Solutions to Exercise Sheet 3

1. The difference quotient is

$$\frac{f(x+h)-f(x)}{h}=\frac{(x+h)^3-x^3}{h}=\frac{x^3+3x^2h+3xh^2+h^3-x^3}{h}=3x^2+3xh+h^2.$$

For this expression, the limit $h\to 0$ can be found by simply setting $h=0$ and thus

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=3x^2.$$

2. Divide the numerator and the denominator by x^3. This gives

$$\lim_{x\to\infty}\frac{2x^3-x+7}{x^3+x^2+100}=\lim_{x\to\infty}\frac{2-\frac{1}{x^2}+\frac{7}{x^3}}{1+\frac{1}{x}+\frac{100}{x^3}}=2.$$

3. (a) We can calculate this as follows:

$$\lim_{x\to\infty}\frac{x^2+6x-7}{3x^2+10}=\lim_{x\to\infty}\frac{1+\frac{6}{x}-\frac{7}{x^2}}{3+\frac{10}{x^2}}=\frac{1}{3}.$$

(b) The exponential function and the sine are continuous. Therefore,

$$\lim_{x\to 2}\sin\left(e^{2-x}\pi\right)=\sin\left(\lim_{x\to 2}e^{2-x}\pi\right)=\sin\left(e^0\pi\right)=0.$$

4. Using the bisection method yields the following table:

x	x^2	interval
1	1	
2	4	[1,2]
1.5	2.25	[1.5,2]
1.75	3.0625	[1.5,1.75]
1.625	2.6406	[1.625,1.75]
1.6875	2.8477	[1.6875,1.75]
1.71875	2.9541	[1.71875,1.75]
1.734375	3.0081	[1.71875,1.734375]
1.7265625	2.9810	[1.7265625,1.734375]

Since both the lower and upper limit of the final interval round to 1.73, we know that $\sqrt{3}\approx 1.73$ to 3 significant figures. In other words, we can be sure that $1.725<\sqrt{3}<1.735$, because $(1.725)^2<3<(1.735)^2$.

5. (a) By the product rule and the sum rule,

$$\frac{dy}{dx} = 6x^5(\sin x + \cos x) + x^6(\cos x - \sin x).$$

(b) Recall that $\cot x = \frac{\cos x}{\sin x}$. By the quotient rule,

$$\frac{dy}{dx} = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = -1 - \cot^2 x.$$

(An alternative way to represent the result is $-\frac{1}{\sin^2 x}$.)

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