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Application of Large-scale Layout Optimization Techniques in Structural Engineering Practice

Matthew Gilbert[1], Wael Darwich[1], Andy Tyas[1], Paul Shepherd[2]

[1]Dept. Civil & Structural Engineering, University of Sheffield, Sheffield, UK, m.gilbert@sheffield.ac.uk, w.darwich@sheffield.ac.uk, a.tyas@sheffield.ac.uk
[2]Buro Happold, Bath, UK, paul.shepherd@burohappold.com

1. Abstract
A number of promising techniques which allow treatment of very large-scale plastic truss layout optimization problems incorporating multiple load cases, self weight and practical nodal stability constraints have been developed recently and collaboration with innovative structural engineering consultants Buro Happold has now provided an invaluable opportunity to investigate the effectiveness of these in practice. In use it was found that the recently developed member adding technique in particular was pivotal in permitting real-world scale problems to be tackled in the first place (this allows single load-case problems containing up to approx. 1,000,000,000 potential truss members to be solved on a desktop PC). However, many practically important issues requiring resolution were identified, and these are briefly discussed in the paper. One specific issue is considered in more detail in the paper; this concerns the specification of external applied loads, something which appears to have received scant attention in the literature to date. Whereas in a conventional layout optimization problem the spatial locations of external applied loads are predefined, in practice, e.g. for roof structures, this may be inappropriate since this strongly influences the form of the resulting 'optimum' structure. In an attempt to overcome this, loads were instead applied to groups of nodes. The technique proved to be remarkably successful, and typical results are presented in the paper.
2. Keywords: Layout optimization, plastic design, trusses, practical applications

3. Introduction

Although discrete (ground structure) layout optimization methods have been available for several decades [1], [2], [3] these are very rarely used in practice by structural engineers, e.g. to identify the optimum forms for skeletal structures such as long span roofs and canopies. This is in contrast to topology optimization methods for continuum type problems, which are now successfully applied in practice in the mechanical and aerospace sectors. Unfortunately a continuum idealization is not particularly appropriate for most structural engineering design problems, partly because the proportion of an initial design domain occupied by the final structure is likely to be extremely small. Thus in the structural engineering sector the initial design stage, which generally has a huge influence on subsequent project costs, tends to be carried out in an ad-hoc manner, with individual engineers' intuition typically being used to initially determine optimum member layouts (though member layouts may subsequently be modified as part of an iterative conceptual design → analysis → re-design loop, this process is time-consuming and costly).

Use of non-classical optimization techniques to optimize the layout of trusses, such as Genetic Algorithms or Simulated Annealing, offer the promise of being able to solve problems with complex real-world design constraints. However, to date such methods seem to have failed to live up to their promise, tending to be highly computationally expensive and/or prone to producing locally optimal solutions, far from the global optimum.

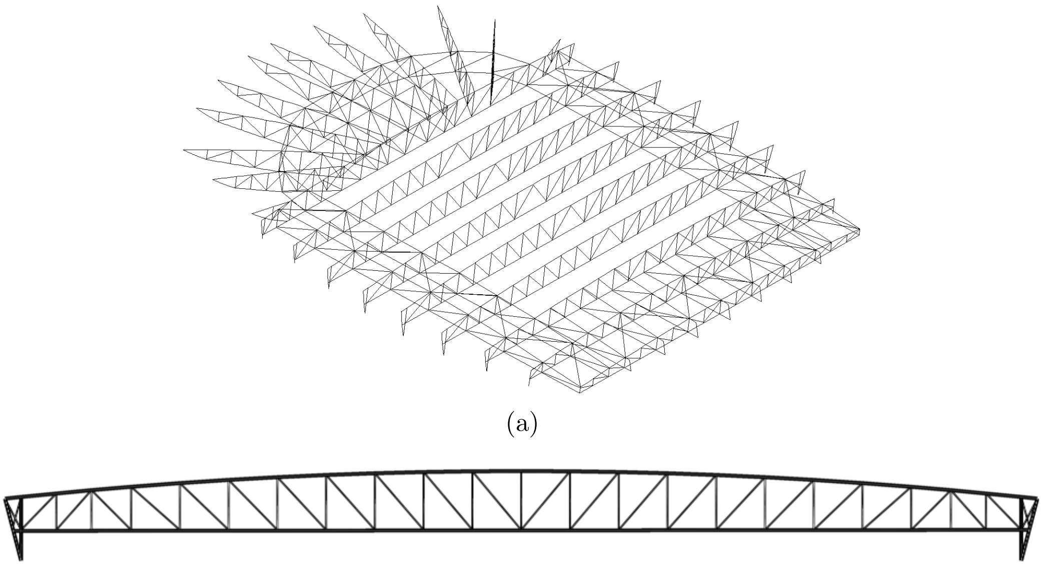
On the other hand, in the field of classical ground structure layout optimization, the preoccupation in recent years of almost all researchers with *elastic* rather than *plastic* formulations has perhaps hindered progress towards the development of practically useful design tools. This is because elastic formulations are generally comparatively difficult to solve when multiple load cases are present (as they invariably will be in real life problems). In contrast, plastic formulations may be solved rapidly using modern interior point based linear programming solvers. Furthermore a number of promising techniques which allow treatment of large-scale plastic layout optimization problems incorporating multiple load cases, self weight and practical stability constraints have recently been developed, e.g. refer to papers elsewhere in the proceedings [4],[5]. It should also be noted that many codes of practice permit the use of plastic design methods and, even in situations where elastic design constraints are instead present, it will often be found that optimum elastic and plastic design topologies are similar. In the special case of single load case problems, elastic and plastic optimum topologies are identical; all example problems contained in this paper fall into this category.

Recent collaboration with innovative structural engineering consultants Buro Happold (designers of the Millennium Dome, London) has provided an invaluable opportunity to investigate the effectiveness of these techniques in practice. Thus this paper describes preliminary findings from the collaboration and focusses on one particular issue which was identified as being important (specification of loading position for roof and other design problems).

4. Practical Considerations

The second author's secondment to Buro Happold provided an invaluable opportunity to evaluate the range of applicability of a current generation layout optimization software tool, and to identify potential limitations - with a view to overcoming these in the future. Many design problems were studied during the secondment. A typical problem was the design for a 90m span exhibition hall. In this case the outline design was complete (Figure 1) and the questions were twofold:

1. How would solutions obtained using the available layout optimization software compare with the outline design obtained manually (both in terms of form and overall volume).
2. Alternatively, could the software usefully be used simply to refine the geometry of the 2D trusses (Figure 1b).



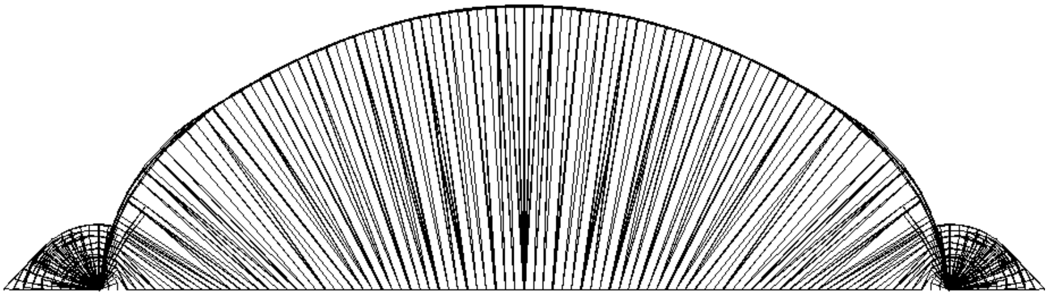
(a)

(b)

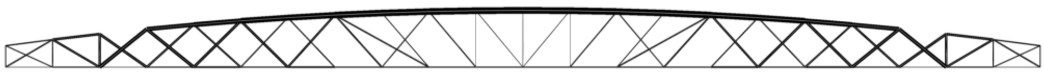
Figure 1: Initial manual design for 90m span exhibition hall roof: (a) 3D view; (b) elevation of sample 2D truss

Early indications were that the currently available layout optimization software tool, whilst powerful for academic problems, was not particularly easy to apply in practice. For example, as expected it was found that rather impractical structures containing numerous members were generated when relatively few design constraints were imposed (e.g. Figure 2a). Imposition of joint cost penalties [6] and/or simply reducing the number of nodes in the design domain led to simpler, more practical forms (Figure 2b, 2c), but of questionable optimality. It was also realised that had the layout optimization solver been incorporated within a user-friendly interactive software package, then it would have been very much easier to rapidly change the design problem in response to feedback from other members of the design team (work on developing such a package is now underway). Attempts were also made to apply the software to a variety of other design problems being tackled in the office at the time (ranging from the roof of a large 60,000 seat football stadium to a temporary canopy roof for a special event). The experience gained led to the following issues being identified:

1. Applying loads to particular specified nodes in the design domain means that these nodes must always be connected by members in the 'optimal' solution. This may be inappropriate in the case of roof structures



(a)



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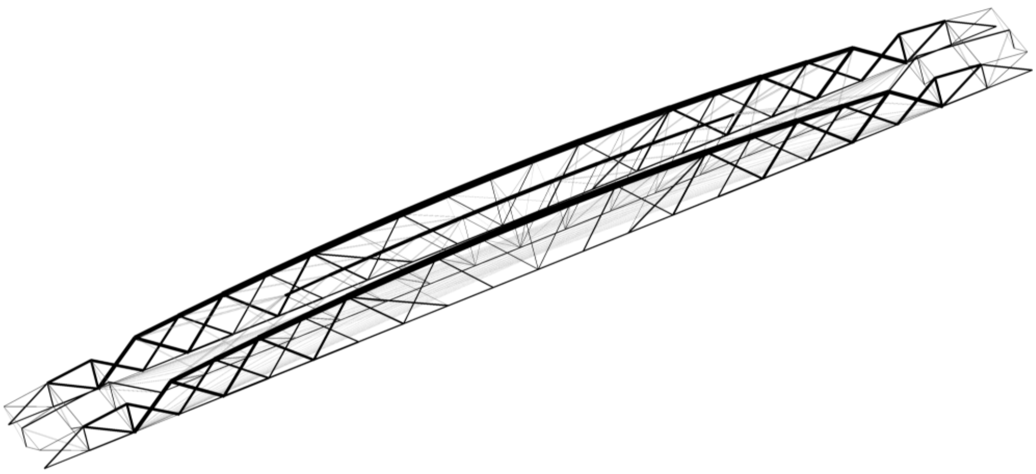

(c)

Figure 2: Initial 'optimized' designs for 90m span exhibition hall roof (10m cantilever side-spans and subject to uniformly distributed vertical load to the bottom chord): (a) sample solution for loosely constrained problem; (b), (c) sample solution obtained using joint costs and coarse nodal refinement

since structural form, and hence the locations where the loads should be applied, will typically not be known in advance of the optimization.

2. Similarly, if loads are applied to nodes in a grid which are closely spaced, all these must be present in the solution. This may lead to a needlessly complex 'optimum' structure.

3. Whereas overlapping/crossing members are not acceptable in practice (especially in the case of compression members), these are often generated by layout optimization software.

4. Roofs made of 2D trusses are typically cheaper than those formed from 3D trusses, even if the 3D truss is lighter (mainly for fabrication and assembly reasons: 2D trusses can readily be assembled on the ground prior to being raised into position).

5. It is often required that in one project relatively few section types and similar trusses are used.

6. Long span trusses should be made of smaller sub trusses that can be fabricated in a factory, then transferred to site for final assembly. The maximum size of those sub trusses is controlled by site access / transportation availability in the project.

7. Supporting systems should be included in the optimization algorithm - otherwise an 'optimum' roof may need very expensive supports.

8. Dynamic behaviour should be considered in long spans. The optimum layout obtained following an initial plastic optimization may typically be assumed to be adequate for serviceability; if the design did not meet specified deflection limits, then all members could be enlarged during a subsequent post-processing phase (in this phase sections obtained by the optimizer could also be upgraded to standard section sizes as necessary).

9. Joints are typically more expensive than members; 3D joints are normally more expensive than 2D joints; having many joints of similar design and dimensions may be better than having a fewer number of joints. Ideally penalties could be applied to joints having members residing in more than one plane.

10. The use of circular hollow sections (CHS) was implicitly assumed by the software since, with commercially available CHS members, there is an approximately linear relationship between applied force and cross-sectional area, as shown in [5]. However these are generally more expensive to use than universal beams and columns, mainly because of joint fabrication costs.

11. Having regular clear spaces between truss members for services is a major issue in design.

Of these, issue 1 has been chosen to be investigated in more detail in this paper. In fact it will be shown that, when using the traditional plastic layout optimization formulation, it is a straightforward matter to specify that that a given specified load is applied to a group of nodes, rather than to a specific node (the use of this technique may also potentially be used to address issue 2 on the list, concerned with the influence of the spacing of loaded nodes on the resulting complexity of the optimized structure).

5. Application of Loads to Node Groups

5.1. The Plastic Layout Optimization Formulation

The classical plastic layout optimization problem for a problem containing n nodes, m members and M load cases is defined in Eq. (1), (2) and (3) as follows:

$$\min V = \mathbf{l}^{\mathbf{T}}\mathbf{a} \tag{1}$$

subject to:

$$\left.\mathbf{B}\mathbf{q}^{\alpha} = \mathbf{f}^{\alpha}\right\} \ \ \alpha = 1....M \tag{2}$$

$$\left.\begin{aligned} a_i \geq \left\{\frac{q_i^+}{\sigma_i^+} + \frac{q_i^-}{\sigma_i^-}\right\}^{\alpha} \\ \{q_i^+\}^{\alpha}, \{q_i^-\}^{\alpha} \geq 0 \\ a_i \geq 0 \end{aligned}\right\} \ \ \alpha = 1....M; i = 1....m \tag{3}$$

Where V is the total volume of the structure, $\mathbf{l}^{\mathbf{T}} = \{l_1, l_2, ...l_m\}$, $\mathbf{a}^{\mathbf{T}} = \{a_1, a_2, ...a_m\}$ and where l_i and a_i are respectively the length and cross-sectional areas of member i. $\mathbf{B}$ is a suitable $(3n \times 2m)$ equilibrium matrix, $\mathbf{q}^{\mathbf{T}} = \{q_1^+, q_1^-, q_2^+, q_2^- ...q_m^-\}$, where q_i^+, q_i^- are the tensile and compressive forces in member i. Also $\mathbf{f}^{\mathbf{T}} = \{f_1^x, f_1^y, f_1^z, f_2^x, f_2^y, f_2^z ...f_n^z\}$ where f_j^x, f_j^y, f_j^z are the x, y and z components of the live load applied to node j $(j = 1...n)$ and σ_i^+, σ_i^- are the limiting tensile and compressive stresses, with α used to denote one of the M separate load cases in the problem.

This problem can be solved using linear programming (LP) algorithms, with the member forces in $\mathbf{q}$ and cross-sectional areas in $\mathbf{a}$ being LP variables.

5.2. Modified formulation

In the normal plastic layout optimization problem formulation described in the preceding section the nodal loads in $\mathbf{f}$ are fixed values, making up the right hand side of Eq.(2). However, consider a variation on the basic formulation in which loads are allocated to groups of nodes rather than individual nodes. In this case, and using a LP formulation, nodal loads in $\mathbf{f}$ may be represented as additional LP variables, with additional constraints introduced to ensure that the total load applied to a given group is appropriate. e.g. for node group s comprising p nodes, and subject to a load in the x direction of magnitude $\hat{f}_s^x$, the requisite constraint is simply:

$$\left.\sum_{j=1}^{p}\{f_j^x\}^\alpha = \{\hat{f}_s^x\}^\alpha\right\} \alpha = 1....M \tag{4}$$

Similar constraints for the y and z directions are clearly also required.

It should be noted that this simple formulation does not stipulate that all loading is necessarily applied to a single node in a given node group (this issue is touched upon again in section 6). Furthermore, additional constraints must be introduced if it is required that: (i) a node which receives a certain proportion of the specified x direction load (say), receives an identical proportion of the y direction load (say); (ii) a node which receives a certain proportion of the specified load from load case α receives an identical proportion of load from load case $\alpha + 1$, etc.

A further feature of this modified formulation is that the dual virtual member strain limit constraints used in the member adding method [4],[7] remain unchanged. This means that the member adding method can be used without modification to solve large problems with loads applied to node groups.

6. Example Problems

In order to test out the efficacy of the procedure described above, two simple 2D benchmark problems were first investigated. The procedure was than applied to a sample 3D design problem.

For all problems, purpose written software was used to set up and then modify the linear programming problem at successive iterations using the member adding procedure [4],[7]. The MOSEK (version 3.2) LP solver which uses a homogeneous and self-dual algorithm (www.mosek.com) was used for the numerical studies. The pre-solve feature was switched off and the problem was initially passed to the solver in memory using the supplied subroutine library. Subsequently it was only necessary to pass changes to the current problem to the solver (rather than the entire revised problem). Problems were run on a 2.4GHz AMD Opteron (64bit) based Linux machine with 16Gb of RAM available. Since the solutions produced by the LP solver contain numerous members of vanishingly small cross-sectional area, a filter was used to remove these prior to plotting the solutions shown in the paper.

6.1 2D Cantilever Benchmark Problem

Hemp [2] provides the exact volume of the optimal cantilever for the problem defined in Figure 3a ($V = 4.34l$, assuming unit compressive and tensile strengths, where l is the cantilever length).

Using a 151×151 node discretisation (259,931,400 potential members), and with a load applied only to the node located at mid-height of the domain remote from the supports, a numerical solution of $4.326l$ was obtained (Figure 3b), which is within 0.3 percent the volume given in [2]*. Instead applying the load to a node group containing all nodes in a vertical group remote from the supports led to exactly the same solution being found.

With the point load again applied only to the node located at mid-height of the domain remote from the supports, but with the tensile strength (only) increased to 3 units, a volume of $2.852l$ and a slightly different optimal form was obtained (Figure 3c). However, specification of a loaded vertical node group remote from the supports coupled with use of the increased tensile strength led to the more optimal volume of $2.808l$ being found, and a rather different optimal form, loaded at a very different vertical elevation (Figure 3d). All these results are summarized in Table 1 together with CPU times. From the table it is evident that the application of load to a node group extended run time by less than 20 percent.

6.2 2D Benchmark Arch Problem

The main purpose of implementing the node grouping functionality was to permit problems involving distributed loads to be tackled. Thus Figure 4a provides details of a simple single span example problem with a design domain comprising 201×101 nodes (206,055,150 potential members), in which a vertically acting uniformly distributed load is applied either to nodes at the bottom of the design domain, or to vertical node groups (applied to inner nodes only, i.e. not over the supports). In the former case an arch form with hangers was obtained (Figure 4b).

*Approximate-discretised solutions of the sort presented here should however always overestimate the exact volume, something which was found not to be the case here. At the time of writing the reason for the small discrepancy identified is unclear.

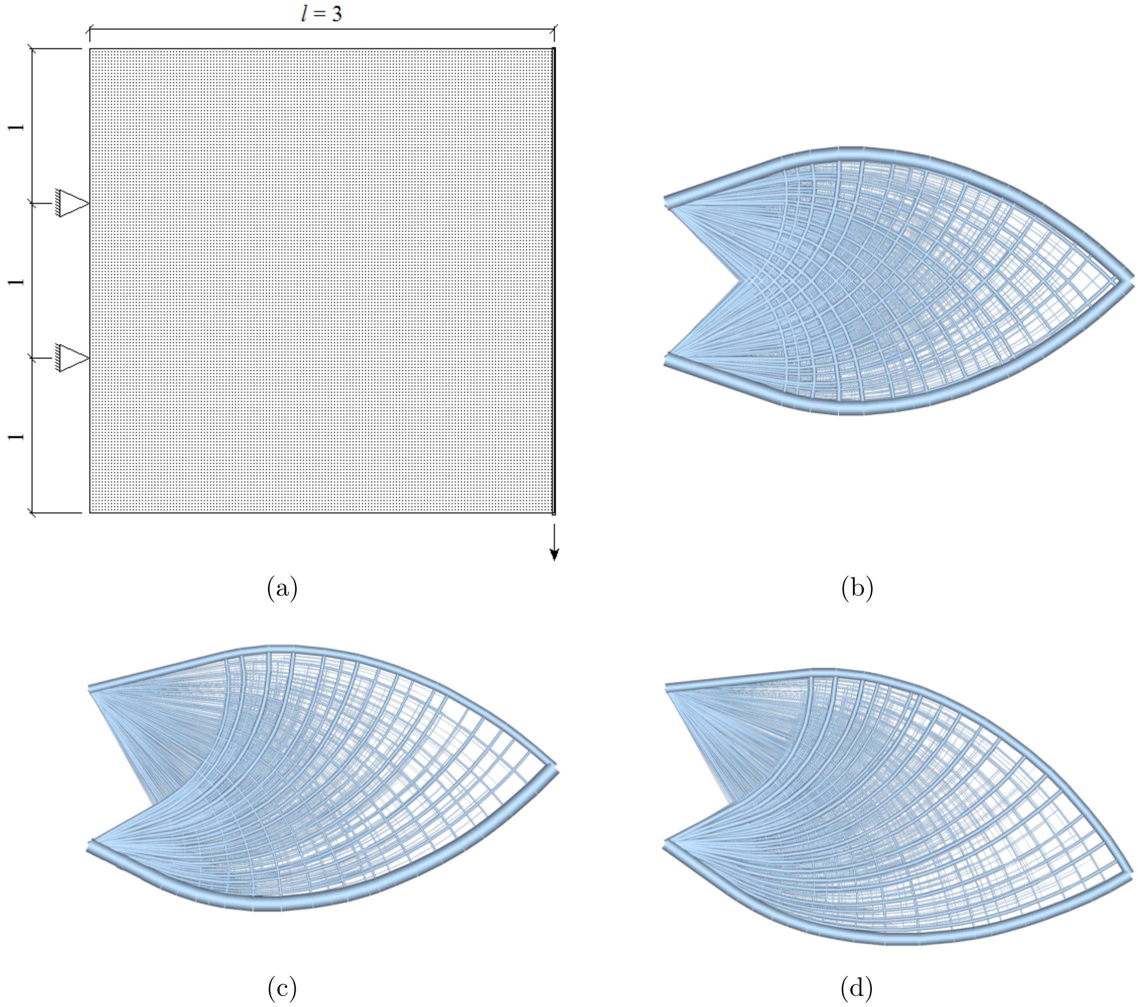

Figure 3: Benchmark 2D cantilever problem: (a) design domain; (b) solution with $\sigma^+ = \sigma^-$; (c) solution with $\sigma^+ = 3\sigma^-$ (load on mid-height node); (d) solution with $\sigma^+ = 3\sigma^-$ (load on tip node group)

In the latter case it was expected that the solution would tend to that of a parabolic arch. The required volume of such an arch can be shown to be:

$$V = \frac{wl^3}{8h\sigma}\left(l^2 + \frac{16h^2}{3}\right) \tag{5}$$

with the volume being minimised when the height is given by:

$$h_{opt} = \sqrt{\frac{3}{16}} \times l \tag{6}$$

where w is the intensity of the distributed load, l is the span and σ is the strength of the material.

The computed optimum design is shown on Figure 4c. This closely approximates to the parabolic form expected (although a number of secondary members do also radiate out from the supports). Computed optimum volumes are summarised in Table 2, together with average mid-span heights (from Eq.(6) this theoretically should be 0.4330 in the case of the parabolic arch without hangers) and CPU times.

By further increasing the number of nodes in the horizontal direction the volume converged towards the theoretical optimum solution ($1/\sqrt{3} = 0.5774$); convergence characteristics are shown in Figure 5.

Thus the indication from this and the previous 2D benchmark problem is that: (i) when a load is applied to a node group typically one or perhaps two constituent nodes will take the vast majority of the specified group

Table 1: Numerical results for benchmark 2D cantilever problem.

Problem	Single loaded node		Load applied to node group	
	Volume	CPU time (hours)	Volume	CPU time (hours)
$\sigma^+ = \sigma^-$	$4.326l$	01:43	$4.326l$	02:04
$\sigma^+ = 3\sigma^-$	$2.852l$	02:10	$2.808l$	02:32

Table 2: Numerical results for benchmark 2D arch problem.

Single loaded nodes			Load applied to node groups		
Volume	Height	CPU time (hours)	Volume	Height	CPU time (hours)
0.7930	0.3350	05:24	0.5788	0.4477	01:34

loading, even though no constraints were explicitly introduced to enforce this; (ii) the procedure can thus be used to identify the optimum position to apply loads.

6.3 3D 'Form Finding' Problem

Figure 6 shows sample 3D 'form finding' solutions using a cuboidal design domain (containing up to approx. 125 million potential members) and with various base support conditions.

The resulting design solutions are reminiscent of the vaulted roofs of historic masonry buildings. Whilst it was not the original purpose of applying loads to node groups to undertake such form finding studies, it appears that the procedure outlined in the paper potentially forms the basis of an interesting and apparently new form finding tool. Such a tool could potentially be useful in the conceptual design stage; the full range of applicability of the tool remains to be investigated.

7. Concluding Remarks

Collaboration with the engineering consultants Buro Happold is providing an invaluable opportunity to investigate how layout optimization software tools may be applied in structural engineering practice. Whilst it has been found that real-world structural engineering design constraints are highly complex, and that the useability of current generation software needs to be significantly improved, it would still appear that practising structural designers, who currently still have to rely on their intuition in order to arrive at sensible structural forms, could potentially benefit enormously from the use of even simplistic automated design tools - both in the initial conceptual design stage and when identifying locations for members in a specific component (e.g in a roof truss). Such tools need not necessarily take account of all possible real-world constraints from day one since the design solutions obtained using optimization software can always be refined as required in the detailed design phase.

The application of loads to *node groups*, described in the paper, appears to extend the usefulness of traditional layout optimization methods when applied to structures such as canopies and roofs (for such applications, specifying the location of the loads has the effect of predetermining the resulting form). Since the overall problem formulation is not significantly changed, the powerful member adding procedure can still be used, potentially permitting large practical problems to be tackled in the future. In this paper it has been demonstrated that the approach is applicable to both 2D and 3D problems, including those where members have differing compressive and tensile strengths. Application of the approach to multiple load case problems is currently being investigated.

8. Acknowledgments

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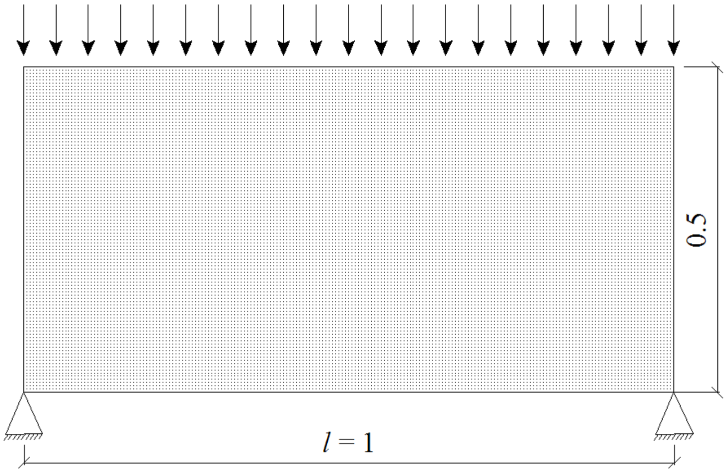
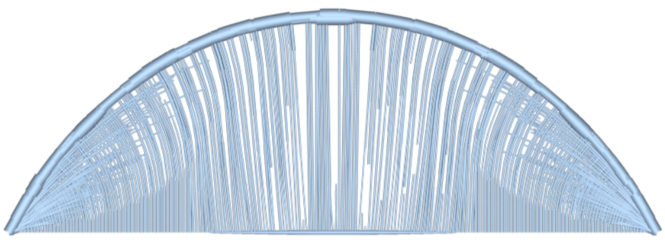


(a)



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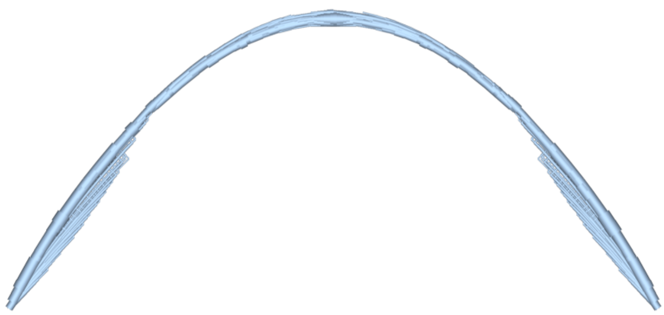

(c)

Figure 4: Benchmark 2D arch problem: (a) design domain; (b) solution with load applied at $y = 0$; (c) solution with load applied to node groups

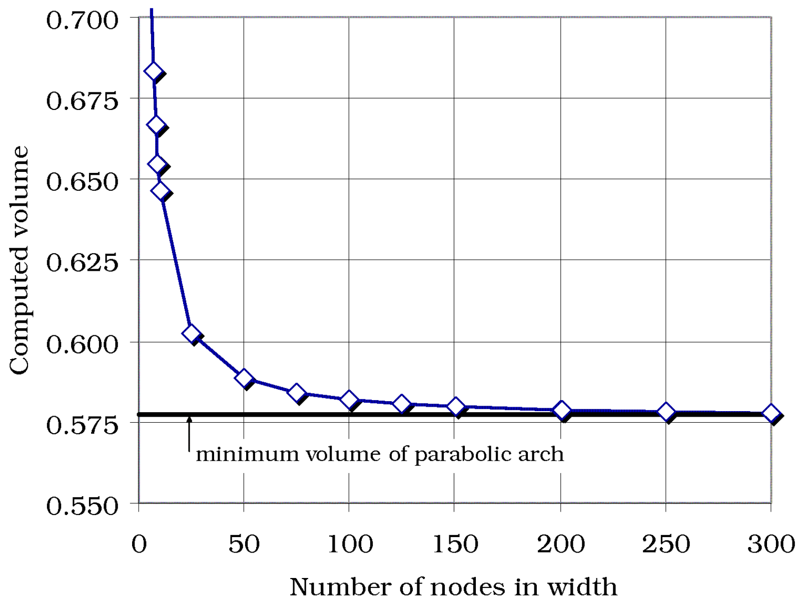

Figure 5: Benchmark 2D arch problem: convergence characteristics when load applied to node groups

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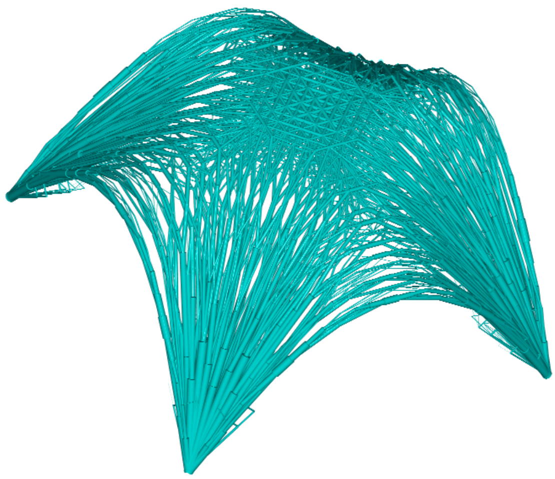
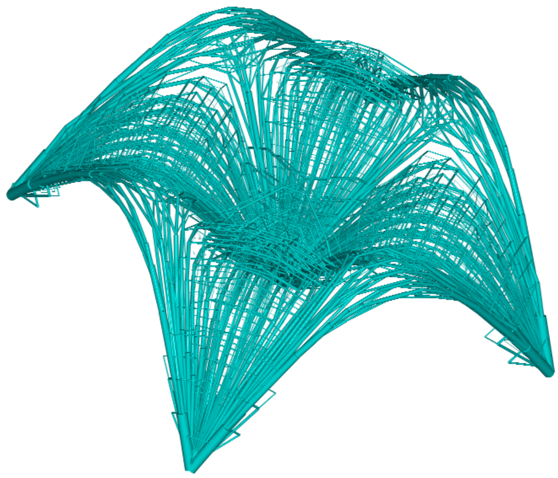
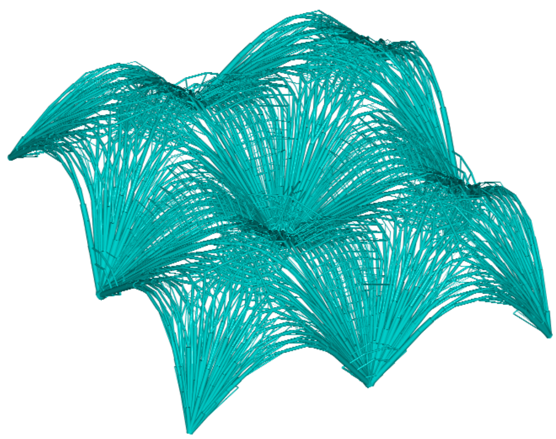


(a) Four supports



(b) Five supports



(c) Nine supports

Figure 6: Optimal solutions for 3D 'form finding' problem