

Inverting A Matrix

- **Swap** rows or columns
- **Multiply** a whole row or column by a number
- **Add** or **subtract** one row or column from another
- Whatever we do to the Left matrix we do to the Right one

$$\begin{array}{c} \mathbf{k} \\ \left[\begin{array}{cccccccc} k_{x_1x_1} & k_{x_1y_1} & k_{x_1x_2} & k_{x_1y_2} & \cdots & k_{x_1x_n} & k_{x_1y_n} \\ k_{y_1x_1} & k_{y_1y_1} & k_{y_1x_2} & k_{y_1y_2} & \cdots & k_{y_1x_n} & k_{y_1y_n} \\ k_{x_2x_1} & k_{x_2y_1} & k_{x_2x_2} & k_{x_2y_2} & \cdots & k_{x_2x_n} & k_{x_2y_n} \\ k_{y_2x_1} & k_{y_2y_1} & k_{y_2x_2} & k_{y_2y_2} & \cdots & k_{y_2x_n} & k_{y_2y_n} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ k_{x_nx_1} & k_{x_ny_1} & k_{x_nx_2} & k_{x_ny_2} & \cdots & k_{x_nx_n} & k_{x_ny_n} \\ k_{y_nx_1} & k_{y_ny_1} & k_{y_nx_2} & k_{y_ny_2} & \cdots & k_{y_nx_n} & k_{y_ny_n} \end{array} \right] \end{array} \parallel \begin{array}{c} \mathbf{I} \\ \left[\begin{array}{cccccccc} 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 \end{array} \right] \end{array}$$

$$\begin{bmatrix}
 k_{x_1x_1} & k_{x_1y_1} & k_{x_1x_2} & k_{x_1y_2} & \dots & k_{x_1x_n} & k_{x_1y_n} \\
 k_{y_1x_1} & k_{y_1y_1} & k_{y_1x_2} & k_{y_1y_2} & \dots & k_{y_1x_n} & k_{y_1y_n} \\
 k_{x_2x_1} & k_{x_2y_1} & k_{x_2x_2} & k_{x_2y_2} & \dots & k_{x_2x_n} & k_{x_2y_n} \\
 k_{y_2x_1} & k_{y_2y_1} & k_{y_2x_2} & k_{y_2y_2} & \dots & k_{y_2x_n} & k_{y_2y_n} \\
 \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
 k_{x_nx_1} & k_{x_ny_1} & k_{x_nx_2} & k_{x_ny_2} & \dots & k_{x_nx_n} & k_{x_ny_n} \\
 k_{y_nx_1} & k_{y_ny_1} & k_{y_nx_2} & k_{y_ny_2} & \dots & k_{y_nx_n} & k_{y_ny_n}
 \end{bmatrix}$$

Divide Row1 through by 1st Diagonal ($k_{x_1x_1}$)

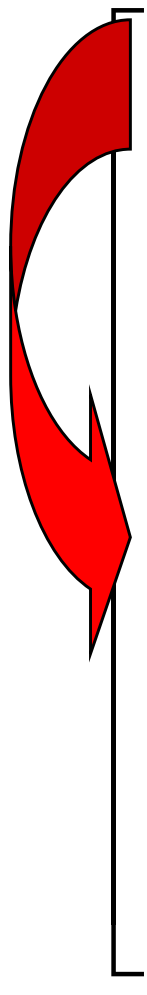
$k_{x_1x_1}$	$k_{x_1y_1}$	$k_{x_1x_2}$	$k_{x_1y_2}$	\dots	$k_{x_1x_n}$	$k_{x_1y_n}$
$k_{y_1x_1}$	$k_{y_1y_1}$	$k_{y_1x_2}$	$k_{y_1y_2}$	\dots	$k_{y_1x_n}$	$k_{y_1y_n}$
$k_{x_2x_1}$	$k_{x_2y_1}$	$k_{x_2x_2}$	$k_{x_2y_2}$	\dots	$k_{x_2x_n}$	$k_{x_2y_n}$
$k_{y_2x_1}$	$k_{y_2y_1}$	$k_{y_2x_2}$	$k_{y_2y_2}$	\dots	$k_{y_2x_n}$	$k_{y_2y_n}$
\vdots	\vdots	\vdots	\vdots	\ddots	\vdots	\vdots
$k_{x_nx_1}$	$k_{x_ny_1}$	$k_{x_nx_2}$	$k_{x_ny_2}$	\dots	$k_{x_nx_n}$	$k_{x_ny_n}$
$k_{y_nx_1}$	$k_{y_ny_1}$	$k_{y_nx_2}$	$k_{y_ny_2}$	\dots	$k_{y_nx_n}$	$k_{y_ny_n}$

$$\begin{bmatrix}
 1 & ? & ? & ? & \dots & ? & ? \\
 k_{y_1 x_1} & k_{y_1 y_1} & k_{y_1 x_2} & k_{y_1 y_2} & \dots & k_{y_1 x_n} & k_{y_1 y_n} \\
 k_{x_2 x_1} & k_{x_2 y_1} & k_{x_2 x_2} & k_{x_2 y_2} & \dots & k_{x_2 x_n} & k_{x_2 y_n} \\
 k_{y_2 x_1} & k_{y_2 y_1} & k_{y_2 x_2} & k_{y_2 y_2} & \dots & k_{y_2 x_n} & k_{y_2 y_n} \\
 \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
 k_{x_n x_1} & k_{x_n y_1} & k_{x_n x_2} & k_{x_n y_2} & \dots & k_{x_n x_n} & k_{x_n y_n} \\
 k_{y_n x_1} & k_{y_n y_1} & k_{y_n x_2} & k_{y_n y_2} & \dots & k_{y_n x_n} & k_{y_n y_n}
 \end{bmatrix}$$

Subtract $k_{y_1x_1}$ times Row1 from Row2

$$\left[\begin{array}{cccccccc}
 1 & ? & ? & ? & \dots & ? & ? \\
 0 & ? & ? & ? & \dots & ? & ? \\
 k_{x_2x_1} & k_{x_2y_1} & k_{x_2x_2} & k_{x_2y_2} & \dots & k_{x_2x_n} & k_{x_2y_n} \\
 k_{y_2x_1} & k_{y_2y_1} & k_{y_2x_2} & k_{y_2y_2} & \dots & k_{y_2x_n} & k_{y_2y_n} \\
 \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
 k_{x_nx_1} & k_{x_ny_1} & k_{x_nx_2} & k_{x_ny_2} & \dots & k_{x_nx_n} & k_{x_ny_n} \\
 k_{y_nx_1} & k_{y_ny_1} & k_{y_nx_2} & k_{y_ny_2} & \dots & k_{y_nx_n} & k_{y_ny_n}
 \end{array} \right]$$

Subtract $k_{x_2 \times x_1}$ times Row1 from Row3 ... and so on

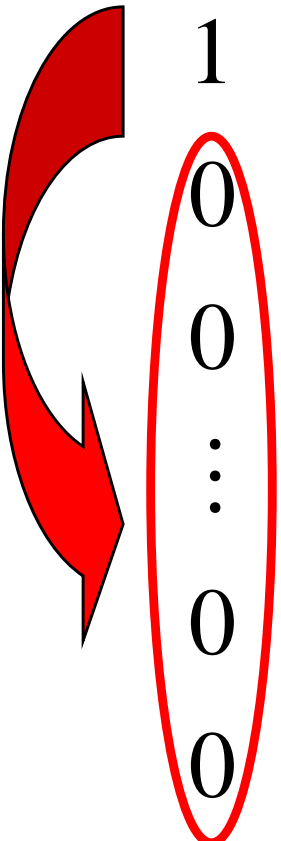

$$\begin{bmatrix} 1 & ? & ? & ? & \dots & ? & ? \\ 0 & ? & ? & ? & \dots & ? & ? \\ 0 & ? & ? & ? & \dots & ? & ? \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & ? & ? & ? & \dots & ? & ? \\ 0 & ? & ? & ? & \dots & ? & ? \end{bmatrix}$$

Divide Row2 through by 2nd Diagonal

$$\begin{bmatrix} 1 & ? & ? & ? & \dots & ? & ? \\ 0 & ? & ? & ? & \dots & ? & ? \\ 0 & ? & ? & ? & \dots & ? & ? \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & ? & ? & ? & \dots & ? & ? \\ 0 & ? & ? & ? & \dots & ? & ? \end{bmatrix}$$

$$\begin{bmatrix}
 1 & ? & ? & ? & \dots & ? & ? \\
 0 & 1 & ? & ? & \dots & ? & ? \\
 0 & ? & ? & ? & \dots & ? & ? \\
 0 & ? & ? & ? & \dots & ? & ? \\
 \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
 0 & ? & ? & ? & \dots & ? & ? \\
 0 & ? & ? & ? & \dots & ? & ?
 \end{bmatrix}$$

Subtract multiples of Row2 from the rows below

$$\begin{bmatrix} 1 & ? & ? & ? & \dots & ? & ? \\ 0 & 1 & ? & ? & \dots & ? & ? \\ 0 & 0 & ? & ? & \dots & ? & ? \\ 0 & 0 & ? & ? & \dots & ? & ? \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & ? & ? & \dots & ? & ? \\ 0 & 0 & ? & ? & \dots & ? & ? \end{bmatrix}$$


$$\begin{bmatrix}
 1 & ? & ? & ? & \dots & ? & ? \\
 0 & 1 & ? & ? & \dots & ? & ? \\
 0 & 0 & 1 & ? & \dots & ? & ? \\
 0 & 0 & 0 & ? & \dots & ? & ? \\
 \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
 0 & 0 & 0 & ? & \dots & ? & ? \\
 0 & 0 & 0 & ? & \dots & ? & ?
 \end{bmatrix}$$

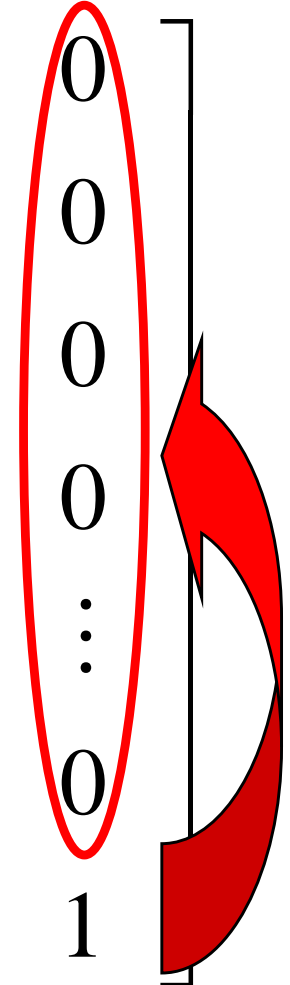
$$\begin{bmatrix}
 1 & ? & ? & ? & \dots & ? & ? \\
 0 & 1 & ? & ? & \dots & ? & ? \\
 0 & 0 & 1 & ? & \dots & ? & ? \\
 0 & 0 & 0 & 1 & \dots & ? & ? \\
 \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
 0 & 0 & 0 & 0 & \dots & ? & ? \\
 0 & 0 & 0 & 0 & \dots & ? & ?
 \end{bmatrix}$$

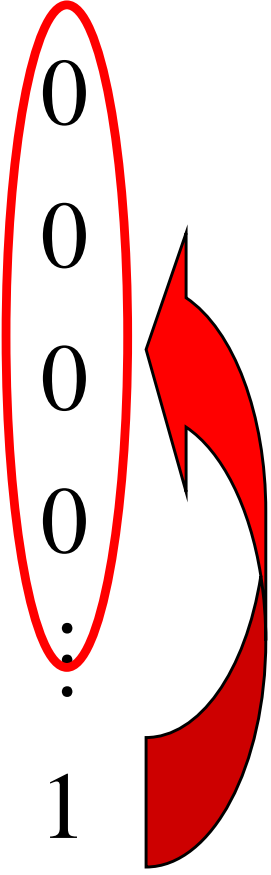
$$\begin{bmatrix}
 1 & ? & ? & ? & \dots & ? & ? \\
 0 & 1 & ? & ? & \dots & ? & ? \\
 0 & 0 & 1 & ? & \dots & ? & ? \\
 0 & 0 & 0 & 1 & \dots & ? & ? \\
 \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
 0 & 0 & 0 & 0 & \dots & 1 & ? \\
 0 & 0 & 0 & 0 & \dots & 0 & ?
 \end{bmatrix}$$

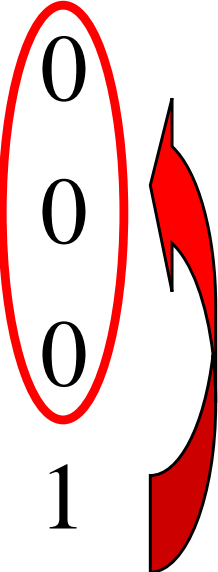
Now it is Upper-Triangular

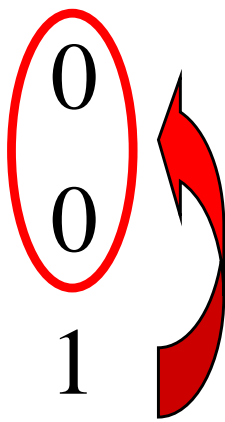
$$\begin{bmatrix} 1 & ? & ? & ? & \dots & ? & ? \\ 0 & 1 & ? & ? & \dots & ? & ? \\ 0 & 0 & 1 & ? & \dots & ? & ? \\ 0 & 0 & 0 & 1 & \dots & ? & ? \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 1 & ? \\ 0 & 0 & 0 & 0 & \dots & 0 & 1 \end{bmatrix}$$

Subtract multiples of LastRow from the rows above

$$\begin{bmatrix} 1 & ? & ? & ? & \dots & ? \\ 0 & 1 & ? & ? & \dots & ? \\ 0 & 0 & 1 & ? & \dots & ? \\ 0 & 0 & 0 & 1 & \dots & ? \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & 0 & 0 & \dots & 0 \end{bmatrix}$$


$$\begin{bmatrix}
 1 & ? & ? & ? & \dots & 0 & 0 \\
 0 & 1 & ? & ? & \dots & 0 & 0 \\
 0 & 0 & 1 & ? & \dots & 0 & 0 \\
 0 & 0 & 0 & 1 & \dots & 0 & 0 \\
 \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
 0 & 0 & 0 & 0 & \dots & 1 & 0 \\
 0 & 0 & 0 & 0 & \dots & 0 & 1
 \end{bmatrix}$$


$$\begin{bmatrix}
 1 & ? & ? & 0 & \dots & 0 & 0 \\
 0 & 1 & ? & 0 & \dots & 0 & 0 \\
 0 & 0 & 1 & 0 & \dots & 0 & 0 \\
 0 & 0 & 0 & 1 & \dots & 0 & 0 \\
 \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
 0 & 0 & 0 & 0 & \dots & 1 & 0 \\
 0 & 0 & 0 & 0 & \dots & 0 & 1
 \end{bmatrix}$$


$$\begin{bmatrix}
 1 & ? & 0 & 0 & \dots & 0 & 0 \\
 0 & 1 & 0 & 0 & \dots & 0 & 0 \\
 0 & 0 & 1 & 0 & \dots & 0 & 0 \\
 0 & 0 & 0 & 1 & \dots & 0 & 0 \\
 \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
 0 & 0 & 0 & 0 & \dots & 1 & 0 \\
 0 & 0 & 0 & 0 & \dots & 0 & 1
 \end{bmatrix}$$


$$\begin{bmatrix}
 1 & 0 & 0 & 0 & \dots & 0 & 0 \\
 0 & 1 & 0 & 0 & \dots & 0 & 0 \\
 0 & 0 & 1 & 0 & \dots & 0 & 0 \\
 0 & 0 & 0 & 1 & \dots & 0 & 0 \\
 \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
 0 & 0 & 0 & 0 & \dots & 1 & 0 \\
 0 & 0 & 0 & 0 & \dots & 0 & 1
 \end{bmatrix}$$

$$\begin{array}{c}
 \mathbf{I} \\
 \left[\begin{array}{ccccccc}
 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\
 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\
 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\
 0 & 0 & 0 & 1 & \cdots & 0 & 0 \\
 \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
 0 & 0 & 0 & 0 & \cdots & 1 & 0 \\
 0 & 0 & 0 & 0 & \cdots & 0 & 1
 \end{array} \right]
 \end{array}
 \parallel
 \begin{array}{c}
 \mathbf{k}^{-1} \\
 \left[\begin{array}{cccccccc}
 f_{x_1x_1} & f_{x_1y_1} & f_{x_1x_2} & f_{x_1y_2} & \cdots & f_{x_1x_n} & f_{x_1y_n} \\
 f_{y_1x_1} & f_{y_1y_1} & f_{y_1x_2} & f_{y_1y_2} & \cdots & f_{y_1x_n} & f_{y_1y_n} \\
 f_{x_2x_1} & f_{x_2y_1} & f_{x_2x_2} & f_{x_2y_2} & \cdots & f_{x_2x_n} & f_{x_2y_n} \\
 f_{y_2x_1} & f_{y_2y_1} & f_{y_2x_2} & f_{y_2y_2} & \cdots & f_{y_2x_n} & f_{y_2y_n} \\
 \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
 f_{x_nx_1} & f_{x_ny_1} & f_{x_nx_2} & f_{x_ny_2} & \cdots & f_{x_nx_n} & f_{x_ny_n} \\
 f_{y_nx_1} & f_{y_ny_1} & f_{y_nx_2} & f_{y_ny_2} & \cdots & f_{y_nx_n} & f_{y_ny_n}
 \end{array} \right]
 \end{array}$$

Summary

$$[K] \parallel [I]$$

- Multiply a whole row by a number
- Subtract multiples of one row from another
- Whatever we do to the LHS we do to the RHS

- Work in a systematic way down the lower triangle
- Then work back up the upper triangle

$$[I] \parallel [K^{-1}]$$