

# Subdivision Surfaces for Integrated Design, Analysis and Optimisation

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## Summary

The research described in this paper combines simple fast analysis and optimisation routines with in-house subdivision surface based parametric modelling software. The aim is to demonstrate a new collaborative environment in which engineers and architects can work closely together, and get instant feedback on the effects of design decisions in terms of structural and environmental performance. A real-life case study project, the design of an ETFE cushion-clad dome structure as an extension to the Aarhus Botanical Garden in Denmark, is used to demonstrate the benefits of adopting a subdivision surface based concurrent design approach in the construction industry.

**Keywords:** *Digital Architectonics, Subdivision Surfaces, Structural, Environmental, Optimisation, Case Study, Aarhus.*

## 1. Introduction

The Digital Architectonics Research Group in the Department of Architecture & Civil Engineering of the University of Bath, with the support of sponsors Informatix Inc., is currently developing new software-based design tools. These tools combine the innovative use of Subdivision Surfaces with multi-objective optimisation routines to provide a collaborative software environment for Architects and Engineers to develop efficient concept-stage designs.

The current building design process is usually very linear. An architect develops an initial design, which is passed on to the structural engineers to make it stand up and the environmental engineers to make it habitable, before being passed further down the line to the contractors to build it. Often this linear process results in the design becoming locked in to a particular scheme very early on, thereby removing many opportunities for innovation and collaboration to develop a truly holistic solution.

The best designs seem to emerge when the whole design team (architects, engineers and contractors) collaborate right from the start and develop schemes together. And with recent developments in commercially available parametric modelling tools this is becoming more common [1, 2]. However, lack of integration between these geometrical modelling environments and the engineers' analysis software means that a well informed interactive discussion cannot take place.

The focus of the research described in this paper is therefore to combine simple fast analysis and optimisation routines within a subdivision surface based parametric modelling software environment. In this way, creativity is not constrained by the computer, but the design team is constantly informed of the repercussions of decisions through graphical feedback in terms of the resulting spatial, environmental or structural performance. This feedback can be provided on a

range of metrics spanning across structural, environmental and project management disciplines, so long as the assessment can be described algorithmically and the results calculated to an acceptable degree of accuracy in a short time. This “concurrent design” approach means that the complete design team can have informed and meaningful discussions on the relative merits of various design options right at the start of a project.

Whilst the combination of modern computing power and closer interoperability of analysis tools offers tantalising opportunities for automatically optimising building designs for many different criteria at the same time, these different design drivers can sometimes be contradictory. Therefore robust methods of determining sensitivity and optimising for multiple objectives are also needed. This paper briefly describes subdivision surfaces and outlines reasons why they should be of particular interest to building designers. It then proceeds to outline the benefits of taking an integrated and collaborative “concurrent design” approach to the building design process. This is then justified through a real-life case study project - a steel and ETFE cushion extension to the Aarhus Botanical Garden in Denmark - which demonstrates the benefits of adopting a subdivision surface based integrated design, analysis and optimisation approach to building design.

## **2. Approach**

### **2.1 Subdivision Surfaces**

Subdivision Surfaces are well established surface modelling tools for computer gaming and animation [3], but have been overlooked by the construction industry despite presenting many benefits over more widespread techniques such as Splines or NURBS. By starting from a relatively coarse triangulated (or quad) mesh, each edge of the mesh is split into two by introducing a new vertex at the middle, and each facet is then re-meshed to incorporate these new vertices. Successive “subdivisions” of this type lead to finer versions of the mesh in a recursive manner, which eventually converge onto a single “limit surface”. The positions of the newly generated vertices are carefully chosen, based on the other surrounding vertices, to result in a mathematically smooth (C2 continuous) limit surface [4]. Such smooth surfaces are aesthetically attractive and the focus of the research described in this paper is on exploiting these surfaces as proposed building envelopes.

The inherent recursive level of detail which subdivision surfaces provide can be exploited in a number of ways for building design. The mesh facets can be used to represent cladding panels, or the mesh edges to represent structural members. The user can then sample the limit surface at any desired level of detail to result in panels or members of the desired size. For example if a surface is to be clad in planar glass triangles, and glass can only be sourced economically in 2m wide sheets, then the surface topology can be easily be sampled at a subdivision level where all facets have edge-lengths less than 2m.

This readily-available hierarchy of multiple levels of detail is even more useful when combining geometric modelling with multi-disciplinary engineering analysis. Finite element analysis for example may require particularly small mesh elements in order to calculate the structural behaviour of a proposed building structure to a reasonable degree of accuracy. However, the thermal or acoustic performance of the same structure might be calculable from a much coarser mesh. By using a subdivision surface as the basis for the model, the proposed building geometry can be sampled separately from a single definition, at exactly the right level of detail for each individual analysis, with very little extra overhead in terms of geometry processing. Subdivision Surfaces can also be constrained to lie along a specified boundary, thereby allowing the designer more control over the resulting form.

If subdivision surface modelling can be seen as a kind of coordinate smoothing algorithm, then other properties, not just coordinates, can be smoothed at the same time. Not only can a newly created vertex position be carefully calculated as a weighted-average of the coordinates of its neighbours. But this weighted-average can also be applied to other properties, such as colour, texture, porosity or louver opening angle, thereby smoothly distributing coarsely defined properties across the entire building envelope.

## **2.2 Integrated Analysis and Optimisation**

In order to facilitate a concurrent engineering approach to building design, software needs to be able to give interactive, graphical feedback on the effects of design decisions. The same framework can then be used to iterate through many design options and this feedback can then be used to choose the best solution to the problem at hand.

In order to be of use to the design team, the feedback provided by the software needs to be virtually instantaneous, in order to allow informed decision-making without stifling creativity. To achieve this, the analysis may need to be approximate and may therefore need to make many assumptions in order to suggest whether the current design is better or worse than an alternative. It is not proposed that such software would ever lead to the role of the building engineer becoming obsolete. Any resulting design would still need to be carefully calculated and engineered to assess and refine its structural and environmental performance and sensible design solutions adopted. However, in the early stages of design, when decisions are being made which quickly become difficult to retract, it makes sense for these choices to be made against some sort of knowledge of the down-stream consequences in terms of building cost, both for construction and in-service operation.

Once this rapid, automatic assessment of performance is available, the designers can then choose to devolve some decision-making to the software. In this sense the computer could generate many design options by changing the values of the parameters defining the geometry, assess the relative performance of each against some pre-defined criteria, and search this vast solution space until a satisfactory, or optimal, set of input parameters is found. This “closing the loop” of the design process is not an inherent part of the concurrent engineering approach, but is certainly a useful addition, which can help speed up the search for an efficient design and plays to the strengths of a subdivision surface modelling framework.

## **3. Aarhus Botanical Garden**

### **3.1 Background**

In November 2008 an architectural competition was launched to generate designs for the construction of a new tropical hothouse as an extension to the existing 1969 glass-house in the botanical garden of the University of Aarhus in Denmark. As part of a team including architects C.F. Moller, who designed the original glass-house, and engineers Soren Jensen, the authors were able to apply the subdivision surface modelling software resulting from this research to a real-life design project for the first time. The aim was to validate the integrated analysis and optimisation approach to building design, whilst at the same time develop an innovative and efficient design proposal for the architectural competition.

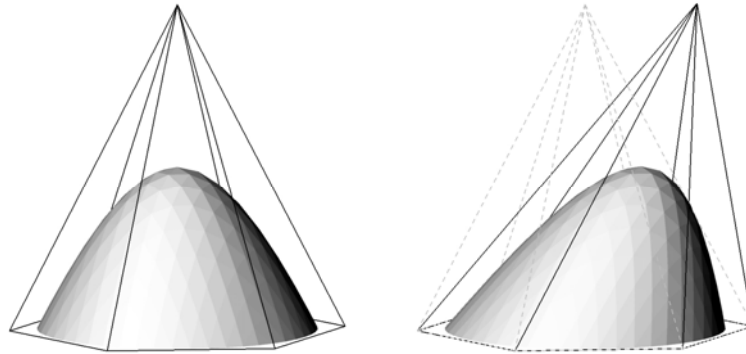
Due to the distributed nature of the design team (the authors were based in the UK and the architects and engineers based near the project site in Denmark) a robust method of collaboration was developed. As well as heavy use of video conferencing and email, scale models of design proposals were transmitted electronically between the UK and Denmark and then directly 3D printed in colour to facilitate further design discussion during face-to-face meetings.

The design development was split into two phases, to establish a surface geometry representing the proposed building envelope, and to develop a supporting structure for this envelope. Ideally, these two tasks would have been performed in parallel, to allow a holistic solution which would be optimised in terms of both environmental and structural performance. However, since the timescale of the design project was very tight, this was not practically possible and the two studies were performed sequentially. Firstly a building envelope geometry was derived based on an optimisation of the environmental performance of the building subject to multiple, and sometimes conflicting objectives. When a shape had been decided upon, a structural grid capable of supporting this building envelope was derived through geometrical and structural performance evaluation.

Since this case-study was hugely valuable in pushing the research forward and testing the approach against real, practical constraints and objectives, the study was continued and further developed after the competition design had been submitted, and this is documented at the end of this section.

### 3.2 Geometry Studies

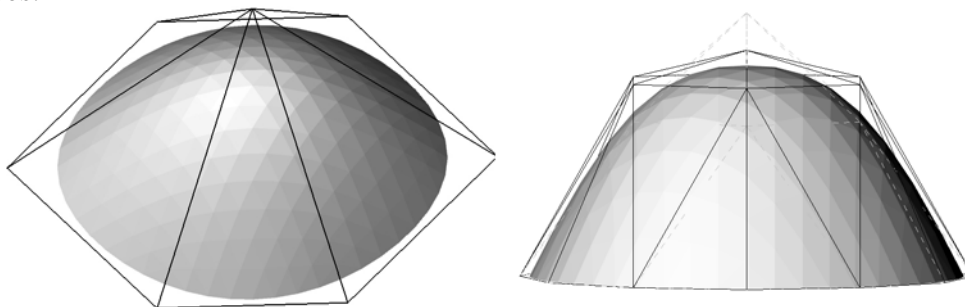
As outlined above, subdivision surfaces define a very smooth surface based on a very coarse control mesh. In order to describe likely dome-shaped building envelopes for the design competition, an initial polygonal mesh of only seven vertices was specified, six on the ground plane in hexagonal form around the coordinate system origin and a seventh directly above the origin, as shown in Fig. 1.



*Fig 1: Control polygons of initial (left) and leaning study (right) and the resulting smooth surfaces.*

The premise of the study was that in order to house tropical plants inside the building, and given that the building was situated in northern Denmark, significant amounts of heat would have to be supplied to keep the plants alive, especially during winter. Therefore the main driver for environmental optimisation was that of maximising solar gain through a very transparent building facade. Given the northerly location of the site, it was expected that the solar gain of the building would be particularly sensitive to the area of the facade facing south and thereby collecting more energy from the southern sun.

In this way, the problem of where to place hundreds of different vertices to describe a smooth dome-shaped building was at first reduced to a single degree of freedom problem, that of how much to move the upper apex vertex towards the north. A second study looked at stretching the dome in the east-west direction, thereby resulting in an elliptical footprint for the building [Fig. 2 left] and again presenting more of the facade towards the southerly sun. This second study introduced another single degree of freedom, which affected only the placement of the two east-west ground-plane vertices.



*Fig 2: Further studies involving an elliptical footprint (left) and more volume (right).*

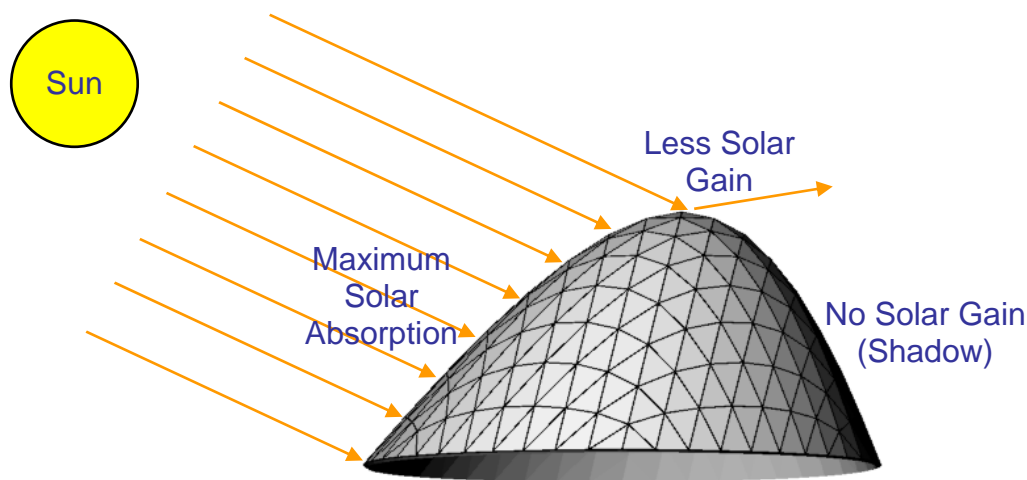
In discussion with the architects, the resulting candidate dome shapes were seen as rather too pointed, the more polite of the comments being that it resembled a “bra”. Therefore, in order to generate more volume for the building without greatly increasing the surface area, the initial control mesh was subdivided once. This in itself simply creates a finer control mesh for exactly the same underlying limit surface. However, newly created vertices of this finer mesh, the ones around the original apex vertex, could be translated vertically upwards, resulting in a more “bulging” and “less pointy” limit surface [Fig. 2 right].

The resulting control polygon therefore contained only 19 vertices and was defined through only three parameters, “lean”, “ellipse” and “bulge”. Using this simple geometrical definition, a smooth and aesthetically pleasing limit surface could be described, capable of being sampled recursively at any desired level of detail. The only question thus remaining was that of which combination of the three variables led to the “optimum” design solution.

### 3.3 Geometry Optimisation

Using the scheme outlined above, many different candidate dome structures could easily and quickly generated, however, in order to be able to assess the performance of each proposed dome, some sort of automatic calculation was needed, so that the design process didn't have to be put on hold whilst each individual option was tested manually.

The dominant measure by which the performance of each dome was being assessed was that of the ability to capture energy from the sun. Therefore, an automatic calculation of solar gain was introduced into the modelling software. This calculation was based on the work of Reda & Andreas [5] to calculate the position of the sun at any given time of day and day of the year, combined with a simple model for calculating the solar transmittance and reflectance of glass using the Fresnel Equations [6]. This information was then used by the software to sum-up the total amount of radiation which would enter each proposed dome geometry over an entire day, taking into account the angle of incidence of the sun's rays on each facet of the dome and the fact that the dome can self-shade, as shown in Fig. 3.



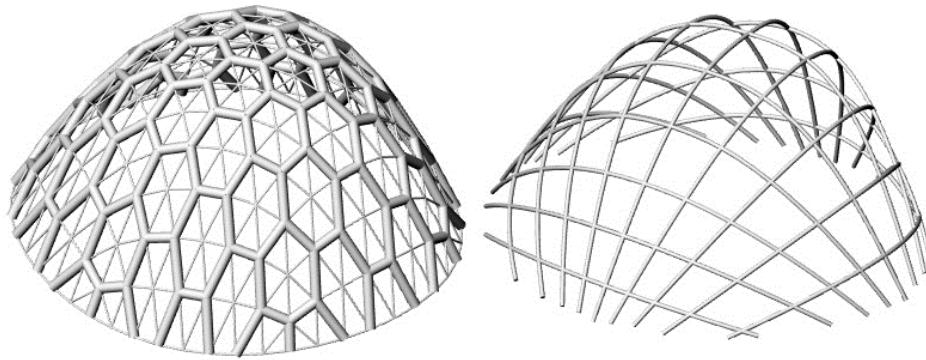
*Fig. 3 Simple model to calculate solar gain of proposed building geometry*

Additionally, the surface area of each proposed dome was included in the optimisation as a variable to be minimised, since it was assumed to be indirectly linked to the cost of construction, and the volume of each proposed envelope was to be maximised to allow more usable space inside.

Within the tight timescale of the competition, the actual optimisation of the form was conducted through only a semi-automatic process, whereby the parametric model was used to automatically produce a number of options exhibiting the full range of domes produced by varying each parameter. Each option was also automatically assessed based on the above described criteria of solar gain, area and volume. The results of these studies were then sent to the environmental engineers as graphs, from which an “optimal” combination of the parameters was chosen by manual inspection. This combination of parameters was then used to generate a final design option as a 3D model, rapid-prototyped into a scale 3D physical model and send to the architects for inclusion in their landscape 3D physical model to be seen in context.

### 3.4 Structure Studies

Once an efficient geometry had been derived, some sort of structure was required to support it. Using a subdivision surface framework facilitates the task of deriving a structural grid in a number of ways. The most obvious structural grid is simply to use the subdivision mesh edges as structural members. The hierarchical nature of subdivision surfaces therefore offers a number of successively finer grids, and a sensible level of mesh refinement can be chosen to respect the construction system and materials. A more complicated approach would use only subsets of these potential members, to give patterns of members, or alternatively a hierarchical structure could be derived with members grouped into primary and secondary elements, each with a different cross-section size [Fig. 4, left].



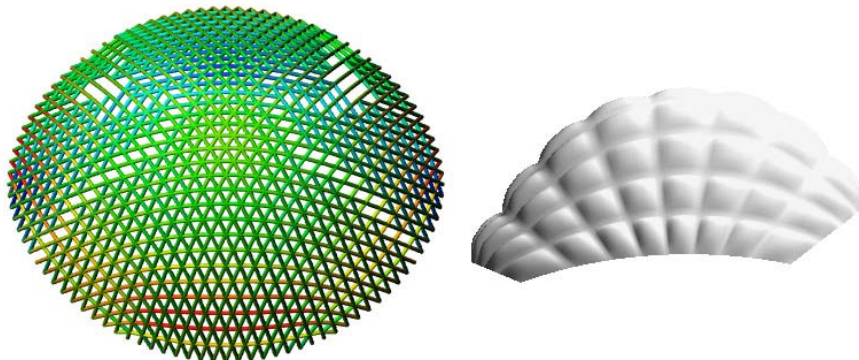
*Fig. 4 Options for structural grid, showing hierarchical members (left) and radial planes (right)*

For the Botanical Garden project, various subsets of the subdivision grid were proposed, and an investigation into growth algorithms was also conducted which recursively “grew” structure from support points along the mesh edges in a recursive, tree-branching-like manner. However, due to constructability requirements, and the fact that by this time the design had moved towards an ETFE cushion solution for the facade, a different approach to structure was eventually followed.

The subdivision surface was parametrically cut with planes to result in planar (singly-curved) structural elements. After a number of parametric studies on the orientation, number, rotational offset and spacing of these planes, two orthogonal “fans” of cutting planes were chosen with a particular angular spacing and centre of rotation to give sensible numbers and sizes of the ETFE cushion panels with sensible member sizes for the supporting steelwork [Fig. 4, right].

### 3.5 Structure Optimisation

Since the modelling software was integrated directly with structural analysis software, a study to identify which members were carrying more load was relatively simple. The parametric model was used to generate a candidate dome geometry this geometry was automatically sent to the finite element analysis software and the member stresses calculated under a nominal self-weight loadcase. The modelling software then received these results and removed any unstressed members before re-submitting the model to analysis once again. In this way, an automatic optimisation of structure was possible, with the resulting structure converging on a proposal where all the members were carrying significant level of stress [Fig. 5, left]. Whilst not eventually used for the derivation of the structural scheme, this exercise was indeed informative in highlighting the areas of structure where members could be removed, to provide a doorway or emergency exit for example.



*Fig. 5 Optimisation results, removing unstressed members (left) and ETFE formfinding (right)*

The inherent analysis capability of the modelling environment to conduct dynamic relaxation formfinding was also of benefit in deriving the likely appearance of the ETFE cushions for the proposal. A coarse initial mesh defining the large quadrilateral panels of ETFE was subdivided further to generate a finer analysis mesh for formfinding. Any vertices lying on the original mesh edges were fixed in position to represent the fact that they would be attached to the stiff structural frame. A force representing an internal pressure was then applied and a dynamic relaxation analysis [7] performed to find the minimal surface which balanced this internal pressure. In this way the outer-, and with a negative pressure the inner-surface of an ETFE cushion could be generated with enough accuracy to create a realistic render [Fig. 5, right] for competition submission.



### 3.6 Competition Entry

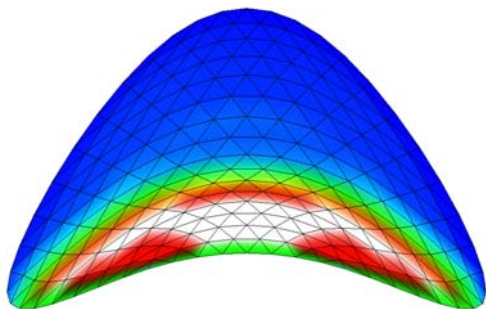


*Fig. 6 Render of successful competition entry*

The resulting design proposal encompassed an environmentally optimised form within an aesthetically desirable, smooth subdivision surface geometry [Fig. 6]. It was the result of a tight collaboration between academia, engineers and architects and was submitted into the design competition as one of six internationally acclaimed and specially invited teams of building professionals. The design won this competition, is currently on-site in Aarhus, and is due for completion in 2012. This demonstrates the success of taking a collaborative and integrated approach to building design, and suggests that a parametric subdivision surface framework has great potential as a design tool for the construction industry.

### 3.7 Post Competition Work

Since the design development process raised a number of interesting questions which could not be investigated fully within the timescale of the design competition, the research was continued after the competition deadline, in order to be able to learn more from the project.



*Fig. 7: New parameter defines curved footprint*

The parametric model used to generate each candidate dome was scripted in the software environment such that design options could be generated by the computer and automatically assessed. The parameter set was also extended to allow the footprint to be curved on-plan [Fig. 7] in order to investigate whether this could be used to collect more solar energy by tracking the sun more in the winter.

The optimal combination of parameters was then determined automatically using simulated annealing after Kirkpatrick et al [8]. This required the measures of success (solar gain, area, volume) to be combined into a single, weighted, measure of performance, based on the environmental engineers' input for the design competition.

## 4. Conclusions

Through the Aarhus Botanical Garden case-study, the benefits of a concurrent engineering approach to design, specifically the integration of quick performance analysis and optimisation routines into a parametric modelling environment have been demonstrated. Furthermore, the many advantages of adopting a subdivision surface based geometry, in terms of a minimal control polygon providing various levels of detail, have also been shown. If more building design professionals adopted this approach, a new collaborative environment could be created in which engineers and architects could work more closely together from the start, and more efficient designs would result from the ability to make informed decisions early on in the design process. The authors are now looking to further develop the range of applicability of their research, through the combination of more powerful parametric modelling tools and a more extensive range of performance evaluation and optimisation algorithms.

## 5. Acknowledgements

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