Exercise Sheet 5

Hand in your work by 6 November.

1. (Warm-up question*) The limits

$$\lim_{h\to 0}\frac{\sin h}{h} \quad \text{and} \quad \lim_{h\to 0}\frac{\cos h - 1}{h}$$

were given in the lectures. Check that l'Hopital's rule will give the same result.

2. (Warm-up question*) Consider the linear function $\ell(x) = x + 1$. Check that the Maclaurin polynomial (Taylor polynomial at 0) of ℓ of order 1 or higher coincides with ℓ.

3. Find

$$\lim_{x\to 0}\frac{\cos x - 1}{2x^2}.$$

4. Use Newton's method to find an approximation to $\sqrt{3}$ starting from $x_0 = 2$. Work to 6 decimal places and stop when the first 5 places don't change.

5. Calculate the Taylor polynomial $T_{3,1}(x)$ of order 3, about $x = 1$, for the function

$$f(x) = x^3 - x.$$

Give your answer in the form

$$T_{3,1}(x) = a_0 + a_1(x-1) + a_2(x-1)^2 + a_3(x-1)^3.$$

Hint: in this case, you can actually check your answer by expanding the terms.

6. Calculate the Taylor polynomial of order 4 at $x = 0$ for the function

$$f(x) = \frac{2\cos(2x)}{1+x^2}.$$

Hint: use the standard series for $\cos x$ and $\frac{1}{1-x}$.

Solutions will be available after the hand-in date at:
`http://people.bath.ac.uk/rm257/MA10192/`

RM, 23/10/2017

*Do not hand in your work for this question.