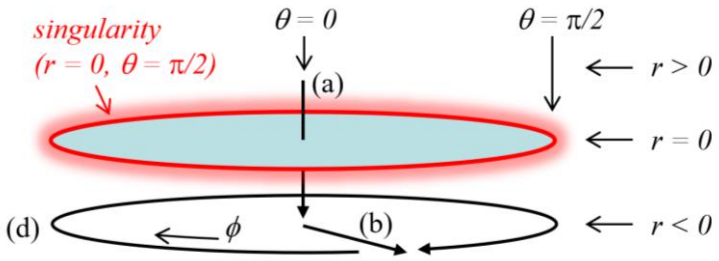
PH30101 – GENERAL RELATIVITY

Optional extra question

Question D10 *closed timelike curves (L17) - this question is quite spectacularly optional!*

NB this question is not about inertial motion: the geodesic equation need not be obeyed, only the requirement that particle worldlines must be time-like ($ds^2 < 0$). Review B4 and D8 for what happens at $r = 0$ in Kerr spacetime.



(a) As discussed in the lectures, the central singularity of a Kerr black holes comprises events with $\rho^2 = 0$. Show that a particle (or an intrepid space traveller we shall identify as "you") on a polar ($\theta = 0$) trajectory can cross $r = 0$ from $r > 0$ to $r < 0$ without encountering the singularity. Congratulations: you have reached $r < 0$ (whatever that means) and $\theta = 0$.

(b) Write down the Kerr metric for worldlines where r and ϕ are fixed (ie, $dr = d\phi = 0$). Show that g_{tt} is guaranteed to be negative if $r < 0$. Hence show that it is possible for you to travel from $r < 0$, $\theta = 0$ to the same $r < 0$ with $\theta = \pi/2$, provided $|d\theta/dt|$ is small enough. You have now (slowly) reached $r < 0$ and $\theta = \pi/2$.

(c) Write down $g_{\phi\phi}$ for $\theta = \pi/2$, and carefully sketch it as a function of r either side of $r = 0$. To do so, consider the behaviour as r grows increasingly positive; as r approaches $r = 0$ from either side; as r grows increasingly negative; and whether the curve is smooth away from $r = 0$. Hence show that there is a range of small negative values of r where $g_{\phi\phi}$ is negative.

(d) Starting from $\theta = \pi/2$, with $r =$ one of the small negative values where $g_{\phi\phi}$ is negative, show that you can travel from your starting value of ϕ to a value of ϕ that is 2π different, while all three of t, r and θ are fixed. (Whether ϕ increases or decreases by 2π will depend on which way your future light cone points, which we need not consider here.)

(e) Since ϕ is periodic with a period of 2π, and t, r and θ did not change, consider the two events where/when you started and ended your journey in part (d). Who will you meet there? What if you murdered them, or (more self-interestedly) persuaded them to go and do something else in exchange for cake? Do you think physics can allow this to happen?