# Optimizing the Cross-Sectional Shapes of Extruded Aluminium Structural Members for Unitized Curtain Wall Facades

Adam D. Lee<sup>a,b,\*</sup>, Paul Shepherd<sup>a</sup>, Mark C. Evernden<sup>a</sup>, David Metcalfe<sup>c</sup>

<sup>a</sup>Department of Architecture and Civil Engineering, University of Bath, Claverton Down, Bath, BA2 7AY, U.K.

## Abstract

Curtain walls are lightweight, weathertight, exterior facades that resist wind loads but do not provide support for the building structures to which they are attached. Although they are used to enclose many different types of modern building, and although they may be designed to carry any of the outward-facing materials an architect might wish to specify, the stereotypical curtain wall is a skyscraper's fully-glazed outer skin.

The materials used in these wall systems, particularly their structural aluminium frames, are produced by energy intensive methods. However, even though there is an environmental motive to reduce this high level of embodied energy by minimizing aluminium content, and despite the obvious commercial incentive, it is a difficult mathematical challenge to find optimal extrusion shapes. The authors believe that because of the inherent complexity of the optimization task, in the curtain walls of real buildings, metal is used inefficiently.

This paper describes the way in which near-optimal shapes for any particular building's curtain wall extrusions may be found using a parametrically-

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<sup>&</sup>lt;sup>b</sup>PTCC Facade Design, Telecom Plaza, 316 Senator Gil Puyat Ave., Makati City, Metro Manila, 1200, Philippines.

<sup>&</sup>lt;sup>c</sup>Centre for Window and Cladding Technology (CWCT), The Studio, Entry Hill, Bath, BA2 5LY, U.K.

<sup>\*</sup>Corresponding Author

*Email addresses:* adamlee@torstencalvi.com (Adam D. Lee), p.shepherd@bath.ac.uk (Paul Shepherd), m.evernden@bath.ac.uk (Mark C. Evernden), david.metcalfe@cwct.co.uk (David Metcalfe)

controlled geometric model in conjunction with a numerical search routine – in this case, a genetic algorithm.

When the curtain walls designed for large and recently-constructed buildings by experienced facade engineers are compared with designs developed using the algorithmic techniques described herein, it is consistently the numericallyoptimized solutions that are more efficient. The magnitude of the metal savings achieved by applying computational methods will vary from building to building, but this study suggests that in many cases aluminium mass may be reduced by 20% or more.

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## 1. Introduction

The most conspicuous features of a modern city's business district – visible for miles around – are the glass-faced, high-rise, office towers. These exterior envelopes are, usually, curtain walls – weathertight enclosures attached to, but <sup>5</sup> providing no support for, the internal structural frames of the buildings to which they are attached. A glance at the metropolitan skyline in established hub cities such as Hong Kong and New York, or a tour of the new commercial areas of Dubai or Shanghai, will confirm that the curtain wall construction method has been popular, and that it remains so. By one estimate [1, p. 82], worldwide spending on unitized curtain wall exceeds US \$12 billion per year.

Examples of contemporary buildings enclosed by unitized curtain wall facades are pictured in Figure 1. The parts of a typical unitized curtain wall panel, as well as the connections between them, are shown in Figure 2.

The structural frames of today's curtain walls are, almost always, made of aluminium, and a significant proportion of the cost of constructing a curtain wall is the cost of the metal [4, pp. 87 & 93; 5, p. 88]. Because the methods by which aluminium is produced are highly energy intensive, the embodied energy



C "v\_mats" [2]. CC-BY-ND Licence [3].

Figure 1: Examples of glazed, unitized curtain walls enclosing buildings at the Moscow International Business Center.



Figure 2: Partially-exploded view of a unitized curtain wall. The extruded framing members are interrupted so that their cross-sectional shapes can be seen. The back pan and insulation normally present in the spandrel area have been omitted for clarity.

within a curtain wall is large. Even when using an energy estimate that takes current recycling practices into consideration [6, pp. 126-127], and ignoring any

- <sup>20</sup> aluminium used in the wall system's non-structural components, around 2 GJ will be consumed in the manufacture of metal for one square meter of curtain wall. The energy spent making a given area of insulated architectural glass is less, but not much less, than the energy put into its extruded aluminium frame. To help place these figures in context, the combined total energy expended in
- the manufacture of aluminium and glass for a building's curtain wall is of the same order of magnitude as the energy required to heat the building, in the UK's climate, for decades. So, if ways can be found to reduce the weight of aluminium in curtain walls, humanity will benefit: new buildings will be less expensive and, environmentally, more benign.
- Since it is a costly material, it is obvious that curtain wall contractors have a financial incentive to minimize the amount of aluminium in their products. At the same time, it is common practice to create new, custom profiles for a specific building, and the extrusion process gives designers a high level of control over the cross-section shapes of their extrusions. Therefore, given that there is a
- <sup>35</sup> strong commercial motive to find efficient solutions, and that there are few technological barriers to the manufacture of new designs, it is only logical to expect that any curtain wall facade of significant scale will have been optimized to make best use of the metal that it contains. Often, in reality, this is not the case. It is a difficult mathematical challenge to find cross-sectional shapes for extruded
- <sup>40</sup> members that meet a particular project's performance requirements using the minimum quantity of metal. Phrased another way, it is design complexity that stands in the way of economy.

# 1.1. Modern Unitized Curtain Wall

The aluminium-framed curtain walls that first became popular in the 1970s were *stick* systems assembled at the construction site from simple box-shaped extrusions. While stick system walls are still being built today, the majority of modern facades now are *unitized* designs [1, p. 82] made up of rectangular panels, each of which is prefabricated and glazed in a workshop. The reasons usually given to explain the popularity of the unitized approach are that the

<sup>50</sup> prefabricated panels can be installed rapidly at site and that it is easier to control quality if parts are cut and assembled in a factory rather than in situ [7 p. 4-5; 8 p. 86]. Other factors are that the joints between adjacent unitized panels can be designed to include more sophisticated defences against water entry, and that they have room for movements larger than those that can be accommodated by a stick wall system [9, pt. 2, p. 23].

Within this paper, the vertical, and only the vertical, structural framing extrusions are referred to as *mullions*: other naming conventions may be in use within the facade industry. The horizontal, two-piece member created when the head (top) of one panel engages with the sill (bottom) of the panel above, is the *stack joint*.

Often, the contractor engaged to supply a large area of curtain wall will develop a bespoke system using new variants of mullion profiles shaped like the letter "E". Many examples of these E-shaped male and female mullion shapes appear in industry publications [e.g. 9, p. 6-51; 4, p. 90; 10, p. 52]. For use in smaller facade areas, for which custom-designed solutions would not be justified, contractors are able to purchase standard E-shaped unitized curtain wall extrusions sold, from stock, by glazing system suppliers [e.g. 11 pp. 6-11; 12].

# 2. Automating the Curtain Wall Design Process

For this study, new software has been created to replace, or indeed improve upon, the services of human designers and engineers whose expertise would otherwise be needed to develop the shapes of extrusions for a bespoke curtain wall system. The decision to write computer code was taken only after searching, unsuccessfully, for a satisfactory alternative. While various industry-specific programs are used by facade fabricators to quantify and machine extrusions, and

<sup>75</sup> while standard structural modelling tools are available to predict stresses and deflections in structures of pre-defined shape, the existing tools are not capable of creating a set of cross-sectional shapes for curtain wall framing members to comply with a given set of codified structural design rules.

For ease of reference the new software has been given an acronym, ACWEDS, the Autonomous Curtain Wall Extrusion Design System. It accepts, as its inputs, the architecturally-defined facade layout, as well as structural performance specifications such as design wind loads and allowable deflections. It then "designs" the cross-sectional shapes for a new curtain wall system containing the minimum possible amount of aluminium. The computational algorithms have

to be able to create new shapes for aluminium profiles that can be assembled to form a watertight framing system, that can be extruded, that are compliant with the structural design codes, and that are optimally efficient in metal usage.

# 2.1. Inputs to Design Software

When a bespoke curtain wall is created for a new building, its general form
and the details of its design are developed in two separate stages, by two different groups of design professionals. Performance criteria for the facade, as well as its layout – the horizontal and vertical spacings between framing members – will be defined in drawings and technical specifications prepared by the building's architects and consultants. The detailed cross-sectional shapes of the extrusions,
<sup>95</sup> however, will be developed later by facade designers and engineers working for

the curtain wall contractor. ACWEDS, is intended to take the place of this second group of people, those appointed by the curtain wall contractor.

A user of the design software, and the "user" may be another computer algorithm, must provide the input data described below: -

- (a) Curtain wall panel geometry: the panel's width and height, the positions of horizontal members, and the location of the mullion's supporting bracket.
  - (b) Structural design criteria: the positive and negative design wind pressures, and also the allowable limits for deflection of framing members.
  - (c) Manufacturing constraints: the values of the maximum and minimum al-
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- lowable metal thickness, as well as the allowable range for the width and depth of the mullion.

# 2.2. Parametric Model

If the task of devising the shapes of aluminium extrusions for a new curtain wall were to be assigned to a living person, rather than to software running on a computer, then it is probable that this human designer would begin by looking at the profiles in an existing and proven curtain wall system. Checks would need to be carried out to determine whether the existing cross-sections satisfy the new building's criteria, and, if not, their sizes or metal distributions would need to be adjusted. While modifying the shapes of the sections, some

dimensions might be changed to alter the structural properties of a member, but other geometric relationships would need to be preserved so that interconnected extrusions remain effective in their functions as air seals, water barriers, movement joints, and so forth. The designer would need to repeat this process of evaluation and shape modification iteratively until an acceptable combination of extrusion shapes had been found.

ACWEDS mimics some of the ways in which human designers work. It manipulates extrusion shapes and then assesses the acceptability of the modified forms. One part of the software, a module that handles parametric shape expression, holds a model of an existing and proven curtain wall. Within this model the various webs and flanges of a unitized split mullion's male and female profiles are represented by rectangular elements, as shown at the left hand side of Figure 3.

The E-shaped male and female mullion profiles represented by the parametric model can be open, boxed or double-boxed shapes. The overall size of the split <sup>130</sup> mullion, and the internal geometry of each profile, can be set to match the performance requirements of just about any conceivable flat, unitized curtain wall facade.

The parametric model is controlled by a total of 21 parametric values. These set the mullion's depth,  $P_d$ , and width,  $P_w$ , as well as the series of dimensions labelled  $P_1$  to  $P_{19}$ . The optimization algorithm also can give an instruction to "switch on" one or more of the web elements associated with dimensions  $P_{04}$ ,  $P_{09}$ , and  $P_{18}$ , or the group of elements associated with dimensions  $P_{13}$ ,  $P_{14}$  and



Figure 3: Parametrically-controlled model of unitized curtain wall mullion extrusions (left), and corresponding transom member (right).

 $P_{15}$ , to change an E-shaped profile's inner chamber, outer chamber, or both chambers, into a hollow rectangular box. The set of dimensions labelled  $K_1$  to

 $K_6$  controls the clearances between adjacent parts, and have been assigned fixed values within ACWEDS. Throughout the optimization process these dimensions are held constant to ensure that the mullion's non-structural functionality is preserved.

Within the parametrically controlled curtain wall system, the only influence that ACWEDS has upon a panel's horizontal members – its transom, shown at the right hand side of Figure 3, and also its head and sill – is to control the lengths of the profiles' webs. These dimensions are adjusted so that the front-to-back depth of each horizontal extrusion matches the depth of the mullion,  $P_d$ .

The axial lengths of the transom, the head and the sill are each equal to the panel width minus the mullion width,  $P_w$ .

# 2.3. Structural Design of a Curtain Wall's Aluminium Members

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The principal structural elements in a conventional, flat, rectilinear curtain wall, are the vertical members, or mullions, which span from floor to floor. Because a curtain wall, by definition, provides no support for the structure to which it is attached, the significant stresses in the mullions are bending stresses induced by seismic accelerations or by the action of wind upon the exterior of the facade. In this study, stresses resulting from seismic motion are ignored because they are always smaller in magnitude than the stresses caused by wind.

When a bespoke curtain wall is being designed for a building, an engineer will compare the theoretical stresses and deflections in each element of each aluminium extrusion, determined using Euler-Bernoulli beam theory, with the limiting values defined in technical specifications and construction codes. In the algorithms implemented in this research study, the magnitudes of allowable stresses are calculated using the rules given in the Aluminum Design Manual (ADM) [13], which is the primary standard for structural design of aluminium in

the United States, and which is a widely used reference within the curtain wall industry elsewhere. The Australian and New Zealand standard, for example, is a rebranded issue of a past edition of the ADM. The ADM was chosen, rather than one of the other established aluminium design codes, because it had been

the basis for each of the reference designs – sets of drawings and calculations for 170 the curtain walls of existing buildings – available to the authors. Because the software implemented the ADM's structural evaluation rules, solutions obtained using ACWEDS could be compared directly with the reference designs.

By programming ACWEDS to look for optimized design solutions that comply with the ADM, the efficiency with which metal is used in a new wall design 175 obtained from the software can be compared directly with the efficiency of an existing wall design created by humans.

The method by which ACWEDS carries out its structural analysis, and the assumptions upon which the analysis method is based, are as follows: -

- (a) The facade is made up of unitized curtain wall panels arranged in a regular, 180 rectangular grid. A mullion at any one floor is connected, structurally, to the mullions at the floor above, to form a continuous beam running vertically, spanning multiple floors. The connections between the mullion and the building's structure are modelled as pin jointed supports, and the connections between adjacent mullions are hinges.
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- (b) The load acting in the direction perpendicular to the plane of the wall is uniformly distributed over the length of the mullion. In other words, wind
- pressure is considered to act upon a tributary strip [4, p. 98; 14, Part VIII, p. 60]. (c) All lateral loads, such as those caused by pressurization of the internal
- 190 cavities of a "pressure-equalized" [15, 16] mullion, are ignored. Within the curtain wall industry, this is the usual analytical approach: in fact, the lateral loads acting upon the webs of a pressure-equalized split mullion are not even mentioned in the literature.
- (d) Stresses caused by axial loads within the mullion profiles, due to the self 195 weight of the wall's components, are small in comparison with the flexural stresses, and are ignored.

(e) Once the cross-sectional properties of the mullions' extrusions have been determined using standard structural formulae [17], the classical beam theory

of Euler [18; 19; 20, pp. 30-36] is used to estimate the magnitudes of stresses and deflections. For the mullions of a multi-floor facade, the patterns of shear force, bending moment and deflection are shown in Figure 4.

- (f) At every point along the length of a mullion, bending moment is divided between the male and the female extrusions. The share of the total moment
- carried by a particular profile is in proportion to that profile's contribution to the total stiffness of the split mullion.
- (g) The structural profiles are extrusions made of 6063 alloy, also named AlMg0.7Si, of T5 temper. This combination of alloy and temper is amongst the most commonly used for the framing members of curtain wall systems [21, p. 19;
- 22, p. 11]. ACWEDS is capable of handling the analysis of other alloys, but in this study only 6063-T5 has been considered.
- (h) Calculations follow the Allowable Stress Design method described in Part IA of the ADM's 2005 [13] edition. The analytical expressions are closely comparable with, although not quite identical, to those published in Part IA of the 1994 and 2000 editions, and in Part I of the 2010 ADM [23].
- (i) Infill materials attached to the curtain wall, such as glass panes or metal sheet, do not stiffen or restrain the aluminium extrusions. In other words, in the analysis of bending about any axis, infill materials are ignored in the structural model. This is the curtain wall industry's usual premise [e.g. 24;
- <sup>220</sup> 13, Part VIII, pp. 56-61].
  - (j) The magnitudes of the bending stresses at the outermost extremities of the male and female profiles are checked to ensure that they do not exceed the allowable maximum for yield-limited designs.
- (k) ACWEDS checks each side of the split mullion for resistance to lateral torsional buckling, considering the horizontal framing elements in a curtain wall panel the head, the sill and the transom members to be discrete torsional braces [14, Part 1, Appendix 6, Section 6.3.2; 25, Section 12.10.2, p. 473]. For the purpose of stability analysis, the mullion's unsupported

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Figure 4: Pattern of deflection, bending moment and shear in the mullions of the multi-storey unitized curtain wall facade. Locations of brackets are marked by horizontal dotted lines, and stack joints by horizontal solid lines.

span is therefore taken as the shortest clear vertical distance between one horizontal extrusion and its neighbour.

- For each of the rectangular elements in the geometric model of the mullion's cross-section, shown in Figure 3, a check is made to ensure that stresses do not exceed the local buckling limits [13, Section VII, Table 2-23, Sections 3.3.15, 3.4.16 & 3.4.18].
- 235 (m) The mullion's maximum out-of-plane deflection is checked to ensure that it is not greater than the specified maximum allowable deflection.

#### 2.4. Manufacturing Constraints

The results of this research will be meaningful only if the shapes of the aluminium structural members considered in the analysis are shapes that could, in practice, be extruded. With regard to metal thickness, the advice given by the Aluminum Extruders Council [26, p. 11] is that:

> "Extrusion allows you to put extra metal where it is needed – in highstress areas, for example – and still save material by using normal dimensions elsewhere in the same piece. Adjacent-wall thickness ratios of less than two-to-one are extruded without difficulty, but large differences between thick and thin areas may create dimensional control problems during extrusion. It is best to maintain near uniform metal thickness throughout a shape if possible."

However, in the authors' experience, it is usually possible to go well beyond this two-to-one thickness ratio limit, particularly if a taper or fillet radius is provided at the transitions between thick and thin elements. So, within ACWEDS, the limiting ratio of thick to thin parts in one cross-sectional shape has been set to four.

The minimum metal wall thickness that can be extruded varies with the diameter of the extrusion's circumscribing circle. For the 6060 and 6063 aluminium alloys, from which curtain wall profiles commonly are made [21, p. 19; 22, p. 11], minimum thickness guidelines published by the American Society for

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Metals [27, table 3.7, p. 133] and by the European Aluminium Association [28, p. 7; 29, p 21] are shown graphically in Figure 5 together with one extrusion firm's recommendations [30, p. 8].



Figure 5: Variation in minimum metal thickness with circumscribing circle size, according to different publications, for open aluminium extrusions (above), and for boxed extrusions (below).

The profiles used to frame curtain wall panels generally have circumscribing circle diameters equal to, or smaller than, 220 mm, and the rule that has been applied in the algorithms described in Section 2, for open and for hollow sections, is that the minimum metal thickness cannot be less than 3 mm.

#### 265 2.5. The Multi-Variable Optimization Task

The task of finding the most efficient shapes for the extrusions is, in mathematical terminology, a constrained, multi-variable optimization problem. For this research, the problem is formulated as a set of algebraic expressions, each of which is a function of the variable lengths in the parametric model of the curtain wall system's mullion. So, the weight of metal in a curtain wall panel, which is the quantity to be minimized, and also the design constraints – for manufacturability, deflection, stress, and so on – have been expressed as functions of the variables labelled *P* in Figure 3. This approach to the optimization of structural shapes, or at least the simple cross-sections of steel members, is already documented in the literature [e.g. 31, pp. 13-16, 41-51]. The characteristics of the objective and constraint functions are described below because their nature has influenced the selection of optimization technique.

Because a curtain wall's extrusions must satisfy multiple criteria, each defined by a different algebraic expression, it cannot be assumed that the surface

- <sup>280</sup> bounding the permissible design space will be smooth. Sharp changes in gradient may be present in those locations where one design constraint becomes dominant over another. Even in the codified description of a single physical phenomenon, where piecewise algebraic expressions may be used to describe the variation in some property, the transitions between the functions are not necessarily smooth or even continuous. For example, the ADM uses three ex-
- pressions to define allowable local buckling stresses over a range of slenderness ratios [13, e.g. Part VII, Table 2-23, Section 3.4.11]. At the junction between one range and another, the stress function is discontinuous both in value and in gradient. When such codes are used as their authors intended, by human analysts who are able to exercise judgement, it is of little consequence if con-

straining curves are not smooth or if they contain small jumps. These features may however interfere with the operation of those classes of optimization algorithms that need to determine the objective function's gradient at such points.

Another characteristic of the design landscape is that it is likely to contain <sup>295</sup> multiple local minima. The parametric model of a curtain wall mullion, shown in Figure 3, allows the optimization algorithm to create profiles in which the number of closed box elements is zero, one or two. So, for a mullion pair – the male and female together – there are sixteen different possible arrangements of rectangular elements, and for each of these there will be at least one local <sup>300</sup> optimum. Because more than one local minimum may exist, and because discontinuities may be present, it follows that the objective function is not necessarily convex and therefore "hill climbing" algorithms are unsuitable.

Other requirements that have influenced the choice of optimization technique are the number of independent variables (the dimensionality of the search), and <sup>305</sup> the degree to which an approximate solution is acceptable. In the context of this practical design problem, knowledge of the exact value of the global minimum aluminium weight is not a necessity: a facade engineer might well be content with a solution that is within a few percent of the mathematical system's absolute minimum. Since the design needs only to be close to the <sup>310</sup> exact optimum, that the parameters controlling the shape of the model need not be continuously variable. Allowable arguments for each parameter have therefore been limited to a set of discrete values.

The final comment on the peculiarities of the search space is that, even if the allowable lengths in the parametric model are limited to a set of discrete values, as described in Section 2, the number of possible solutions –  $2^{96}$  – is far too large to search exhaustively. The time that would elapse if each possible combination of input values were to be evaluated in sequence, using a modern computer, would be several million times the age of the universe. Even if the evaluations were to be carried out in parallel on a supercomputer, the solution time would be impractically large.

# 2.6. Genetic Algorithms

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The Genetic Algorithm (GA), a numerical optimization method, was pioneered by Holland [32] and later refined by his students, De Jong [33], Goldberg [34] and Mitchell [35]. A concise description of the GA procedure may be found in a summary by Judson [36].

Although a GA will be not able to find a solution for every type of optimization problem [37, p. 50], the search method does not require an objective function that varies smoothly or continuously, and so it can be said to be more robust than the classical, calculus-based techniques [e.g. 34, p. 10]. It is to be expected that curtain wall designs obtained using a GA will be only near-optimal, rather than mathematically precise solutions. Nonetheless, for practical engineering purposes, approximate solutions are still valuable. If in the future there were to be a requirement for greater accuracy then a classical, gradient-following algorithm could be programmed to begin its search from the near-optimal location identified by the genetic algorithm.

The configuration of the GA used in this study is summarized in Table 1. Each of the candidate curtain wall system designs is defined by a set of 25 numerical arguments, 21 of which control parametric dimensions, and 4 of which change the number of rectangular elements in the cross-section. These dimensions are encoded as binary strings, and each parametric variable's string length is shown in Table 2. In total, the number of binary digits in the chromosome is 96.

A parameter that is controlled by a four-bit binary string will have sixteen possible discrete values. If the gene's binary string is 0000 then the corresponding dimension in the geometric model (Figure 3) will be set to its minimum allowable value. Conversely, if the binary string is 1111 then the dimension will be set to the maximum allowable.

The parametric inputs numbered 22 to 25 in Table 2 are single bit switches. Each of these four arguments is associated with one of the cavities in the mullion's cross-section – the interior and exterior chambers in the male and female profiles. Only if its value is 1 will the extrusion profile form an enclosed tube or box around the cavity.

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A genetic algorithm can be configured to allow the population's best solutions to pass, unaltered, into the succeeding generation. This practice, known as "elitism", may reduce the computational effort required to arrive at a solution, but it is thought to be detrimental to the algorithms ability to find a global optimum [33, pp. 101-102, 192]. Elitism is not implemented within ACWEDS.

Table 1: Details of the configuration of the genetic algorithm used to find optimal curtainwall designs.

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Feature	Implementation
Genetic encoding:	Binary string.
Fitness function:	$f(i) = (1000/\{m_{al}(i)\})^5$ for compliant designs.
	f(i) = 0 for non-compliant designs.
	(For $i$ , an individual design, $m_{al}(i)$ is mass of aluminium in
	kg per m <sup>2</sup> .)
Population size:	N = 1000 individuals.
Initial population:	Random string genotypes, tested for viability (for individuals
	$i = 1$ through N, the fitness functions $f(i) \neq 0$ ).
Mutation rate:	0.001 per bit per generation [34, p. 14].
Reproduction:	Single-point crossover at randomly-selected locus, creating
	two "child" chromosomes.
Selection:	The chance that any one individual will be chosen to repro-
	duce is equal to the selection probability,
	$P_s(i) = f(i) / \{\sum_{i=1}^N f(i)\}.$
Crossover probability:	$P_c(i) = 1.$
Genes per individual:	25. (See Table 2.)
Chromosome length:	96 bits. (For sequence and lengths of genes see Table 2.)
Termination:	Search halts after evaluation of 10,000 generations.

Genetic Algorithm Configuration

Gene	Parameter	Description	$\mathbf{Length}$
	(See Figure 3)		(bits)
1	$P_1$	Male mullion interior flange thickness.	4
2	$P_2$	Male mullion interior air seal flange thickness.	4
3	$P_3$	Male mullion interior web.	4
4	$P_4$	Male mullion interior boxing web thickness.	4
5	$P_5$	Male mullion intermediate flange thickness.	4
6	$P_6$	Male mullion intermediate rain screen flange thickness.	4
7	$P_7$	Not used in this model.	4
8	$P_8$	Male mullion exterior flange thickness.	4
9	$P_9$	Male mullion exterior boxing web thickness.	4
10	$P_{10}$	Male mullion exterior flange thickness.	4
11	$P_{11}$	Female mullion interior flange thickness.	4
12	$P_{12}$	Female mullion interior web thickness.	4
13	$P_{13}$	Female mullion innermost boxing flange thickness.	4
14	$P_{14}$	Female mullion interior boxing web thickness.	4
15	$P_{15}$	Female mullion intermediate boxing flange thickness.	4
16	$P_{16}$	Female mullion intermediate web thickness.	4
17	$P_{17}$	Female mullion exterior web thickness.	4
18	$P_{18}$	Female mullion exterior boxing web thickness.	4
19	$P_{19}$	Female mullion flange thickness.	4
20	$P_d$	Front-to-back depth of mullion.	8
21	$P_w$	Overall width of split mullion.	8
22	$P_{mi}$	Web / no web at interior of male.	1
23	$P_{mo}$	Web / no web at exterior of male.	1
24	$P_{fi}$	Web / no web at interior of female.	1
25	$P_{fo}$	Web / no web at exterior of female.	1

Table 2: Length and sequence of genes in chromosome of a curtain wall system.

Total Chromosome Length

 $96\,\mathrm{bits}$ 

# 2.7. Computer Programming & Algorithmic Efficiency

The numerical methods used by ACWEDS are computationally demanding. Therefore, while developing the software, steps were taken to make the algorithms efficient, and to implement them in fast running machine code. The choice of programming language was influenced by research [38] showing that mathematically intensive computer programs execute most quickly if coded in C++. A software profiler [39] – a tool capable of monitoring a program, while

- <sup>365</sup> running, to determine the time taken to execute each line of code, and the number of occasions on which each line of code is called was used to gather information about ACWEDS' internal processes, and the insights gained in this way made it possible to improve the program's logical flow. With care, non-ISO-compliant compilation methods (such as GCC's -ffast-math option [40, [41]])
- p. 144]) were applied, resulting in a fourfold increase in computational throughput.

Using a modern but unremarkable desktop personal computer, and executing instructions in a single thread on a 2.7 GHz processor, approximately 31,000 prospective curtain wall design solutions were evaluated each second. At this rate, the time taken to find an optimized curtain wall design was just over five minutes.

# 3. Evaluating the Computer-Generated Designs

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The effectiveness of the automated design process was appraised in the following ways: -

- (a) Human appraisal: the principal author has reviewed sample designs, created using ACWEDS, to ensure that they are rational and practical. Attempts to use engineering judgment to create better designs – that is to say, to find acceptable solutions using having less metal than the machine-generated solutions – were unsuccessful.
- <sub>385</sub> (b) Design repetition: the designs initially considered in a genetic search are generated by random selection, and some of the choices made during the

design evolution procedure also are randomized. It is therefore possible that, when the same algorithm is applied to solve a problem on more than one occasion, that the returned solutions may differ from one another.

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The automated design algorithm was tested by applying it repeatedly to the same task – to find optimized cross-sectional shapes for the extruded aluminium framing members of the curtain wall panel shown in Figure 6, subject to design wind pressures of +2.8 kPa and -3.5 kPa – and variability within the set of results was measured. After a series of 150 design optimization trials, during which a total of 1.5 billion candidate solutions had been evaluated, the best design contained 8.4095 kg of aluminium per square meter of facade. When the GA was configured in the manner described in Table 1, the mean mass of metal in its designs was found to be approximately 1% more than in the best result. The worst of the designs found using the GA was approximately 2.3% heavier than the best.

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The authors judged that the output from the GA is adequately consistent for this engineering study.

- (c) Comparative case studies: the mass of aluminium in two dozen existing curtain wall systems each one of them custom-designed for a different high-rise tower building, by professional facade engineers has been compared with the mass of aluminium in curtain wall systems designed, to matching specifications, by ACWEDS. Amongst the curtain wall systems with which ACWEDS' output was compared, some had been developed by curtain wall contractors, and some by facade consultants working for the building owner. In every case, ACWEDS' solution was found to contain less metal, and generally much less metal, than the existing designs created by experienced humans. In Figure 7, examples of the mullion profiles conceived by the facade industry's design professionals are shown, side by side, with the shapes obtained algorithmically.
- The facts set out here, in Section 3, support the claim that, in the development of bespoke curtain wall systems, it is complexity that stands in the way



Figure 6: Geometry of the unitized curtain wall considered in the numerical study described in Section 3b.



Figure 7: Cross-sections of unitized curtain wall mullions, drawn in idealized form, without non-structural features such as gasket raceways. The split mullions on the left hand side were designed, each for a specific building, by a curtain wall contractor or specialist facade consultant, while the pairs of profiles on the right are numerically-optimized solutions complying with the same performance criteria. In each case the amount of metal in the machine-generated curtain wall system is less than that in the professionally-designed solution. In the comparative studies presented at the top, middle and bottom of this figure, the magnitudes of the savings are 27 %, 24 % and 14 % respectively.

of efficient design. Further, the observations above suggest that the mass of metal in a design obtained using ACWEDS will be within a couple of percent of the global optimum, and that the software's solutions are consistently and often significantly superior – that is to say lighter in weight – than the designs of professional facade engineers.

#### 3.1. Mullion Shapes

A parametrically-driven and numerically-optimized geometric model, of the sort described in this paper, is a powerful tool capable of finding efficient crosssectional shapes that might, at first, appear strange or irrational, even to an experienced curtain wall designer. Some design features are more prevalent in the solutions obtained from ACWEDS than in the extrusions developed by people, and humans might therefore abstract and learn from the machine-generated solutions. Listed below are several examples optimization strategies revealed by 430 computer-generated features that might appear strange: -

- (a) The internal features of a split mullion do not have to be symmetrical on the male and female sides. Flange thicknesses and boxing arrangements need not necessarily be uniform on the two sides.
- (b) In some instances, when the governing structural design consideration is
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- lateral torsional buckling, it may be more efficient to thicken a profile's webs, and hence increase bending stiffness about the minor axis, rather than thicken the flanges.
- (c) More efficient designs may be achieved if the designer has a good understanding, and is prepared to make full use, of the extruder's capacity to vary metal thickness within a profile.

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Figure 8 shows examples of each of the above points in one optimized split mullion design, created using ACWEDS for a curtain wall system in which the vertical unbraced span of the mullion is almost equal to the panel height.



Figure 8: Optimized male and female profiles, created using ACWEDS, for a curtain wall in which the vertical unbraced span of the mullion is nearly equal to the panel height.

## 4. Conclusions

- If a bespoke curtain wall system is to be created for a particular building, then a set of numerical tools – a parametrically-driven geometric model of the curtain wall, a structural evaluation procedure, and a robust optimization algorithm – may be used together, in combination, to find well-optimized crosssectional shapes for the wall system's extruded framing members. For each one
- of 24 different building facades, existing curtain wall designs conceived by professional facade engineers have been compared with algorithmically-determined solutions. Consistently, the machine-generated extrusion profiles meet the specified performance criteria with less aluminium than the corresponding wall systems developed by experienced human designers. The magnitude of the metal
- $_{455}\,$  saving will vary from case to case, but in this survey it has been easy to find instances in which computational shape optimization techniques can reduce a facade's metal mass by 20 % or more.

The approach to metal minimization adopted in this study does not affect

the number of extrusion profiles in a curtain wall system, nor does it increase the

design's complexity in other ways. If a contractor intends to create a customdesigned wall system for a particular building, then the additional cost associated with optimization of the extrusion shapes is negligibly small: only a little computational time is required.

The observations suggest that widespread adoption of numerical design methods within the curtain wall industry would result in aluminium savings in the hundreds of millions of kilograms per year. Eliminating the need to manufacture this metal would bring sizeable environmental benefits [6, p. 10]: the reduction in annual greenhouse gas emissions, expressed as a mass of CO<sub>2</sub>, would be trillions of kilograms, and the annual energy saving would be in tens of quadrillion

 $_{470}$  (or, expressed another way, more than  $10^{16}$ ) Joules.

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