac{\bar{c}_3 \otimes c_3}{o} \right\}$ </div> <p style="text-align: center; color: red;">$c_1 \triangleleft \bar{c}_2$</p>	$H_3\{c_2\}\{o\}\{\bar{c}_3\} \otimes \bar{H}_3\{o\}\{\bar{c}_3\}\{c_3\}$ <p style="text-align: center;"> </p> <div style="border: 1px solid black; padding: 5px; margin: 5px auto; width: 80%;"> $H'_3\{c_2\}\{\bar{c}_3\} \left\{ \frac{\bar{c}_3 \otimes c_3}{o} \right\}$ </div> <p style="text-align: center; color: red;">$c_2 \triangleleft \bar{c}_3$</p>
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This is a standard construction in SBV.

The conclusion is unprovable, as a direct consequence of cut elimination.

SUMMARY

SKV has been designed starting from a **natural** notion of normalisation:

Splitting

+

Decomposition

~ yanking

linear

no complexity

term-based

~ push contractions

structural

exponential

based on non-tree diagrams

Why natural? Inference rules have direct translations to **process algebras**.
The induction measures to control normalisation are **trivial**
(if you know where to look).

SKV is just an example: the methodology is **general**.

The **subatomic perspective** offers guidance and unification.

OPEN PROBLEMS

Is SKV Turing-complete?

Can we extend it?