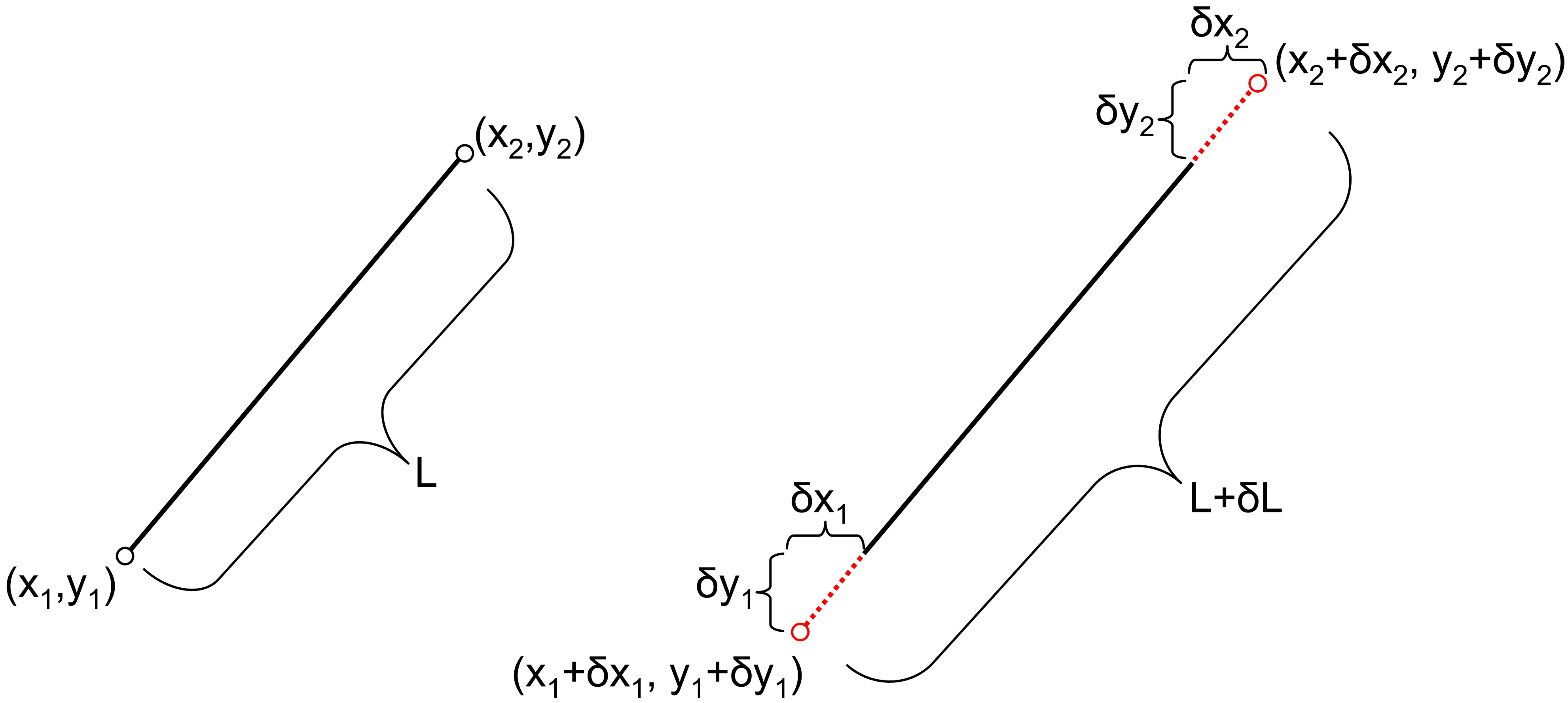
Calculating Member Forces



Binomial Expansion Trick

$$(a+b)^r = \sum_{k=0}^{\infty} \binom{r}{k} a^{r-k} b^k$$

Newton's Generalised Binomial Theorem

$$(a+b)^{\frac{1}{2}} = \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} a^{\frac{1}{2}-k} b^k$$

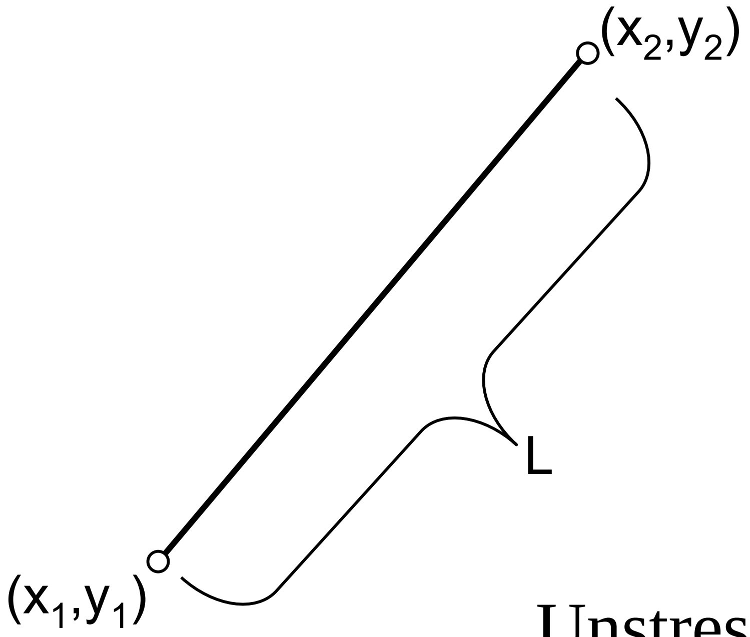
Remember √ is just *to-the-power-half*

$$= \overset{=1}{\binom{\frac{1}{2}}{0}} a^{\frac{1}{2}} + \overset{=\frac{1}{2}}{\binom{\frac{1}{2}}{1}} a^{-\frac{1}{2}} b + \cancel{\binom{\frac{1}{2}}{2} a^{-\frac{3}{2}} b^2 + \ldots}$$

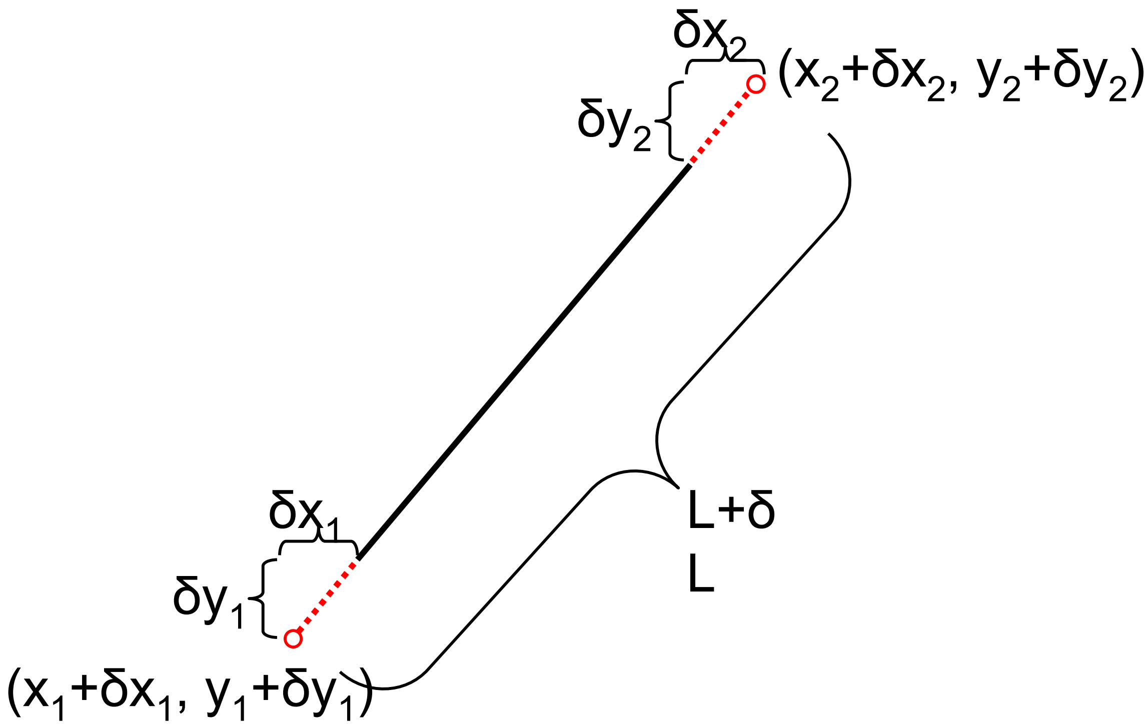
$$\approx a^{\frac{1}{2}} + \frac{1}{2} a^{-\frac{1}{2}} b$$

$$\approx \sqrt{a} + \frac{b}{2\sqrt{a}}$$

The square-root of two small things added together is roughly the square-root of the first thing plus the second thing divided by two times the square-root of the first thing



$$\text{Unstressed Length} = L = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$$



$$\text{Stressed Length} = L + \delta L = \sqrt{\left[\left(x_2 + \delta x_2\right) - \left(x_1 + \delta x_1\right)\right]^2 + \left[\left(y_2 + \delta y_2\right) - \left(y_1 + \delta y_1\right)\right]^2}$$

$$= \sqrt{\left[\left(x_2 - x_1\right) + \left(\delta x_2 - \delta x_1\right)\right]^2 + \left[\left(y_2 - y_1\right) + \left(\delta y_2 - \delta y_1\right)\right]^2}$$

$$= \sqrt{\left[(x_2 + \delta x_2) - (x_1 + \delta x_1)\right]^2 + \left[(y_2 + \delta y_2) - (y_1 + \delta y_1)\right]^2}$$

$$= \sqrt{\left[(x_2 - x_1) + (\delta x_2 - \delta x_1)\right]^2 + \left[(y_2 - y_1) + (\delta y_2 - \delta y_1)\right]^2}$$

$$= \sqrt{\left[(x_2 - x_1)^2 + 2(x_2 - x_1)(\delta x_2 - \delta x_1) + \cancel{(\delta x_2 - \delta x_1)^2}\right] + \left[(y_2 - y_1)^2 + 2(y_2 - y_1)(\delta y_2 - \delta y_1) + \cancel{(\delta y_2 - \delta y_1)^2}\right]}$$

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + 2(x_2 - x_1)(\delta x_2 - \delta x_1) + 2(y_2 - y_1)(\delta y_2 - \delta y_1)}$$

$$= \sqrt{L^2 + \left[2(x_2 - x_1)(\delta x_2 - \delta x_1) + 2(y_2 - y_1)(\delta y_2 - \delta y_1)\right]}$$

$$\sqrt{a + b} = \sqrt{a} + \frac{b}{2\sqrt{a}} + \ldots$$

$$= \underbrace{\sqrt{L^2}}_{=L} + \frac{\cancel{2}(x_2 - x_1)(\delta x_2 - \delta x_1) + \cancel{2}(y_2 - y_1)(\delta y_2 - \delta y_1)}{\cancel{2}\underbrace{\sqrt{L^2}}_{=L}} \cancel{+ \ldots}$$

$$\approx L + \frac{(x_2 - x_1)(\delta x_2 - \delta x_1) + (y_2 - y_1)(\delta y_2 - \delta y_1)}{L}$$

$$\cancel{L} + \delta L \approx \cancel{L} + \frac{(x_2 - x_1)(\delta x_2 - \delta x_1) + (y_2 - y_1)(\delta y_2 - \delta y_1)}{L}$$

$$\delta L \approx \frac{(x_2 - x_1)(\delta x_2 - \delta x_1) + (y_2 - y_1)(\delta y_2 - \delta y_1)}{L}$$

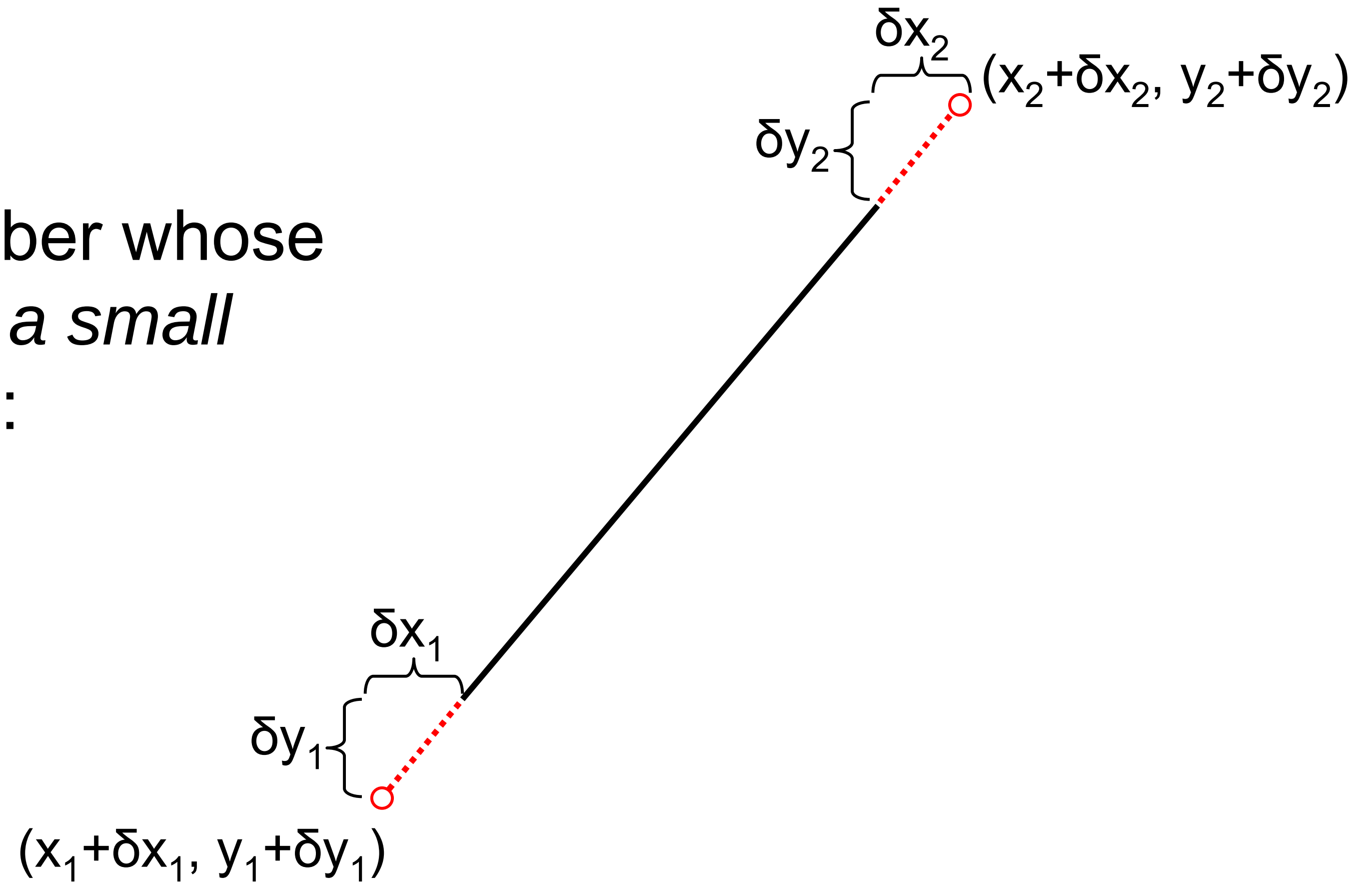
$$E = \frac{\sigma}{\varepsilon} \qquad \sigma = E\varepsilon \qquad \frac{Force}{Area} = E\frac{Extension}{Original\ Length} = E\frac{\delta L}{L}$$

$$\text{Member Force} = \text{F} = \frac{EA}{L}\delta L$$

$$F = \frac{EA}{L}\left[\frac{(x_2 - x_1)(\delta x_2 - \delta x_1) + (y_2 - y_1)(\delta y_2 - \delta y_1)}{L}\right]$$

Summary

The force in a member whose nodes have moved *a small amount* is given by :



$$F = \frac{EA}{L^2}\left[(x_2 - x_1)(\delta x_2 - \delta x_1) + (y_2 - y_1)(\delta y_2 - \delta y_1)\right]$$