THE STATISTICAL SCALING HYPOTHESIS IN THE LANDAU-DE GENNES Q-TENSOR THEORY OF NEMATIC LIQUID CRYSTALS

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Abstract. In this talk I shall present an analysis of a parabolic partial differential equation taken from physics, and shall ask questions about the long-time behaviour of associated *correlation functions*. We shall work with a macroscopic continuum theory of thermotropic nematic liquid crystals, in which the liquid crystal structure is captured by the Q-tensor order parameter – a theory proposed by the physicist Pierre-Gilles de Gennes. The model we employ is the *Q-tensor equation*, given to be

$$\partial_t Q = \Delta Q - a\,Q + b\left(Q^2 - \frac{1}{3}\mathrm{tr}(Q^2)I\right) - c\,\mathrm{tr}(Q^2)Q,$$

where Q is a map into the space of traceless and symmetric 3×3 real matrices $\mathcal{Q}_0$, the Q-tensor order parameter manifold. When studying the evolution of a liquid crystal profile, one often analyses the behaviour of the correlation function C associated with the system. In our work, we shall define the correlation function C to be

$$C(r,t) := \frac{\int_H \int_{\mathbb{R}^3} \mathrm{tr}\left(Q(x+r,t)Q(x,t)\right)\, dx d\mu_t}{\int_H \int_{\mathbb{R}^3} \mathrm{tr}\left(Q(x,t)Q(x,t)\right)\, dx d\mu_t},$$

where $\{\mu_t\}_{t\geq 0}$ is an appropriate one-parameter family of Borel measures on the phase space H. Physicists claim that for a large class of systems, following a temperature quench, the correlation function assumes for large time a self-similar form i.e. $C(r,t) \sim f(r/L(t))$ as $t \to \infty$, a claim that we shall call the **statistical scaling hypothesis**. The exact form of the characteristic length scale $L(t)$ for nematic liquid crystal systems is of some debate in the physics literature. I shall present a result which confirms that, if the initial data of the Q-tensor equation are small enough and decay sufficiently quickly at spatial infinity, then the correlation function indeed assumes asymptotically a self-similar form and $L(t) = t^{1/2}$.

This is joint work with Eduard Kirr and Arghir Zarnescu.