Resonant radiation and collapse of ultrashort pulses in planar waveguides

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We study the spatiotemporal dynamics of ultrashort pulses close to a point of zero group-velocity dispersion in planar waveguides with focusing nonlinearity. We find that the process of pulse collapse enhances the emission of so-called resonant radiation, providing an efficient mechanism of energy transfer from solitonic to dispersive waves and leading to suppression of the collapse. © 2005 Optical Society of America

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Collapse of spatial and spatiotemporal two- and three-dimensional beams and pulses is one of the classic problems in optics. Over the past few years there has been a renewal of interest in this topic, in particular in the physical mechanisms of collapse suppression, as a result of active experimental efforts to observe spatiotemporal solitons. Planar waveguides with Kerr-like nonlinearities are potentially a simple system in which to observe the nonlinear spatiotemporal evolution of ultrashort pulses, including the collapse situation. Experiments with ultrashort pulses in planar silica waveguides have demonstrated, however, the absence of pulse collapse for input powers well above the critical level. Numerical modeling and measurements have suggested that higher-order dispersion plays an important role in collapse suppression. In particular, it has been demonstrated that small fourth-order dispersion or diffraction suppresses the collapse of two-dimensional solitons. However, third-order dispersion typically dominates fourth-order dispersion in planar waveguides. Furthermore, the spectral region of anomalous group-velocity dispersion (GVD), required for soliton formation, is usually bounded by zero-GVD points. It is clear that close to these points third-order dispersion becomes an important factor, strongly affecting pulse evolution.

It is known that third-order dispersion leads to resonant radiation from temporal solitons in optical fibers. The intensity of this radiation is directly proportional to the spectral amplitude of the soliton at the resonant frequency. It is natural to expect that the same effect takes place in planar waveguides. The main objectives of this work are to investigate resonant radiation from the spatiotemporal pulses and to demonstrate that this effect can serve as the sole mechanism for collapse suppression and pulse broadening observed in planar waveguides with zero-GVD points. Indeed, the spectrum of a pulse propagating in a planar waveguide with a focusing nonlinearity and anomalous GVD will broaden substantially at the initial stages owing to collapse. Simultaneously, the spectral amplitude of the pump pulse at the resonant frequency will grow and stimulate emission of resonant radiation. Thereby, energy will flow from the localized to nonlocalized parts of the field and, as shown below, lead to collapse suppression.

First we introduce a general model for pulse propagation in planar waveguides:

\[ i \frac{\partial}{\partial z} E + \int \frac{dT'}{T'} l(X', T') E(X - X', T - T') + \gamma \left(1 + \frac{i}{\omega_p} \frac{\partial}{\partial T} \right) \int \frac{dT'}{T'} R(T') |E(X, T - T')|^2 = 0. \]

Here \( E \) is the amplitude of the fundamental waveguide mode and \( Z \) and \( X \) are the longitudinal and transverse coordinates, respectively. \( T \) is the time measured in the reference frame moving with the group velocity at the pump frequency. \( R(T) \) is the dimensionless nonlinear response function, which accounts for the instantaneous nonlinear Kerr and delayed Raman response. \( \gamma = 2 \pi n_2 / (\lambda_p \hbar) \) is the nonlinear parameter of the waveguide, \( \lambda_p \) is the pump frequency, \( \hbar \) is the effective thickness of the waveguide, and \( n_2 \) is the nonlinear index. \( L(X, T) \) is the linear response function, such that \( L(X, T) = 1 / (4 \pi^2) \int \kappa(\Delta, K_x) \exp(i K_x X - i \Delta T) d \Delta d K_x \), where \( \kappa \) is the shifted propagation constant of a linear guided wave: \( \kappa(\Delta, K_x) = [\beta(\omega_p + \Delta)^2 - K_x^2]^{1/2} - \beta(\omega_p) - \Delta \delta \omega \beta(\omega_p) \). Here \( \Delta = \omega - \omega_p \) is the detuning from pump frequency, \( \omega_p = 2 \pi \lambda_p / c \), where \( c = 3 \times 10^8 \text{ m/s} \). We chose as an example a 2-\( \mu \text{m} \)-thick silica waveguide surrounded by air and calculated the frequency dependence of propagation constant \( \beta \) for a TE fundamental mode (polarized parallel to the waveguide plane). Under these conditions a zero-GVD wavelength is found at \( \approx 0.84 \mu \text{m} \). We chose the pump wavelength sufficiently close to the zero-GVD point and on the anomalous side; e.g., for \( \lambda_p = 0.87 \mu \text{m} \), we have \( \beta(\omega_p) = \frac{\beta_0^2 \beta}{2} = -4.6 \times 10^{-5} \text{ ps}^2 / \text{m} \), \( \beta_0^2 \beta = 7.2 \times 10^{-5} \text{ ps}^2 / \text{m} \), and \( \delta \omega \beta = -4.96 \times 10^{-8} \text{ ps} / \text{m} \). For a pulse duration of 50 fs the ratio of the fourth- to the third-order dispersion length is \( \approx 300 \), indicating strong dominance of third-order effects. Equation (1) accounts exactly for all the dispersion and diffraction orders and for the mixing of these two effects. The integral representation of the interplay between diffraction and dispersion used in Eq. (1) and introduced previously in Refs. 11 and 12 is valid in the entire frequency and wave-number domain unlike.
the truncated Taylor expansion \(^1\) [see discussion after Eq. (2)]. The \((i/\omega_p)\partial_T\) term describes the self-steepening effect.\(^1\)

For our model we introduce normalized variables \(z\), \(x\), and \(t\): \(z = Z/L_{df}, x = X/w,\) and \(t = T/\{w[\beta(\omega_p)]\beta_2(\omega_p)]^{1/2}\}\), where \(w\) is the beam width and \(L_{df} = w^2\beta(\omega_p)\) is the diffraction length. A dimensionless field \(A\) is given by \(A = E(\gamma L_{df}/w)^{1/2}\). Choosing \(w = 100 \, \mu m\) and \(\lambda_p = 0.87 \, \mu m\), we find that the scaling coefficient for power is \(w(\gamma L_{df}) = 8.3 \, kW\), the scaling for distance is \(L_{df} = 10.4 \, cm\), and the scaling for time is \(\approx 20 \, fs\). If in the Taylor expansion of \(\kappa(\Delta, K_x)\) one neglects all the derivatives higher than the second, then the Hamiltonian for field \(A\) is given by \(H_{nls} = \int dx dt ([[\partial_x A]^2 + [\partial_t A]^2 - |A|^4]/2\) and the idealized nonlinear Schrödinger equation is \(i\partial_t A = \delta H_{nls}/\delta A^*\). Condition \(H_{nls} < 0\) is sufficient for collapse development.\(^1\)\(^,\)\(^13\) We take the initial pulse in the form of \(A = a \exp[-x^2/(2\sigma^2) + t^2/(2\tau^2)]\) and choose \(a = 6.43, r = 0.212,\) and \(\tau = 0.863,\) yielding a dimensionless energy \(Q = \int dt dx |A|^2 = 23.7\), whereas critical energy \(Q_c = \pi(r^2 + \tau^2)/(r\tau) = 13.55\) and the collapse distance is \(z_c = [\pi r \tau / (Q - Q_c)]^{1/2} \approx 0.237.\)\(^1\) Spectral and temporal plots obtained from numerical modeling of Eq. (1) are shown in Figs. 1(a) and 2(a). As one can see, no collapse happens, but instead the pulse develops a well-pronounced radiation tail. This tail has a conical shape in the spectral domain and belongs to the region of normal GVD [see Fig. 1(a)]. Note that in the space–time domain the radiation has a conical shape as well.

To explain and explore the physics behind these conically shaped patterns of radiation and collapse suppression, it suffices to take into account the third-order terms in the Taylor expansion of \(\kappa(\Delta, K_x)\) around \(\Delta = K_x = 0\). After substitution of the Taylor expansion into Eq. (1) and neglect of the Raman and self-steepening effects, we are left with the well-known generalization of the nonlinear Schrödinger equation:\(^1\)

\[
i\partial_z A + \frac{1}{2}(\partial_x^2 + \partial_t^2 - i\epsilon \partial_t^2 - i\sigma \partial_t^2)A + |A|^2 A = 0,
\]

where \(\epsilon = \partial_x^2 + 3\beta_2(\omega_p)[L_{df}\beta_2(\omega_p)]^{1/2}\) and \(s = \partial_x^2 \partial_t \kappa[L_{df}/|\beta_2(\omega_p)|]^{1/2} / (3w^2)\). The limited range of validity of Eq. (2) in the spectral domain causes, however, some problems linked to the \(\partial_t \partial_x^2\) term describing coupling between dispersion and diffraction in the leading order. Indeed, the exact diffraction coefficient given by \(-\partial_x^2 \kappa\) is naturally positive for any frequency detuning \(\Delta\). However, substituting \(A \sim \exp(i\beta z + ik_x x - i\delta t)\) into truncated model (2), we obtain diffraction coefficient \(-\partial_x^2 \beta = -s\delta\), which changes its sign for \(\delta = 1/s\). Here, \(\delta\) and \(k_x\) are the dimensionless frequency detuning and the transverse wave numbers, respectively. It is known that the presence of the frequencies at which the GVD vanishes results in energy transfer from the quasi-solitonic pulses in the anomalous GVD region to the dispersive waves in the normal GVD region.\(^8\)\(^,\)\(^9\)\(^,\)\(^10\) Modeling Eq. (2) with \(s \neq 0\), we found

Fig. 1. Spectral profiles of the Gaussian pulse after propagation distance \(z = 0.2\). (a) is calculated from Eq. (1) and (b) from Eq. (2). Normalized initial conditions given in the text correspond to the initial pulse with a duration of 53 fs, an aperture of 60 \(\mu m\) and a peak power of 180 kW. Thin dashed lines show the zero-GVD frequency. The solid curves are calculated from the approximate conditions of resonance: \(\kappa(\Delta, K_x) = 0\) for (a) and \(\delta^2(\epsilon \delta - 1) = k_x^2\) with \(\epsilon = 0.24\) for (b).

Fig. 2. Changes in the temporal profiles of the Gaussian pulse at \(x = 0\) as it propagates inside the waveguide, calculated from (a) Eq. (1) and (b) Eq. (2). The vertical panel in (b) shows the evolution of the Hamiltonian integral calculated over only the localized part of the field. All the parameters are the same as in Fig. 1.
that the analogous effect happens in the presence of the zero-diffraction points. However, in reality, diffraction is always positive, and the change in sign of the GVD coefficient $\frac{\partial^2}{\partial z^2} \beta = -1 + 3\epsilon \delta$ for $\delta = 1/(3\epsilon)$ is physical. To eliminate spurious loss of energy to the unphysical part of the spectral domain with negative diffraction, one should either use Eq. (1) or assume $s = 0$ in Eq. (2). We found that numerical solutions of Eq. (1) and Eq. (2) with $s = 0$ have good qualitative and quantitative agreement, providing that the third-order temporal dispersion is the dominant correction (see Fig. 1). Therefore we can use Eq. (2) with $s = 0$ for interpretation of the emission of the conical radiation.

The frequency of the resonant radiation can be estimated from the condition that the propagation constant of the Fourier component of the collapsing spatiotemporal wave packet. This condition can be derived and used in the same way as in the fiber case $^9,^{10}$ but in the planar waveguide the resonant frequency will be parameterized by transverse wave number $k_x$. It is known that for $s = \epsilon = 0$ the collapsing solution of Eq. (2) can be sought in the form of $\tilde{A} = f(z)Q(\xi, z)\exp(iz\Sigma)$, where $\xi = f(z)(t^2 + x^2)^{1/2}$, $f$ and $Q$ are some functions, and $q$ is the $z$-independent shift of the wave number $^{13}$ The matching condition of the wave numbers of dispersive wave $\beta$ and of the collapsing wave packet is given by $q = (-k_x^2 - \delta^2 + \epsilon \delta^2)/2$. One can show that for $\epsilon \neq 0$ and fixed $k$, the above equation has one real-valued root. For $\epsilon > 0$ the radiation at all the values of $k_x$ is blueshifted with respect to the spectral center of the soliton, and for $\epsilon < 0$ it is redshifted. For the full model the resonance condition is $q/L_{df} = \kappa(\Delta, K_x)$. Note that the third-order dispersion, Raman, and self-steepening effects generally make $q$ dependent on $z$. Therefore resonance frequency $\delta_0$ is expected and indeed changes with propagation. However, these changes are usually small. Furthermore, the values of $\delta_0$ are of the order of $1/\epsilon^{3/4}$ and condition $|\delta_0|^2 >> q$ is easily satisfied. Therefore the resonance conditions can be simplified to $\kappa(\Delta, K_x) = 0$ and $\delta^2(\epsilon \delta - 1) = \kappa_x^2$, respectively. The blue radiation observed in our case is well described by the above conditions (see Fig. 1). Thus we conclude that the radiation observed in the full and truncated models is indeed the same resonant radiation.

One can see from Fig. 2 that at the first stage of the evolution, i.e., $z \leq z_c$, the pulse is compressed by the collapse mechanism, but at $z = z_c$ it starts to emit radiation. Considering reduced model (2), we checked that Hamiltonian $H = H_{\text{els}} + eH_e$, where $H_e = i \int dx dt(\hat{A}^* \frac{\partial}{\partial x} \hat{A} - c.c.)/4$, calculated over the entire computational domain, is conserved. However, if we compute $H$ as a function of $z$ by integrating only over the localized part of the field, we observe a change in the sign of $H$ from negative to positive [see Fig. 2(b)]. We also checked that, for a localized part of the field, $H$ and $H_{\text{els}}$ are close in value, and therefore $H_e$ is in fact important only for the radiation. The sign change of $H$ happens approximately at $z = z_c$, when the resonant radiation is already strong and the soliton is practically destroyed. The positiveness of $H$ indicates that the derivative terms dominate over $|A|^4$, and it strongly suggests that all the chances for collapse have ceased. Thus it is clear that collapse is indeed suppressed by the energy transfer from the localized solitonic part of the field to resonant radiation. Increasing the power, we observe that the Gaussian initial condition can keep its localized form for longer propagation distances and can undergo several oscillations. However, eventually all the energy is transferred from the localized solitonic field to the dispersive radiation. The radiation reported here should be distinguished from the Cherenkov-type optical radiation reported in plasma waveguides induced by intense laser pulses propagating in air. $^{14}$ In the latter case the collapse is suppressed by the plasma corrections to nonlinearity, not by the radiation. The radiation is emitted into continuum modes and out of the plasma waveguide. $^{14}$ In our work, however, radiation is emitted into the same guided modes as those guiding the pump pulse.

In summary, we have investigated the dynamics of ultrashort optical pulses propagating in planar waveguides in the proximity of the zero-GVD point. Our results indicate that previously observed collapse suppression under conditions similar to ours $^4$ can be attributed solely to the third-order dispersion, which is responsible for the energy transfer from the solitonic part of the field into resonant radiation.

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