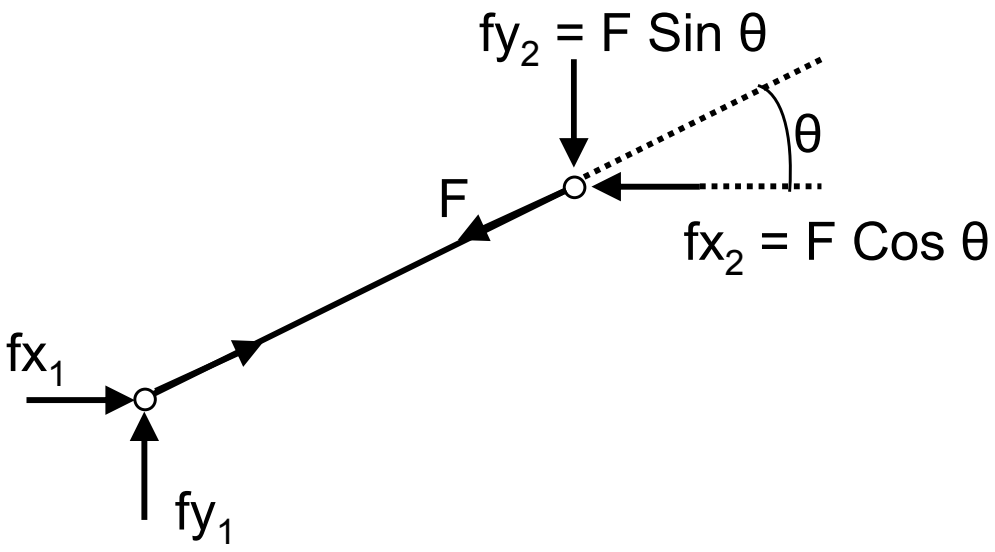
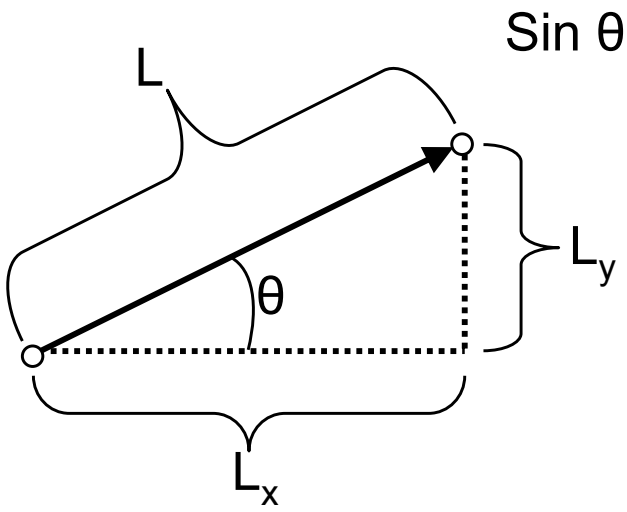
Forming The Stiffness Matrix

$$F = \frac{EA}{L}\left[\frac{(x_2 - x_1)(\delta x_2 - \delta x_1) + (y_2 - y_1)(\delta y_2 - \delta y_1)}{L}\right]$$

$$= \frac{EA}{L^2}\left[L_x(\delta x_2 - \delta x_1) + L_y(\delta y_2 - \delta y_1)\right]$$

$$= \frac{EA}{L^2}\begin{bmatrix} -L_x & -L_y & L_x & L_y \end{bmatrix}\begin{bmatrix} \delta x_1 \\ \delta y_1 \\ \delta x_2 \\ \delta y_2 \end{bmatrix}$$





Sin θ = L_y / L

Cos θ = L_x / L

fx_1 = Force node1 exerts on member in X-direction
fy_1 = Force node1 exerts on member in Y-direction
fx_2 = Force node2 exerts on member in X-direction
fy_2 = Force node2 exerts on member in Y-direction

$$fx_1 = -F\ Cos\theta = -F\ \frac{L_x}{L} \qquad fx_2 = F\ Cos\theta = F\ \frac{L_x}{L}$$

$$fy_1 = -F\ Sin\theta = -F\ \frac{L_y}{L} \qquad fy_2 = F\ Sin\theta = F\ \frac{L_y}{L}$$

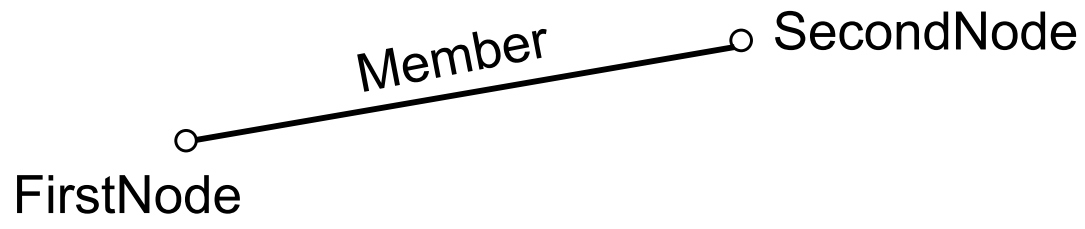
Forming The Stiffness Matrix

$$\begin{bmatrix} fx_1 \\ fy_1 \\ fx_2 \\ fy_2 \end{bmatrix} = \frac{F}{L}\begin{bmatrix} -L_x \\ -L_y \\ L_x \\ L_y \end{bmatrix} = \frac{EA}{L^3}\begin{bmatrix} -L_x \\ -L_y \\ L_x \\ L_y \end{bmatrix}\begin{bmatrix} -L_x & -L_y & L_x & L_y \end{bmatrix}\begin{bmatrix} \delta x_1 \\ \delta y_1 \\ \delta x_2 \\ \delta y_2 \end{bmatrix}$$

$$\begin{bmatrix} fx_1 \\ fy_1 \\ fx_2 \\ fy_2 \end{bmatrix} = \frac{EA}{L^3}\left[\begin{array}{cc:cc} L_xL_x & L_xL_y & -L_xL_x & -L_xL_y \\ L_yL_x & L_yL_y & -L_yL_x & -L_yL_y \\ \hdashline -L_xL_x & -L_xL_y & L_xL_x & L_xL_y \\ -L_yL_x & -L_yL_y & L_yL_x & L_yL_y \end{array}\right]\begin{bmatrix} \delta x_1 \\ \delta y_1 \\ \delta x_2 \\ \delta y_2 \end{bmatrix}$$

F = K_e d

This is our Element Stiffness Matrix



$$\frac{EA}{L^3}\begin{bmatrix} -L_xL_x & -L_xL_y \\ -L_yL_x & -L_yL_y \end{bmatrix}$$

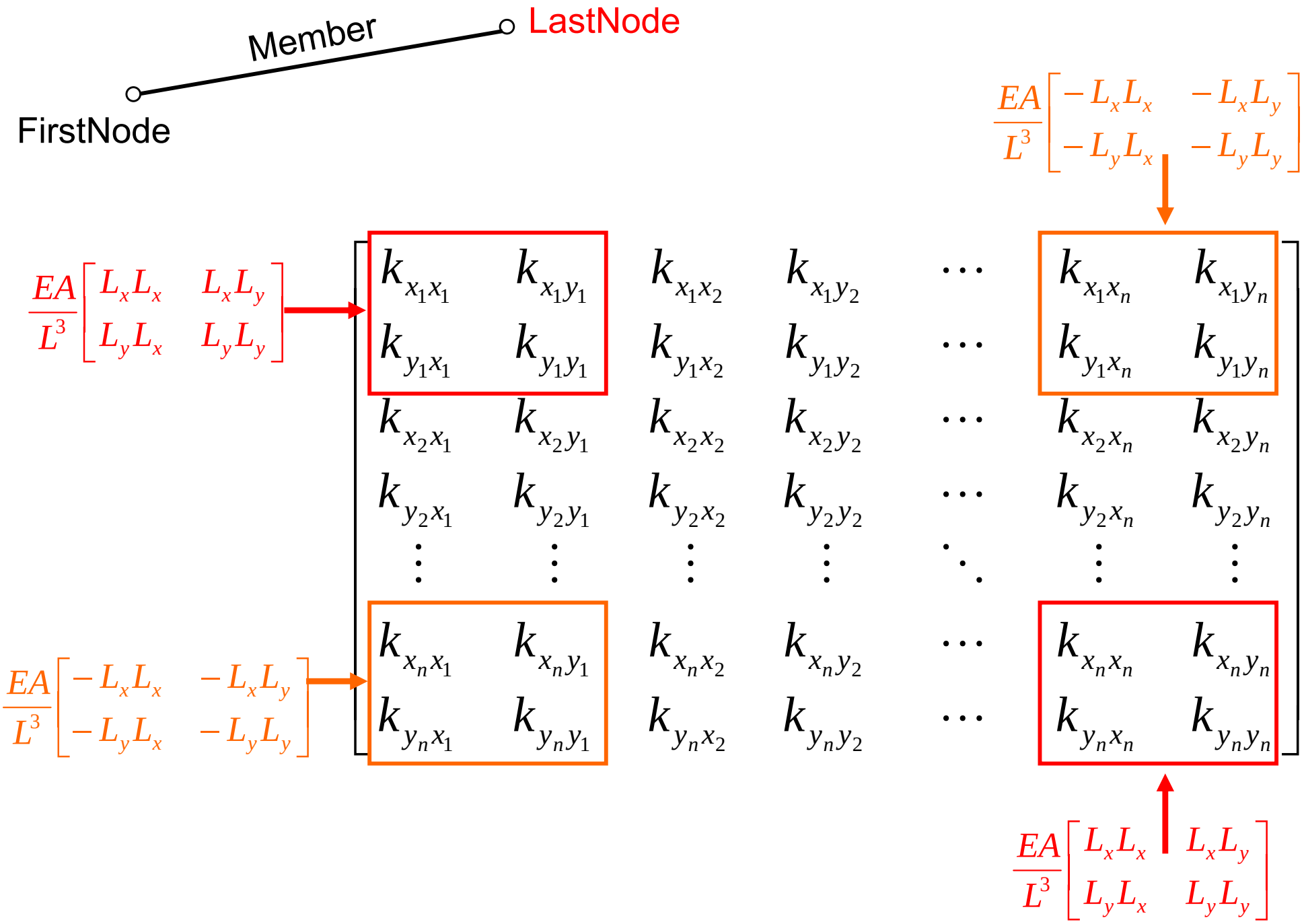
$$\frac{EA}{L^3}\begin{bmatrix} L_xL_x & L_xL_y \\ L_yL_x & L_yL_y \end{bmatrix}$$

$$\frac{EA}{L^3}\begin{bmatrix} -L_xL_x & -L_xL_y \\ -L_yL_x & -L_yL_y \end{bmatrix}$$

$$\begin{bmatrix} k_{x_1x_1} & k_{x_1y_1} & k_{x_1x_2} & k_{x_1y_2} & \cdots & k_{x_1x_n} & k_{x_1y_n} \\ k_{y_1x_1} & k_{y_1y_1} & k_{y_1x_2} & k_{y_1y_2} & \cdots & k_{y_1x_n} & k_{y_1y_n} \\ k_{x_2x_1} & k_{x_2y_1} & k_{x_2x_2} & k_{x_2y_2} & \cdots & k_{x_2x_n} & k_{x_2y_n} \\ k_{y_2x_1} & k_{y_2y_1} & k_{y_2x_2} & k_{y_2y_2} & \cdots & k_{y_2x_n} & k_{y_2y_n} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ k_{x_nx_1} & k_{x_ny_1} & k_{x_nx_2} & k_{x_ny_2} & \cdots & k_{x_nx_n} & k_{x_ny_n} \\ k_{y_nx_1} & k_{y_ny_1} & k_{y_nx_2} & k_{y_ny_2} & \cdots & k_{y_nx_n} & k_{y_ny_n} \end{bmatrix}$$

$$\frac{EA}{L^3}\begin{bmatrix} L_xL_x & L_xL_y \\ L_yL_x & L_yL_y \end{bmatrix}$$

This is our Global Stiffness Matrix



Summary

The stiffness matrix for each element is made up of lots of combinations of :

$$\frac{EA}{L^3} \quad \pm L_x \quad \pm L_y$$

The global stiffness matrix is compiled by adding in the stiffness of each member to the correct part of the global stiffness matrix