Solutions to Exercise Sheet 1

1. Add 8 and then divide by 4. This gives $x = 3$.

2. By the definition of $\log_5$, the solution is $x = \log_5 2$.

3. Factorising, this can be rewritten as $(x+1)(x-4) = 0$, so the solutions are $x = -1$ and $x = 4$.

4. Using exponential rules, this can be rewritten as $e^{4x} = 2$. Solve by taking the natural logarithm of both sides to get $4x = \ln 2$. Thus $x = \frac{1}{4}\ln 2$.

5. Using the substitution $u = \ln x$ gives $u^2 + 2u - 3 = 0$. Factorisation gives $(u+3)(u-1) = 0$, with solutions $u = -3$ and $u = 1$. Thus the solutions in the original variable x satisfy $\ln x = -3$ and $\ln x = 1$, respectively. So the solutions are $x = e^{-3}$ and $x = e$.

6. Using the logarithm rule, the equation becomes
$$\ln(x+1) + \ln(x-2) = \ln[(x+1)(x-2)] = 0.$$
Taking exponentials of both sides gives
$$e^{\ln[(x+1)(x-2)]} = e^0.$$
That is,
$$(x+1)(x-2) = 1.$$
Expanding and rearranging gives the quadratic equation $x^2 - x - 3 = 0$, which has solutions (given by the quadratic formula) $x = \frac{1\pm\sqrt{13}}{2}$.
However, note that in the original equation, we must have $x > 2$ for $\ln(x-2)$ to make sense. But $\frac{1-\sqrt{13}}{2} < 0$, so the only solution is $x = \frac{1+\sqrt{13}}{2}$. Note that $\frac{1+\sqrt{13}}{2} > \frac{1+\sqrt{9}}{2} = 2$.

RM, 14/09/2017