Comparison of AC losses, magnetic field/current distributions and critical currents of superconducting circular pancake coils and infinitely long stacks using coated conductors

Weijia Yuan, A M Campbell, Z Hong, M D Ainslie and T A Coombs

Electronic, Power and Energy Conversion Group, Electrical Engineering Division, Engineering Department, University of Cambridge, Cambridge CB3 0FA, UK

E-mail: wy215@cam.ac.uk

Received 11 May 2010, in final form 14 June 2010
Published 12 July 2010
Online at stacks.iop.org/SUST/23/085011

Abstract
A model is presented for calculating the AC losses, magnetic field/current density distribution and critical currents of a circular superconducting pancake coil. The assumption is that the magnetic flux lines will lie parallel to the wide faces of tapes in the unpenetrated area of the coil. Instead of using an infinitely long stack to approximate the circular coil, this paper gives an exact circular coil model using elliptic integrals. A new efficient numerical method is introduced to yield more accurate and fast computation. The computation results are in good agreement with the assumptions. For a small value of the coil radius, there is an asymmetry along the coil radius direction. As the coil radius increases, this asymmetry will gradually decrease, and the AC losses and penetration depth will increase, but the critical current will decrease. We find that if the internal radius is equal to the winding thickness, the infinitely long stack approximation overestimates the loss by 10% and even if the internal radius is reduced to zero, the error is still only 60%. The infinitely long stack approximation is therefore adequate for most practical purposes. In addition, the comparison result shows that the infinitely long stack approximation saves computation time significantly.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Only in recent years have long-length coated conductors become commercially available. This makes it possible to wind pancake coils for large scale applications, e.g. superconducting magnetic energy storage (SMES), superconducting fault current limiter (SFCL), MRI, etc. Therefore an understanding of magnetic field and current density distribution, and the ability to predict AC losses and critical currents of the coated conductor coils are necessary before designing these devices [1–5].

This paper builds on work reported in [6–8] which introduce a relatively fast model to calculate the losses. However, in [6–8] an infinitely long stack of tapes was used to approximate the circular coils.

This paper presents an exact model of circular coils using elliptic integrals. The AC losses, penetration depths and critical currents of the coil with both different transport currents and different radii are calculated. A comparison is made between the results of the circular coil and the infinitely long stack.

2. The model

2.1. Assumptions of the model

In the previous model [7, 8], a circular coil was treated as an infinitely long stack of tapes. If the radius of the coil is much
larger than the thickness of the coil, this approximation would be accurate enough for engineering applications. Figure 1 shows how this approximation is made. These tapes are insulated from each other and are all carrying the same current $I$.

Although the infinitely long stack model is easy to compute, it’s not so accurate especially when the coil radius is comparable to the coil thickness. In this paper, we will treat the coil as an exact circular coil. Due to the symmetry, we only need to analyse the cross section of the coil. Figure 2 gives the cross section view of a circular coil. Each tape is carrying the same transport current. The width and thickness of each tape are $2a$ and $D$ respectively, the thickness of the coil is $2b$, the mean radius of the coil is $r$. We set the origin at the centre of the cross section.

Unlike a single tape, the magnetic field can penetrate into the middle of the stack. However, the flux lines still cannot penetrate each individual tape, thus they have to lie parallel to the wide surface of each tape in the coil [6]. This result is also achieved by another model [9]. Therefore we can assume there are two regions in the coil: penetrated and unpenetrated regions. In the penetrated region, the flux lines can penetrate in every direction. A critical current density subject to the local magnetic field by the Kim model will flow there [10]. In the unpenetrated region, the flux lines can only lie parallel to the wide surface of the tapes. A uniform current density will flow there in each tape since the magnetic field only has one component. Figure 3 shows the two regions in the coil. In the penetrated area, the current density is the critical current density $J_c$; in the unpenetrated area, the current density is $J_m$, the magnetic field is parallel to the tapes. Due to the symmetry along the $x$-axis we assume the red curves representing critical boundaries can be described by two parabolae,

$$x(z) = \pm \left( \frac{c_1 + c_2 - c_3}{2b} z + \frac{(c_2 + c_3)/2 - c_1^2}{b^2} \right)$$

(1)

where $c_1$ is the $x$-coordinate value of the intersection point of the right parabola with the $x$-axis, $c_2$ and $c_3$ are the $x$-coordinate values of the intersection points of the right parabola with the top and bottom boundaries of the cross section area.

Usually the vertices of the parabolae will not lie on the $x$-axis, also $c_2$ is not the same with $c_3$ either, since there is an asymmetry along the $x$-axis; however, when the radius is much larger than the thickness of the coil, it will behave like an infinitely long stack of tapes, the vertices will then lie on the $x$-axis, and $c_2$ will be equal to $c_3$.

We need to point out that there are other more complicated functions which can describe the critical boundary more accurately. However, a parabola is the simplest 3-parameter fit to describe the critical boundary, thus to gain an acceptable computation speed we use the parabola in this paper.

2.2. The solution methodology

2.2.1. Numerical division of the coil. This problem has to be solved numerically, thus we need to divide the cross section into small blocks, each of which carries a different uniform current density. In the previous paper [7], we made this division dynamically by changing the subdivision for each parabola as the solution progressed. Although this dynamic division copes with extreme situations very well, i.e. when the penetration depth is too small or too large, generally its accuracy and computation speed is not as good as the following division method. In this paper we use a fixed division method. Since the locations of the blocks are fixed, we can produce a look-up table for every computation which will save a lot of time, hence we can divide the cross section into smaller blocks and gain computation speed.
2.2.2. The interrelationship of the current density \( J_y \) and the detailed division example is given in figure 4 with \( M \) this region. However it still has point \((x_0, z_0)\) along the \( z \)-axis, with a current lines parallel to the \( x \)-axis. We regard the blocks intersected by the curves as the critical state boundary. The inner part (including the blocks intersected by the red dashed curves) is the unpenetrated region, which is considered as one block along the \( x \)-axis since the current density is uniform in this region. However it still has \( M \) blocks along the \( z \)-axis, thus it has a current density as \( J_m(z) \). The shaded area shows the inner blocks. The outer part is the critical region with the critical current density \( J_c(x, z) \) varying with each block. A detailed division example is given in figure 4 with \( M = 8 \) and \( N = 20 \).

Figure 4. Division of the cross section of the circular coil.

Firstly we divide the cross section into \( M \) equal sections along the \( z \)-axis by drawing \( M - 1 \) equally spaced straight lines parallel to the \( x \)-axis. Secondly we divide the region into \( N \) equal sections along the \( z \)-axis by drawing \( N - 1 \) equally spaced straight lines parallel to the \( z \)-axis. We regard the blocks intersected by the curves as the critical state boundary. The inner part (including the blocks intersected by the red dashed curves) is the unpenetrated region, which is considered as one block along the \( x \)-axis since the current density is uniform in this region. However it still has \( M \) blocks along the \( z \)-axis, thus it has a current density as \( J_m(z) \). The shaded area shows the inner blocks. The outer part is the critical region with the critical current density \( J_c(x, z) \) varying with each block. A detailed division example is given in figure 4 with \( M = 8 \) and \( N = 20 \).

\section*{2.2.2. The interrelationship of the current density \( J_y \) and the magnetic field \( B_z \).}

We consider a ring located at \((x, z)\) as shown in figure 5, with a current \( I \) flowing in it. The vector potential and radial component of the magnetic field at the point \((x_0, z_0)\) due to the ring are [11],

\[ A_y = \frac{\mu_0 I}{\pi k} \left( \frac{a}{r_1} \right)^2 \left[ \left( 1 - \frac{1}{2} k^2 \right) K - E \right] \]  \( (2) \)

\[ B_z = \frac{\mu_0 I}{2 \pi r_1} \left( \frac{d}{d^2 + (a + r_1)^2} \right)^2 \left[ -K + \frac{a^2 + r_1^2 + d^2}{(a - r_1^2)^2 + d^2} E \right] \]  \( (3) \)

\[ k^2 = 4 a r_1 d^2 + (a + r_1)^2 \]  \( (4) \)

where \( K \) and \( E \) are complete elliptic integrals of the first and second kind.

A block in figure 4 carrying a current density \( J_y \) within the area \( x_1 < x < x_2, z_1 < z < z_2 \) produces a vector potential and \( z \) direction magnetic field at the point \((x_0, z_0)\) as in (6) and (7),

\[ A_y(x_1, x_2, z_1, z_2) = \int_{x_1}^{x_2} \int_{z_1}^{z_2} A_y(x, z) \, dx \, dz \]  \( (6) \)

\[ B_z(x_1, x_2, z_1, z_2) = \int_{x_1}^{x_2} \int_{z_1}^{z_2} B_z(x, z) \, dx \, dz \]  \( (7) \)

where \( A_y \) and \( B_z \) are given by (2)–(5), in which \( I \) is replaced by \( J_y \).

By summing the contributions from each block we can find the total \( A_y \) and \( B_z \) at every point of the cross section. Unlike the infinitely long stack model, \( A_y \) in (2) is zero on the axis of the coil for all rings in the pancake. Therefore we do not need to subtract the contribution of \( A_y \) at \((0, 0)\) from each block.

To save computation time a grid of \( A_y(x_1, x_2, z_1, z_2) \) and \( B_z(x_1, x_2, z_1, z_2) \) values of each block in section 2.2.1 for unit current density was calculated and a look-up table was used to find the values for each computation.

To make \( J_y \) consistent with \( B \) in the penetrated region, we use the Kim expression. Since for coated conductors the magnetic field perpendicular to the wide face has a much larger impact on the critical current density than the parallel magnetic field, we neglect the parallel field when calculating \( J_y \), therefore we have (8) below,

\[ J_y(x, z) = J_0 \frac{B_0}{B_0 + |B_z(x, z)|} \]  \( (8) \)

To make sure each tape carries the same transport current \( I \), the current density in the unpenetrated region needs to be,

\[ J_m(z) = \frac{I / D - \int_{c(z)}^{a} J_0 \frac{B_0}{B_0 + |B_z(x, z)|} \, dx}{2c(z)} \]  \( (9) \)

Combining (6)–(9), we can get an equation set as follows,

\[ J_y = f(B_z) = \begin{cases} J_y(x, z) = J_0 \frac{B_0}{B_0 + |B_z(x, z)|} & \text{when } c(z) < |x| < a \\ J_m(z) = \frac{I / D - \int_{c(z)}^{a} J_0 \frac{B_0}{B_0 + |B_z(x, z)|} \, dx}{2c(z)} & \text{when } |x| < c(z) \end{cases} \]  \( (10) \)

\[ B_z = f(J_y) = \sum \sum B_z(J_y) \]  \( (11) \)
If we know the expression for the parabolae describing the critical boundaries as (1), we can put it into the equation set (10) and (11), and then solve them. Therefore we can get the distribution of \( J_y \), \( A_y \) and \( B_z \) at every point of the cross section from a current \( I \) and a given set of \( c_1, c_2 \) and \( c_3 \).

As the flux lines are parallel to the \( x \)-axis in the unpenetrated region, in theory \( B_z \) will be zero there. Thus we define an objective function \( f_c = 2 \int_0^c \int_{-b}^b B_z^2 \, dx \, dz \). For a given transport current \( I \), we vary \( c_1, c_2 \) and \( c_3 \) to search for the minimum \( f_c \) to make it approximately zero. This gives the nearest solution to the ideal which has \( B_z = 0 \) at all points in the unpenetrated region. Having found \( c_1, c_2 \) and \( c_3 \), we can get the distribution of \( J_y, A_y \) and \( B_z \) in the circular coil.

To get the AC losses of a coil on applying a transport current, we firstly need to get the current and perpendicular magnetic field distribution at the peak \( I \). The AC losses per cross section area on charging the coil from the virgin state to the first peak \( I \) are [6, 12],

\[
Q_0 = 2 \int_{c(a)}^a \int_{-b}^b J_{\text{peak}} |B_{\text{peak}}| (a - x) \, dx \, dz. \tag{12}
\]

For an infinitely long stack with constant \( J_c \) the AC losses of a full cycle is \( Q = 4Q_0 \). In [8] it was calculated that \( Q \) is about 7% less than the full cycle AC loss due to the dependence of \( J_c \) on \( B_z \), and computing the complete cycle adds significantly to the complexity. Therefore we will use this approximation in this paper to compare the infinitely long stack and circular coil.

\( Q \) is the AC losses of the cross section area during a full cycle, if we multiply it by the mean circumference of the circular coil, we can get the AC losses of the whole coil.

Equation (12) gives a very convenient method to calculate the AC losses of a full cycle since it only requires the solution at the peak current \( I \). For most engineering applications, 7% is acceptable and (12) can give a much quicker estimate compared to finding solutions for a full cycle.

### 3. The calculation result

#### 3.1. The configuration of the pancake coil

We are using a pancake coil wound with Superpower’s SCS12050 tapes. This tape has a very good performance in field. The width of the tape is 12 mm, the thickness of the tape including insulation and epoxy is 0.15 mm, the critical current at self-field, 77 K is 240 A. This coil will be operating at 22 K. From Superpower’s data we can get the analytical expression of the critical current density in field as the following (only the perpendicular magnetic field is considered),

\[
J_c = \frac{J_0}{B_0 + |B_z|} \tag{13}
\]

where \( J_0 = 7.69 \times 10^8 \) A m\(^{-2}\), \( B_0 = 1.58 \) T.

This coil is wound with a long continuous superconducting tape, and will carry a 600 A (peak value) AC transport current. The detailed configuration is listed in table 1 and figure 6. Also the half tape width \( a \), tape thickness \( D \), mean radius of the coil \( r \), and half width of the coil \( b \) are marked in figure 6.

#### 3.2. Comparison of an infinitely long stack and a circular coil with a transport AC current

To gain an accurate solution with an acceptable computation speed, we divide the coil into blocks with \( M = 10 \) and \( N = 200 \). We should notice that this division is much finer than the dynamic division in [7]. We will use this new fixed division method in the following computations. The coil configuration in table 1 and figure 6 was used in the computation. This a small circular coil with \( r = 1.2b \) (when \( r = b \) there is no central core so this is quite a small radius coil). For comparison we also computed an infinitely long stack, in which the cross section area and tapes are the same as the circular coils.

The comparison figures are given in figures 7–14. From figures 7 to 10 we can find the flux lines are parallel to the tape wide face in the unpenetrated area. The perpendicular magnetic field is nearly zero in the unpenetrated area as shown in figures 11–14. These results are in good agreement with the model assumptions.

Figures 7 and 9 present the asymmetry effect of the circular coil along the \( x \)-axis with \( c_2 \) smaller than \( c_3 \). In comparison \( c_2 \) is equal to \( c_3 \) in the infinitely long stack as shown in figures 8 and 10 since it is symmetric along the \( x \)-axis. From figure 7 it is clear that when the radius \( r \) is small, there will be more flux lines in the inner half than the outer half of the coil, while in an infinitely long stack the flux lines are symmetric in each half. Due to the cancelling effect of the other half of the circular coil which has not been depicted in the figures, \( B_z \) will also be smaller than that in the same place.
of the infinitely long stack as shown by figures 9–12. Thus the penetration depth in the circular coil will be smaller than the infinitely long stack.

We need to point out that since in the circular coil model the zero of the vector potential is along the coil axis, however in the infinitely long stack we define the centre as the origin, the values of the contour of vector potentials are very different between figures 7 and 8.

The AC losses of the circular coil on the cross section area in a full cycle would be 590.3 J/cycle/m. The AC losses of the infinitely long stack on the cross section area in a full cycle is 843.3 J/cycle/m. Table 2 gives the comparison results of the AC losses and computation time. The $z$-component magnetic field in the coil will partly be compensated by the other half of the coil, so the modulus of $B_z$ will be smaller than that in an infinitely long stack, the AC loss will thus be smaller. The program was running in a PC with Intel Core2 Duo 2.00 GHz and 3 GB memory. We can find that the AC losses of the circular coil is 70% but the computation time is more than three times of those of an infinitely long stack.

### Table 2. Comparison of the AC losses and computation time between the circular coil and infinitely long stack.

<table>
<thead>
<tr>
<th></th>
<th>AC losses (J/cycle/m)</th>
<th>Computation time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circular coil</td>
<td>590.3</td>
<td>68.8</td>
</tr>
<tr>
<td>Infinitely long stack</td>
<td>843.3</td>
<td>20.5</td>
</tr>
</tbody>
</table>

#### 3.3. Comparison of the circular coil and infinitely long stack with different transport currents

Now we apply transport AC currents with different peak values into the coil, find solutions individually, and observe how the penetration depth and AC losses will change with the transport current. We define $\lambda = \frac{1}{3} \left( |a-c_1| + |a-c_2| + |a-c_3| \right)$ as the penetration depth. Again we use $M = 10$ and $N = 200$ to divide the cross section area.

For comparison we also computed solutions for the infinitely long stack model, in which the cross section area and
Figure 11. Contour of $B_z$ in the coil when $r = 1.2b$.

Figure 12. Contour of $B_z$ in the infinitely long stack.

Figure 13. $J_y$ at different levels in the coil when $r = 1.2b$.

Figure 14. $J_y$ at different levels in the infinitely long stack.

tapes are the same as the circular coils. Figures 15 and 16 present the results:

We find that the AC losses increase approximately with the cube of the current $I$, the penetration depth increases from 0 to nearly 1 as the transport current increases from 0 to 700 A. We can also find both for AC losses and penetration depth, the circular coil model gives smaller values than the infinitely long stack model. This is already explained in section 3.2.

3.4 Effect of the radius of the coil

Although normally there is an asymmetry along the $z$-axis of the circular model, for a relatively large value of the radius $r$ the coil will rather behave like an infinitely long stack. We now keep all the configurations the same as in section 3.1, except for changing the radius $r$ from 1.2b to 100b. The computation result shows that $c_1$, $c_2$ and $c_3$ are 1.9 mm, 2.8 mm and 2.8 mm, respectively. The AC loss is 843.3 J/cycle/m. For an infinitely long stack with the same configuration and transport current, $c_1$, $c_2$ and $c_3$ will also be 1.9 mm, 2.8 mm and 2.8 mm, and the AC loss will be 843.3 J/cycle/m. Thus when $r$ is very large, we can use the relatively simple model of an infinitely long stack to approximate circular pancake coils.

To understand how the radius of the coil affects the AC losses, penetration depths and critical currents (in [7] it was defined that the critical current would be the minimum transport current when the critical regions penetrate into the centre of one of the tapes), we vary $r$ but keep the cross section and transport current the same as in section 3.1. Obviously the minimum $r$ is 1, in which case there is no central core. The normalized computation results are given in figure 17, figures 18 and 19. Also we produced a look-up table for the computation of AC losses as in table 3.

From figure 17 we can find that the AC losses of the circular coil will be approximately the same as an infinitely
Table 3. Look-up table of correction factors of the coil AC losses at different radii. (Note: Correction factor = the AC losses divided by the AC losses of the infinitely long stack.)

<table>
<thead>
<tr>
<th>Coil size ((r/b))</th>
<th>Correction factor</th>
<th>Coil size ((r/b))</th>
<th>Correction factor</th>
<th>Coil size ((r/b))</th>
<th>Correction factor</th>
<th>Coil size ((r/b))</th>
<th>Correction factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.60</td>
<td>1.7</td>
<td>0.84</td>
<td>2.4</td>
<td>0.91</td>
<td>4</td>
<td>0.96</td>
</tr>
<tr>
<td>1.1</td>
<td>0.66</td>
<td>1.8</td>
<td>0.85</td>
<td>2.5</td>
<td>0.91</td>
<td>5</td>
<td>0.97</td>
</tr>
<tr>
<td>1.2</td>
<td>0.70</td>
<td>1.9</td>
<td>0.87</td>
<td>2.6</td>
<td>0.92</td>
<td>6</td>
<td>0.98</td>
</tr>
<tr>
<td>1.3</td>
<td>0.74</td>
<td>2.0</td>
<td>0.88</td>
<td>2.7</td>
<td>0.93</td>
<td>7</td>
<td>0.99</td>
</tr>
<tr>
<td>1.4</td>
<td>0.77</td>
<td>2.1</td>
<td>0.89</td>
<td>2.8</td>
<td>0.93</td>
<td>8</td>
<td>0.99</td>
</tr>
<tr>
<td>1.5</td>
<td>0.80</td>
<td>2.2</td>
<td>0.90</td>
<td>2.9</td>
<td>0.94</td>
<td>9</td>
<td>0.99</td>
</tr>
<tr>
<td>1.6</td>
<td>0.82</td>
<td>2.3</td>
<td>0.90</td>
<td>3.0</td>
<td>0.94</td>
<td>10</td>
<td>0.99</td>
</tr>
</tbody>
</table>

Figure 15. Comparison of the AC losses of the coil/stack with different currents.

Figure 16. Comparison of the penetration depth of the coil/stack with different currents.

Figure 17. AC losses of the coil with different radii normalized to those of an infinitely long stack.

Figure 18. Penetration depth of the coil with different radii normalized to those of an infinitely long stack.

When the radius is relatively small, the z-component magnetic field will partly be compensated by the other half of the coil, so the modulus of \(B_z\) will be smaller than that in an infinitely long stack, the AC loss and penetration depth will thus be reduced. As the radius increases, this impact will decrease until the circular coil behaves like an infinitely long stack. However, the critical current is larger than those of an infinitely long stack as shown in figure 19. Even quite small coils can be treated as infinitely long stacks, the error is 10% when \(r = 3b\). And this approximation will save computation time significantly.
4. Discussion and summary

Starting from the original infinitely long stack model, we adapted it for an exact circular coil model. By changing the dynamic divisions to the fixed divisions, we gained the computation speed and accuracy in all but very small and very large penetrations.

The computation results are in good agreement with the assumptions. In the penetrated region flows the critical current density which is given by the Kim expression (13) at the local perpendicular magnetic field $B_z$. In the unpenetrated region, the magnetic flux lines are parallel to the wide face of tapes and the perpendicular magnetic field is approximately zero. A smaller uniform current density flows in each tape there.

We should point out that although we did not divide the cross section area into the real number of tapes along the $x$-axis, each tape is still carrying the same current due to (9). The critical boundaries are described by two parabolae which lack symmetry along the $x$-axis.

For a small value of the radius $r$, the effect of the other half of the coil will reduce the perpendicular field and thus the AC losses and penetration depth, and the critical current will increase. Also there is an obvious asymmetry along the $z$-axis. $B_z$ in the outer half of the coil will be less than the inner half. As $r$ increases, this effect will gradually become negligible. If $r$ is larger than 10$b$, the coil will behave like an infinitely long stack, and the parabolae will become symmetric along the $z$-axis. Our computation results of the AC losses, penetration depth and critical currents confirmed this result.

In conclusion, the infinitely long stack approximation for a circular pancake coil is therefore adequate for most practical purposes since the error is small and the computation time is saved significantly. We find that if the internal radius is equal to the winding thickness the infinitely long stack approximation overestimates the AC losses by 10% and even if the internal radius is reduced to zero, the error is still only 60%. In addition, the computation time for calculating the infinitely long stack is less than one third of that for a circular pancake coil.

Acknowledgments

The authors thank Mark Husband from Rolls Royce for his great help and support through this work. The authors also thank Yingsong Zhang and Kai Yu from the Engineering Department for their help in numerical methods.

References