Form-Fitting Strategies for Diversity-Tolerant Design

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Abstract

Conventional structural design proceeds by establishing a rough geometry for a structure, determining internal forces, and sizing members to resist these forces. This design process assumes effectively *infinite* supplies of *standardised* elements. There are some cases, however, where structural materials are available in strictly finite quantities, or may not be processed into standard sections. In these instances, conventional design approaches are slow or ineffective, requiring time-consuming trial and error to develop viable designs, or not finding solutions at all. This paper proposes new computational design strategies to help designers match *finite* sets of *diverse* structural elements with desired structural forms. The methods proposed build on algorithmic techniques developed for the Bin-Packing Problem. Two key applications are discussed: the reuse of steel elements from deconstructed structures, and the use of unsawn round timbers in spatial structures.

Keywords: component reuse, structural design, structural optimization, form-fitting, computational tools, algorithms.

1. Introduction

When extracted carefully and inspected for quality, steel elements from deconstructed structures can be reused safely in new structures (Gorgolewski [5]). Milford [8] estimated that reuse of steel in new construction could result in CO_2-equivalent emissions reductions as high as 80-90%, accounting for 10-20% member over-specification by mass. However, a 2012 survey of UK demolition contractors found that only 7% of primary structural steel from deconstructed buildings was reused (Sansom and Avery [13]). Among the barriers to increased reuse identified by Gorgolewski [5] was a need for greater flexibility in the design process to deal with the uncertainties posed by not being able to specify "off-the-shelf" components. New design tools to address this challenge could increase the reuse of structural steel elements in new buildings.

Minimally processed round timbers are an architecturally exciting, low-impact structural material with potential applications in spatial spanning structures (Wolfe [15]). Round timber is more dimensionally stable than sawn timber, so does not require kiln-drying, resulting in a savings of 66-72% CO_2-equivalent emissions per kilogram compared to sawn timber (Hammond *et al.* [7]). Recent structures (Mollica and Self [9], Sahu and Wang [12], Festival Foods [3]) have demonstrated the use of low-cost LIDAR and photogrammetry for 3D survey of round timbers. These projects also used digital design and fabrication tools to address the challenges of generating and fabricating complex geometries using geometrically diverse round timbers. However, the challenge of designing structures with finite inventories of non-resizeable elements still presents a major challenge to wider adoption.

These two key applications point to an exciting opportunity in structural design. New digital survey and database tools are increasingly allowing designers to explicitly consider inventories of unique, diverse structural elements in their design process. Digital fabrication tools allow for the cost-effective

realisation of designs with complex geometries. This research addresses the gap between these tools – the discovery and optimisation of viable structural forms given finite inventories of diverse elements.

2. Diversity and Scarcity Tolerant Design

2.1. State of the Art of "Form-Fitting"

The problem addressed in this paper can be described as "form-fitting" – finding good "fits" between a finite inventory of available structural elements and a desired structural form. Several authors have considered this problem (Mollica and Self [9], Monier *et al.* [10], Sahu and Wang [12], Stanton [14]). Each used fully- or semi-automated computational workflows to make the form-fitting process faster and more effective at finding "good" solutions. Their approaches can be roughly grouped into "growth", "attraction", and "fitting" methods. In growth methods, elements are added to one another sequentially to achieve a geometric fit, usually guided by target geometry (points, curves, or surfaces). Attraction methods relax this connectivity constraint by placing elements onto a target geometry, such as a target surface, and then "attracting" them together to achieve a geometric fit. Fitting methods require the designer to specify a pre-defined structural geometry. Elements in this geometry are then progressively replaced by elements in the inventory. A key limitation of growth and attraction methods is that they provide no information about structural behaviour until a non-mechanism structure is found, assuming one is found. Fitting methods provide an analysable structure throughout the design process, meaning that elements from the inventory can be selected based on the required resistance at that location in the structure. Fitting methods, however, restrict solutions to those which match the initial pre-defined topology. This paper will focus on fitting method approaches.

2.2. The Basic Form-Fitting Problem

The basic problem of form-fitting using the fitting method can be framed as a generalisation of the classic Bin Packing Problem. The objective of the bin-packing problem is to find an assignment of "items", each with some size $0 < s_i \leq 1$, into a set of "bins", each size 1, such that the total number of bins used is minimised, and no bin is overfilled (Coffman *et al.*[2]).

This problem can be easily adapted to the case of form-fitting an inventory of linear bars to a statically determinate pin-jointed truss (Fig. 1). In this case, items represent elements in the structure, and bins represent elements in the inventory. Items are cut from bins to fit into the structure. As in the conventional bin-packing problem, no splicing of bins is allowed. For this problem, bins are also assigned a "resistance", and items are assigned an "effect", representing their axial compressive/tensile strength and axial compressive/tensile force, respectively. A new constraint is also added to the problem: the effect of an item must be lower than the resistance of the bin it is placed in. If formulated with the same objective as the classic bin packing problem, the optimal solution would be the one which used the minimal number of inventory elements to satisfy the geometric and structural constraints of the given structure.
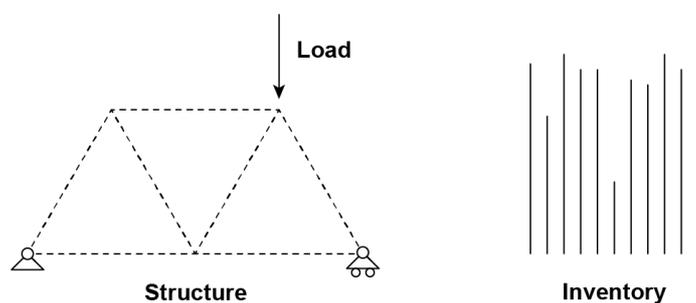
Figure 1: The Basic Form-Fitting Problem. Left: structure containing "Items". Right: inventory containing "Bins."

The bin packing problem is in the computational complexity class NP-Hard (Coffman *et al.* [2]). Specifying variable bin sizes, and adding a structural check can be made without loss of generality, meaning that, like the bin packing problem, this formulation is also NP-Hard. The answer to this problem is polynomial in size with respect to the size of the problem inputs and can be checked in polynomial time. Therefore the form-fitting problem is also in NP, making it NP-Complete. This means that, as with all NP-Complete problems, no efficient (polynomial time) algorithm can theoretically exist for this problem, unless P = NP (P vs. NP Problem [11]).

2.3. Algorithmic Approaches to the Form-Fitting Problem

Because this problem is NP-Hard, heuristic algorithms are most likely to achieve a good trade-off between speed and quality of results. Two basic heuristics are First-Fit (FF) and Best-Fit (BF) (Coffman *et al.* [2]). In First-Fit, items are considered one by one and placed into the first bin which can fit them, based on some unsorted ordering of bins. In Best-Fit, items are considered one by one and placed into the bin which will result in the lowest remaining capacity in that bin. The quality of solutions of these algorithms can be improved by pre-sorting the list of items. First-Fit Decreasing (FFD) refers to First-Fit where the items have been sorted in decreasing order by their size. Best-Fit Decreasing (BFD) refers to the same pre-sorting applied to items in Best-Fit.

The constraint of finite, unique bins of variable sizes can be considered directly using conventional FF, BF, FFD, and BFD. This paper proposes several straightforward extensions which can be added to these heuristics to try to guide them towards more optimal solutions in the presence of the additional structural constraints of the form-fitting problem. The first is to change the conditions for "Best-Fit" from the minimisation of remaining length in a bin to the maximisation of Effect / Resistance. The second is to change the pre-sorting criterion of FFD and BFD to Effect, as opposed to Length. A third modification which can be explored is sorting bins by their Resistance, or their Length. Finally, a fourth approach, which will be considered in future work, is to construct the algorithm such that rather than trying to find a fit for a given item, an algorithm would try to find items to fit a given bin.

2.3. Form-Fitting of Non-Standardised Elements

So far in this paper, the form-fitting problem has been posed only for elements which are straight and have uniform cross-section. The second key problem for form-fitting is the fitting of elements with *diverse* geometries – i.e. non-straight members with varying cross-section. Round, unsawn timbers often have significant deviations from straightness, and always taper along their length. Previous authors have used a "skeleton" representation (Fig. 2) to capture the geometry of round timbers (Mollica and Self [9]). A skeleton is comprised of a graph of nodes and edges, where nodes contain oriented cross-sections, and edges indicate connectivity between nodes. This representation can be generated from a 3D surface representation of a timber, as produced by LIDAR or photogrammetry surveys (Hackenberg *et al.* [6], Mollica and Self [9]). The structural behaviour of this representation is also easily analysed using finite element methods, by treating each edge as a linear beam element with stiffnesses corresponding to the cross-sections defined at its adjacent nodes.
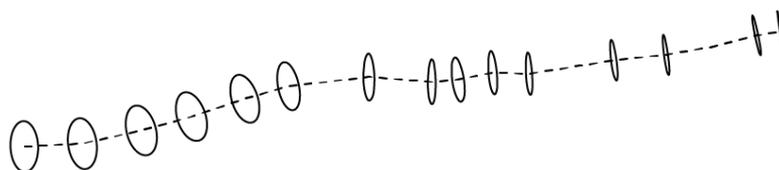
Figure 2: Skeleton representation of a round timber with significant eccentricities. Circles indicate cross-sectional area. The dashed line represents the area centroid of the round timber at all points along its length.

Clearly, checking for geometric fits when using the skeleton representation is more complex than the simple one-dimensional variable-sized bin packing case. This paper proposes a new representation for geometric constraints on structural elements within the desired structure. These are informally called "gates" (Fig. 3). Gates are geometric constraints or objectives defined as oriented boundary regions or volumes on the topology of the structure. These, when used alongside constraints such as connection locations and target geometry, are intended to help designers specify buildability constraints, while allowing for some geometric diversity in structural elements. For example a gate might correspond to the limits on the depth of a packer which connects a primary structural member to a roof purlin.
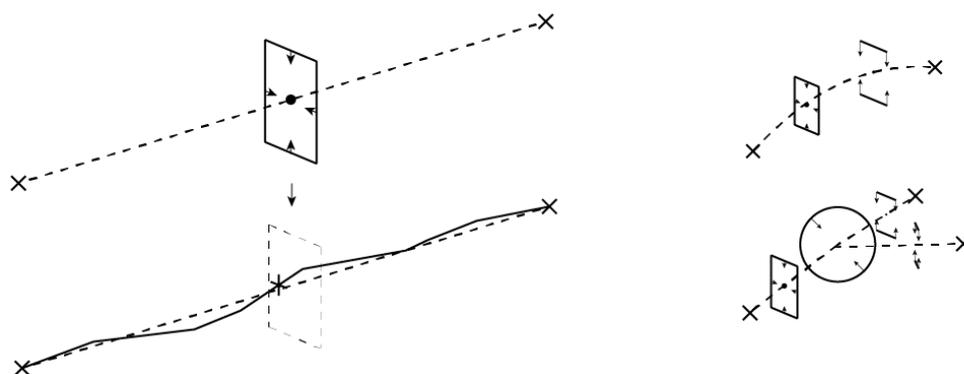
Figure 3: "Gate" constraints. Left: rectangular region constraint with element skeleton shown passing through. Right: rectangular region, double half-plane, and spherical volume constraints on curved and branching elements.

2.4. Additional Structural Checks

In the simple case described in Section 2.2, the only structural constraint was axial crushing or tension failure in a bar. For "automatic" form-fitting algorithms to be useful in more general cases, including the design of real steel and round-timber structures, they must be able to efficiently perform additional structural checks on elements being placed, such as bending, shear, buckling, and connection checks. Also, fitting algorithms must consider the effect of individual element placement on global structural behaviour. This is important in statically indeterminate structures subject to load redistribution, and when considering global buckling, dynamic, and serviceability limit states. Future work will address these factors in the design of form-fitting algorithms.

The simple linearised buckling checks specified in Eurocode 5 are not applicable to timber elements with significant eccentricities (BS EN 1995-1-1 [1]). One approach might be to carry out a fully non-linear buckling analysis for each element placement, but this would likely require too much processing time to be efficiently included in an automatic form-fitting process. The author is currently engaged in a pilot study which aims to correlate physical tests of the buckling capacity of eccentric round timbers to analytical buckling equations which account for eccentricity, such as Ylinen's equation (Zahn [16]).

3. Heuristic Algorithms for the Basic Form-Fitting Problem

3.1. Methods

The first pilot study related to this work was an investigation of the performance of a number of variations on the classic First-Fit and Best-Fit heuristics when applied to the basic form-fitting problem. The following strategies were considered:

First-Fit Strategies:

- **FF** – First-Fit with no pre-sorting.
- **FFDL** – First-Fit with pre-sorting of items by length (decreasing), and no sorting of bins.
- **FFDE** – First-Fit with pre-sorting of items by effect (decreasing) and no sorting of bins.
- **FFDL/L** – First-Fit with pre-sorting of items by length (decreasing), and pre-sorting of bins by length (increasing).
- **FFDL/E** - First-Fit with pre-sorting of items by length (decreasing), and pre-sorting of bins by resistance (increasing).
- **FFDE/L** – First-Fit with pre-sorting of items by effect (decreasing) and pre-sorting of bins by length (increasing).
- **FFDE/E** – First-Fit with pre-sorting of items by effect (decreasing) and pre-sorting of bins by resistance (increasing).

Best-Fit Strategies:

- **BF(L)** - Best-Fit, where "Best" is defined by minimising remaining length.
- **BF(E) –** Best-Fit, … by maximising item effect / bin resistance *(i.e. utilisation)*.
- **BFDL(L) –** Best-Fit, with items pre-sorted by length (decreasing), min. remaining length.
- **BFDL(E)** – Best-Fit, With items pre-sorted by length (decreasing), max. effect / resistance.
- **BFDE(L) –** Best-Fit, with items pre-sorted by effect (decreasing), min. remaining length.
- **BFDE(E)** – Best-Fit, With items pre-sorted by effect (decreasing), max. effect / resistance.

The bins and items in this test did not correspond to an actual structural geometry. Instead their lengths and resistances were each pseudo-randomly generated in a uniform distribution between 0 and 1. This approach provides a baseline measure of performance independent of any given structural form. 100 bins and 50 items were generated. This was intended to represent the likely real-world case where the inventory is several times larger than the number of elements in a structure. The tests were run 100 times with newly generated input values for each iteration. The mean and standard deviations of the results over these iterations were recorded.

The following results were recorded for each algorithm

1. Number of bins used to fit the items.
2. Number of items which were not successfully fitted.
3. Total wasted length (sum of remaining length of bins with at least one item placed in them).

Running time was not recorded, as algorithms were not implemented with the most efficient data structures possible, and therefore any results would not be representative of running speeds in a real design tool.

3.2. Results

Table 1: Performance of variations on First-Fit for the basic form-fitting problem for a set of 100 bins and 50 items.

	FF		FFDL		FFDE		FFDL/L		FFDL/E		FFDE/L		FFDE/E	
	μ^*	σ^*	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ
Bins Used:	**34**	2.8	**38**	3	**34**	3	**42**	2.8	**41**	2.7	**45**	2.4	**38**	2.7
Items Remaining:	**7**	2.7	**6**	2.8	**6**	2.6	**7**	3	**4**	2.4	**4**	2.4	**7**	2.6
Wasted Length**:	**3.4**	0.7	**2.6**	0.6	**3.7**	0.8	**1.6**	0.5	**6.3**	1.2	**2.1**	0.6	**7.7**	1.3

Table 2: Performance of variations on Best-Fit for the basic form-fitting problem for a set of 100 bins and 50 items.

	BF(L)		BF(E)		BFDL(L)		BFDL(E)		BFDE(L)		BFDE(E)	
	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ
Bins Used:	**40**	2.8	**39**	2.6	**38**	2.6	**41**	2.7	**41**	2.7	**38**	2.7
Items Remaining:	**6**	2.6	**6**	2.5	**7**	2.9	**4**	2.4	**4**	2.4	**7**	2.6
Wasted Length**:	**1.4**	0.4	**7.2**	1.2	**1.1**	0.4	**6.3**	1.2	**1.3**	0.4	**7.7**	1.3

*Mean values for bins used and items remaining were rounded to the nearest integer. Mean values for wasted length and all standard deviations were rounded to 2 significant figures.

**Lengths are expressed in terms of unit length 1, the upper bound on length for both items and bins.

3.2. Discussion

The results of this test (Tables 1,2) showed, surprisingly, that the performance of these algorithms, at least for this particular input case, did not differ significantly. The largest differences observed were in the total length of wasted material. Algorithms which either sorted bins in increasing order by their resistance (FFDL/E, FFDE/E), or which found best fits by maximising effect / resistance (BF(E), BFDL(E), BFDE(E)) all produced significantly higher wasted length – up to 7 times more than other heuristics.

Interestingly, the number of bins used was lowest for the simplest strategy (FF) – this is likely because more sophisticated strategies found better matchings between lengths of items and bins, and therefore were more likely to find whole bins which were just above the required size for a given item. Future studies should investigate the effect such "greedy" algorithms have on the quality of the remaining material set.

Some of the best-performing algorithms, based on their results in all three metrics, were BFDE(L), and the relatively simple BF(L) and FFDE. Although running time is not considered here, this will be an important criterion in the design of form-fitting algorithms for design tools. If performance is similar between relatively simple algorithms and more complex ones, it may be advantageous to implement the simpler algorithms in the interest of fast fitting during the initial design process. For design refinement processes where longer running times may be tolerated, slower implementations with higher quality results may be considered.

4. Form-Fitting of a Round-Timber Roof Truss

4.1. Methods

A preliminary study was also carried out to test the application of some basic heuristics to a real-world form-fitting problem – the form-fitting of round timbers to a pin-jointed planar roof truss. The structure to be fitted was based on a truss design developed by WholeTrees Architecture and Structures for the Festival Foods Grocery store, completed in 2016 (Festival Foods [3]). The trusses at Festival Foods had continuous timbers in their top and bottom chords, meaning that there was moment transfer through these connections, and the structure was statically indeterminate. For the purposes of this study, the top and bottom chords were modelled as discontinuous, and with all connections as

pins, making the structure statically determinate (Fig. 4). This allowed a relatively straightforward application of variations on the algorithms discussed in Section 3.1 to this problem. Future studies will consider indeterminate structures. Connections were modelled as dowel-type bolted flitch plates, sized according to Eurocode 5 specifications (BS EN 1995-1-1 [1]).
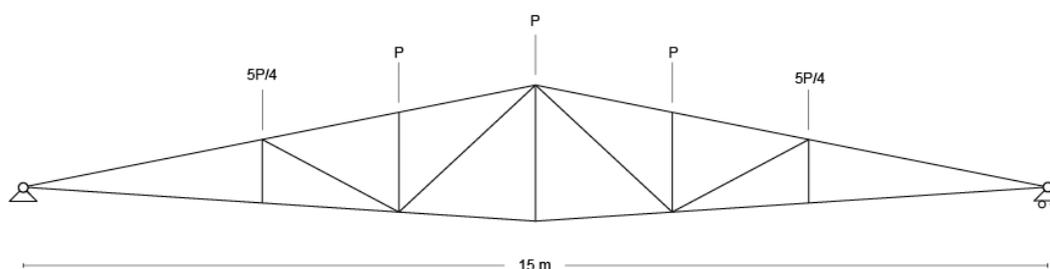
Figure 4: Standardised truss module with uniformly distributed roof loads represented as concentrated nodal loads.

A set of artificial trees (Fig. 5) between 10 and 30 meters in height and with base diameters between 100 and 300 millimeters was generated as the inventory for this study, based on realistic values of height and taper derived from Fonweban *et al.* [4].
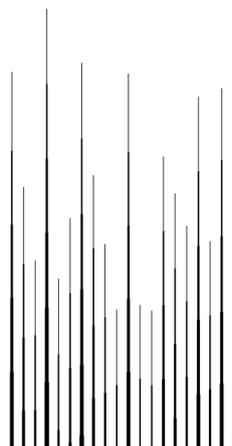
Figure 5: Artificially generated trees based on data from Fonweban *et al.* [4].

The algorithm implemented was a variation on First-Fit with pre-sorting (Table 3). Structural elements were either sorted by their axial force (corresponding to their "effect") or were randomly sorted. Timbers were either sorted by their base diameter (roughly corresponding to their "resistance") or were randomly sorted. Due to implementation issues, this algorithm differs from those implemented in the study described in Section 3 in that timbers and structural elements were resorted (or in the case of random strategies, reshuffled) after successful element placement. Elements were always cut from timbers at their base. The minimum number of placement attempts required to fit (to the nearest 25) was recorded, as well as the real time taken to execute.

Table 3: Pre-sorting strategies applied to First-Fit for form-fitting of planar truss in round timber.

Test Case	Structure Elements Sorting Criterion	Timber Sorting Criterion
1	Random	Random
2	Random	Base Diameter
3	Axial Force	Random
4	Axial Force	Base Diameter

4.2. Results

Test Case 1: Random-Random

Time: 40 s | Placement Attempts: 300

Test Case 2: Random-Base-Diameter

Time: 141 s | Placement Attempts: 400

Test Case 3: Axial Force-Random

Time: 39 s | Placement Attempts: 225

Test Case 4: Axial Force-Base Diameter

Time: 91 s | Placement Attempts: 300
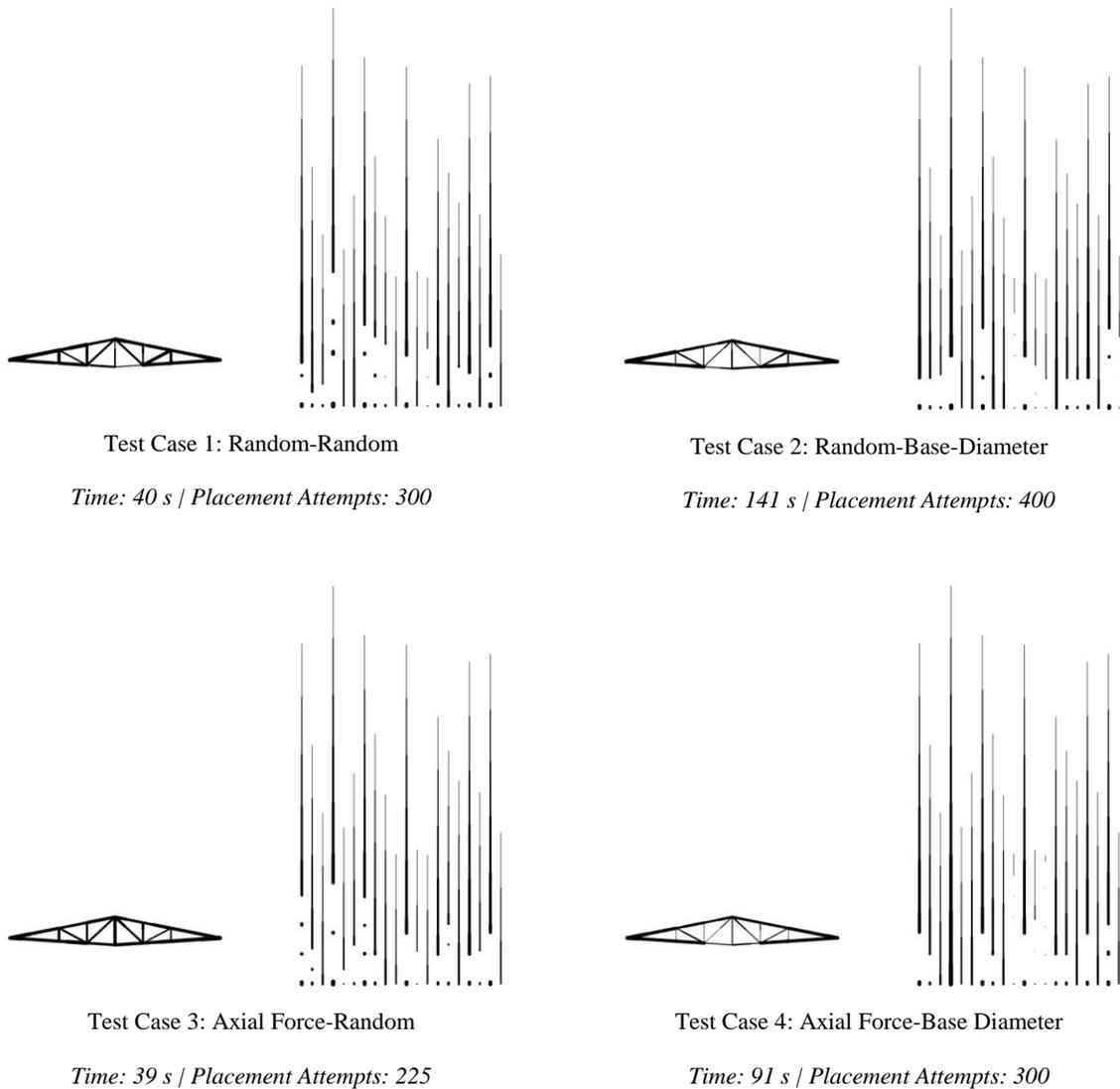
Figure 6: Fitting results for a planar round timber truss for four cases of First-Fit with various pre-sorting strategies.

4.3. Discussion

It is difficult to draw conclusions based on the results of this preliminary study (Fig. 6). Sorting timbers by base diameter results in longer run times, likely because a greater number of elements must be considered before a fit is found, but it is not clear why these times were proportionately longer per element placement. All algorithms used segments from 12 to 14 trees out of the total of 20, but varied in the degree to which a given tree was used. Future studies will consider the objectives and constraints in round timber fabrication more closely, to determine the relative importance of the number of cuts, number of timbers used, or amount or quality of offcuts. Based on visual inspection, the Axial-Force/Base-Diameter strategy produced the best matching of inventory and structure – smaller elements are clearly placed in locations of lower internal force, and some of the smallest (i.e. lowest value) trees were nearly completely used.

6. Conclusions and Future Work

6.1. Graphical "Ease of Fit" Feedback

A key limitation of fitting methods, as described in Section 2.1, is that algorithms will only ever find matches to the structure pre-defined by the designer. This structure may be poorly suited to use the available material – i.e. elements may be too long or short. Additionally, stresses may be too high, or so low that elements aren't efficiently utilised. A key focus for future work will be to develop a graphical feedback method for the form-fitting design process (Fig. 7), which will provide information to the designer about the "ease of fit" in terms of both geometric fit and structural constraints. As it is iterating through potential fits, the fitting algorithm will record information about fit attempts, and output these results in a graphical way to the designer. This will allow the designer to infer where the structural geometry could be modified to better suit the available material.
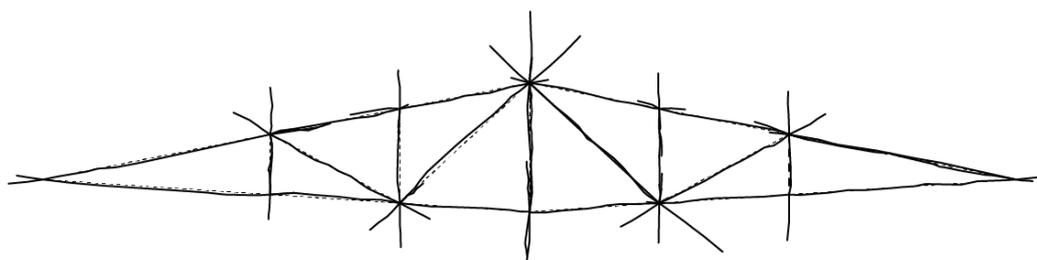
Figure 7: Graphical "ease of fit" feedback. Inventory elements are shown superimposed on the structure, indicating whether a given element might be lengthened or shortened.

6.2. Conclusions

This paper has demonstrated the potential for automatic form-fitting algorithms to help designers design structures using finite sets of diverse elements. Simple extensions to basic heuristic algorithms for bin packing were shown to be effective for basic formulations of the form-fitting problem. Key future challenges are form-fitting of indeterminate structures, and checking and optimising for a broader range of structural constraints. A second major research focus will be a graphical display of "ease of fit" which will guide the designer towards more optimal matches between structure and material. Ultimately, these methods could make it easier for designers to develop efficient and expressive structures using reused steel elements and round-timber.

Acknowledgements

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