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Layout optimization of space frame structures

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Abstract
Despite the evident advantages of double layered grid structures to produce freeform and aesthetically pleasing designs, the challenge in their application lies in the computational cost associated with the optimization of their layout and limitations in their fabrication. Conway operators are an efficient tool for the manipulation of such structures, producing complex yet aesthetically pleasing topological layouts. This paper presents a computational workflow for the evaluation of double layered grid structures generated using Conway operators. After defining the operators applied, the structure is optimized in compliance with structural design criteria. The inherent modularity of such layouts significantly reduces the computational resources required, while at the same time facilitates the fabrication process. The tool is applied on an existing structure available in literature on which different topologies are evaluated. As a result, a decrease in the overall mass of the structure is achieved, while insightful information for the structural behaviour of the different operators is obtained. The potentials for future evolution of this approach are hence highlighted.

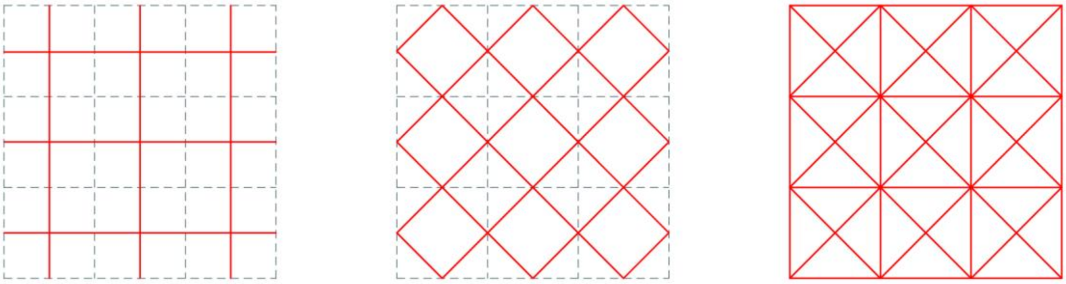
Keywords: Layout optimization, topology, space frame, Conway operators, computation, geometry.

1. Introduction
Interest in doubly-curved space frame structures has increased significantly in recent years due to the freedom of design they provide, the aesthetically pleasing designs and the continuous development of computational and fabrication tools to generate them. This development has led to the advancement of a series of tools for the optimization of their members' connectivity, their mass and geometrical properties, as in Darwich [4] and Ahrari and Kalyanmoy [1]. However, such optimization processes are very expensive in terms of the computational resources required, thus limiting the scale of problems they are capable of dealing with (Gilbert *et al* [5] and Gilbert and Tyas [6]). Moreover, even when identified, the numerically optimum solution for a given problem often fails to be applied in practice, either due to fabrication limitations or the increased cost associated with bespoke designs.

This paper presents a method to manipulate the layout of double layered grid structures using Conway operators. John Conway developed a method of polyhedral notation using the five Platonic solids as a seed and applying a number of defined topological operations on them, Conway *et al* [3]. For the scope of this paper, focus will be placed on the three principle operators, "*dual*", "*ambo*" and "*kis*", visualized in Figure 1. Through a defined set of instructions on a seed topology, these operators generate new layouts. The detailed process followed for each and the effect this has to the number of faces, edges and vertices of the initial configuration are described in Table 1. The relationship between the vertices assumes they are applied on closed, convex polyhedral, with no boundary conditions. However, they could practically be applied on any surface as a tiling, considering it is a part of a larger sphere.

Table 1: Description of the three Conway operators studied and the relations between the total number of vertices (v), edges (e) and faces (f) of the seed and generated grid respectively [3].

Operator	Description	Vertices (v)	Edges (e)	Faces (f)
Dual	Each vertex is replaced by a face and each face by a vertex	f	e	v
Ambo	New vertices are added at mid-edges, while the seed vertices are removed	e	2e	f+v
Kis	A vertex is added at the centre of each n-sided face, dividing it to n triangles	v+f	3e	2e



a. Dual b. Ambo c. Kis

Figure 1: Geometrical representation of Conway operators. The seed topology is a quadrilateral grid represented with a dashed grey line, while the generated topology with solid orange lines.

Despite the simplicity of their formulation, Conway operators can lead to highly complex configurations, especially when applied consequently on a grid. Their algorithmical implementation for analysing and generating such complex topologies has been explored extensively in academia, as in Hart [7] and Nonaka [8], but remains outside the scope of this paper.

Their application on double-layered grid structures has been studied by Shepherd and Pearson [10], who generated the top and bottom layer of a space frame using Conway operators and applied proximity constraints to define the inter-layer connectivity. The structure was optimized using the member adding technique, described in Darwich [4]. Despite the material saving achieved in the final design, it disregarded the structure's modularity.

This paper presents a computational workflow of evaluating double-layered grid structures generated by Conway operators. When uniformly applied on regular grids, those operators produce modular layouts. Aim of this study is to generate final configurations in which this property is preserved throughout the structure. Such an approach could address fabrication limitations faced in the current optimization techniques, as well as decrease the computational cost required. Additionally, this allows for quick comparative studies of different topologies to be carried out at an early design stage, thus leading to structurally informed and aesthetically pleasing final designs. In order to evaluate the reliability of this workflow, the tool developed will be applied on an existing structure from literature. The Oguni dome in the Kumamoto prefecture of Japan is chosen for this case study, described in detail in Chilton [2].

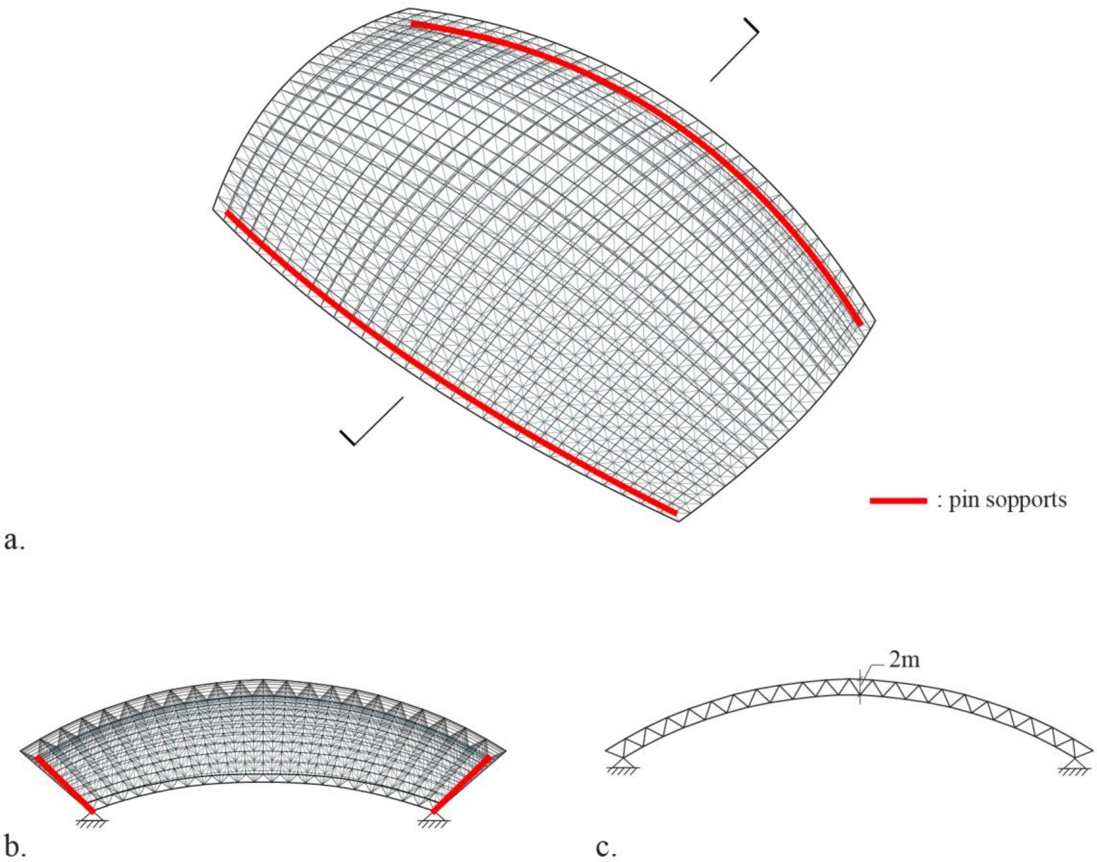


Figure 2: The Oguni dome: a. Three-dimensional representation of the structure and the supports conditions, b. The structure's elevation, c. A section across the dome demonstrating the relative position between the top and bottom layer and their connectivity. The geometry was regenerated using the information provided in Chilton [2].

2. Benchmark study

Designed by Shoei Yoh in 1988, the Oguni dome is a double layered, doubly-curved timber dome covering an area of 63m by 47m with a uniform depth of just 2m. The material used is the Japanese cedar, *sugi* (cryptomeria japonica) with an external steel skin, as in Chilton [2].

The top layer of the structure is a quadrilateral grid, while the bottom layer is its topological dual. The connectivity between them is achieved by joining each vertex of the bottom layer to all the vertices of the top layer's face it was produced from. Offset perpendicularly to the top surface of the dome, the bottom layer is shorter than the top by half a face on both ends, as shown in Figure 2c. Supports are placed along the long edges of the bottom layer, thus defining the structure's boundary condition.

The timber has a rectangular solid section throughout the structure, with differentiated sizes for each layer respectively; 0.11m by 0.15m for the top, 0.09m by 0.125m for the web and 0.11m by 0.17m for the bottom layer. Pin joints are placed at each vertex, creating a fully pin-jointed truss, according to Chilton [2].

3. Method

To evaluate the structure's behavior, the geometry of the dome was recreated in the 3D CAD modeling software Rhinoceros and its parametric modeling plug-in Grasshopper. The cross-sectional geometry and the material properties of Japanese cedar were assigned to the model (Young's modulus= 9,600,000kN/m^2, specific weight=4.9kN/m^3, Modulus of rupture=53,000kN/m^2, according to Ramsay and Macdonald [9]). The load cases applied for the analysis are the structure's self-weight and a combination of additional dead and live loads of 1.7kN/m^2. Both values were factored up for the ultimate and service limit state analyses, in compliance with the EN1991 standards. The external loading was applied as an area load on the top layer vertices. The analysis was performed using the structural analysis plug in for Grasshopper Karamba3D from which and the tensile and compressive forces of each member were extracted. The utilization was calculated as the ratio of the stress applied on each member to either the yield strength of the material or the critical buckling stress for members in tension or compression respectively. For a structure to be deemed safe this ratio needs to be smaller than 1. Custom components were created to perform those calculations and optimize the members' cross section.

Results of the analysis of the remodelled existing structure show a maximum utilization in tension of 13.2% in the web layer and of 75.9% in compressive resistance at the top layer, therefore validating the reliability of the computational analysis tool developed.

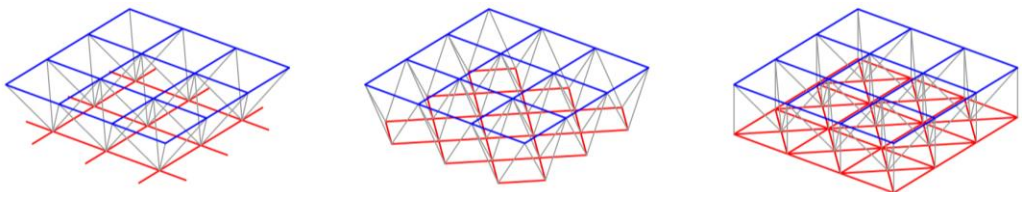
Before moving on to the formulation and evaluation of different grid topologies an intermediate analysis was carried out, in which the rectangular sections of the original dome were replaced by circular solid ones. This makes comparison easier, since it guarantees inherently uniform structural properties in all directions, irrespective of the Conway operator applied and the orientation of the final geometries' members. To guarantee an equal moment of inertia for the two section geometries a section modulus was used, leading to larger circular cross sectional areas respectively. This inevitably led to a slight increase in the structure's self-weight and therefore to a rise of its maximum utilization from 75.9% to 77.7% (its mass increased from 112,498 kg to 137,602 kg respectively).

With the geometry with the circular solid section as a benchmark, a strategy for generating different grid topologies was determined. The top layer was kept constant for this process, while different Conway operators were applied on the bottom layer. A crucial role in the generation of these topologies was played by the method of defining the connectivity between the two layers, as it directly affects the structure's global stability. More specifically, the top layer is a quadrilateral grid, with no in-plane shear resistance. The web layer is therefore always required to be stable to secure the structure's static determinacy, irrespectively of the stability of the bottom layer.

After generating the topologies of the bottom layer, the web layer is produced by connecting vertices in top and bottom layer in two steps. Firstly, if a vertex of the bottom layer was generated on a respective face or an edge of the top layer, it is connected to all the vertices (either the four vertices of the face or the start and end point of the edge it lies on). Secondly, if a vertex is a duplicate of an existing vertex of the top layer, they directly connect to each other. Even though this approach generates dense web layers, it is required by the existence of pin joints at the members' intersection. More precisely, despite the fact that a sparser web might be able to resist the loading, the existence of pin jointed vertices, which are not connected vertically, would render them prone to out-of-plane buckling.

After establishing the method of generating the double-layered grids, different topologies were evaluated. Their optimization problem can be formulated as a minimizing function of their mass, in which the members' cross sectional area was the driving variable. This process was subject to the following set of constraints: a maximum utilization limit for each member of 77.7%, which is the maximum utilization of the Oguni dome members, when circular cross sections were applied; uniform cross sections throughout each layer, in order to ensure the final result's modularity; and, finally, a constant offset distance of 2m for the whole structure, as in the original dome.

Three topologies were evaluated for the bottom layer, the dual, ambo and kis, with their three-dimensional configurations demonstrated in Figure 3. Dual is the topology of the original dome, but it was re-evaluated in order to explore the potential of further decreasing its mass. The reason for choosing these topologies and not a more complex combination is the interesting relationships created between the three layers. As far as their orientation is concerned, both the top and bottom layer of the dual have the same directionality, while the bottom layer of the ambo is diagonally rotated relative to the top; the kis topology has members in both directions. Furthermore, the three topologies show a diversity in the relative relationship of their total members' length for each layer (Figure 4). While the web layer is the heaviest in all cases, the bottom layer varies from being the layer with the least members' length in the dual, maintaining an average value in the ambo topology, slightly heavier from the top layer, and finally reaching a value almost as high as the web layer in the kis topology. Taking those observations into consideration, it is interesting to see how these relations translate to structural performance.



a. Dual b. Ambo c. Kis

Figure 3: Detail of the connectivity between the three layers for the three topologies in 3d. Top layer is denoted in blue, web in grey and bottom in red.

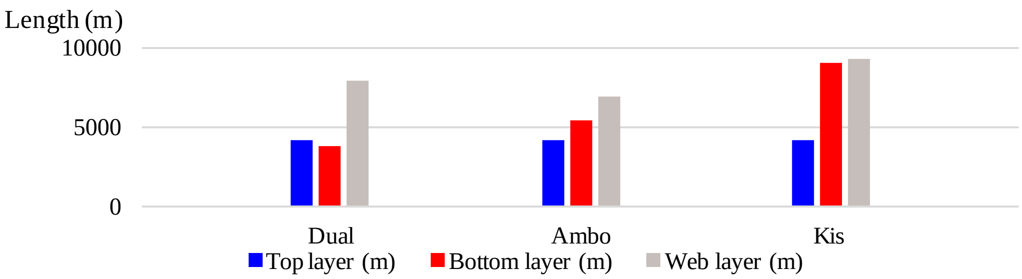


Figure 4: The three layers' total members' length in the final structures.

4. Results and discussion

According to the results of the optimization process dual is the optimal layout of the dome, for which a 6% saving in mass is achieved (from 137,602kg in the original model with circular cross sections to 130,117kg when optimized). As far as kis and ambo topologies are concerned, kis performed more efficiently, despite the relative total lengths of their members discussed above, with a final mass of 173,215kg compared to 190,105kg. The performance of the different layouts in relation to their member's utilization is demonstrated in Figure 6.

Regarding the utilization of the members of each layer, there seems to be a direct relationship between the distribution of the tension and compression areas throughout the structure and its performance (Figure 5). When comparing the original dome analysis to its optimized version, the areas of pure tension or compression are minimized. In contrast, the creation of distinct zones of tension and compression resistance in the ambo generates greater stresses in- those members, hence they require larger cross sections per layer. Finally, the densest kis topology presents a better distributed utilization across its members, a fact which favors the use of smaller cross sections. It therefore has a smaller final mass compared to ambo, inverting the initial relative ranking of the three topologies in terms of their layer's density.

Figure 5: Distribution of the members' utilization of the three layers for the different topologies optimized. Red surface corresponds to compression and blue to tension members.

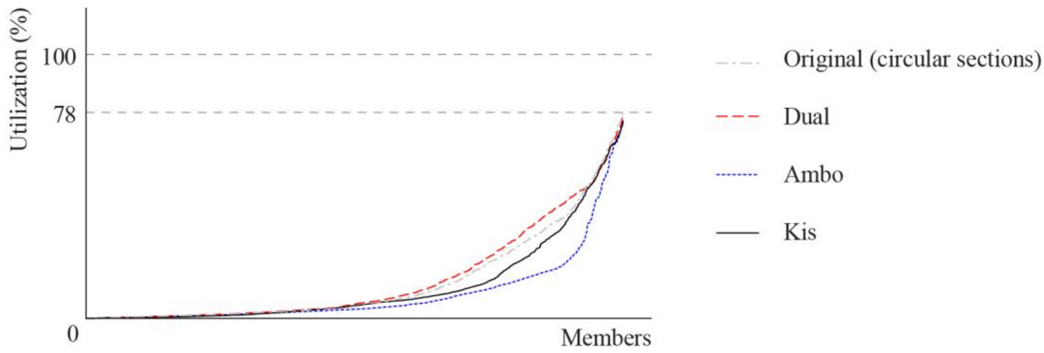


Figure 6: Sorted utilization of individual members for each topology.

Further analysis was made in the performance of the members of the optimal dual topology. It is evident in the slenderness graph in Figure 8 that the optimization was driven by a small number of elements for the top and bottom layer, which are close to the buckling limits, while the majority of members are utilized at a much smaller level. The web layer however shows a more efficient distribution across its members. It is hence suggested that a further exploration of the application of Conway operators could highlight potentials for an optimized distribution of the utilization for all three layers, i.e. it is evident that different combinations of Conway operators can produce topologies with a range of efficiency levels. The exploration of diverse topologies for the seed grid, such as shear resistant triangular grid, could further inform this approach.

When the members in tension and compression are respectively highlighted in this graph (Figure 9), it becomes clear that compression members were the drivers, defining the cross section of each layer. A reduction in the tension members' cross section is therefore possible, should their evaluation be performed separately. Such an approach, of course, would have important implications on the workflow described. First of all, the driver for the evaluation of the structures and their cross sections' definition has been ultimate limit state analysis. Removing material from the tension members, however, would increase the structure's flexibility and potentially question the dominance of the ultimate limit state over the service limit state analysis. Secondly and more importantly, such an approach would sacrifice the structure's modularity and all the derived advantages of constructability, something which is not desired for the scope of this research.

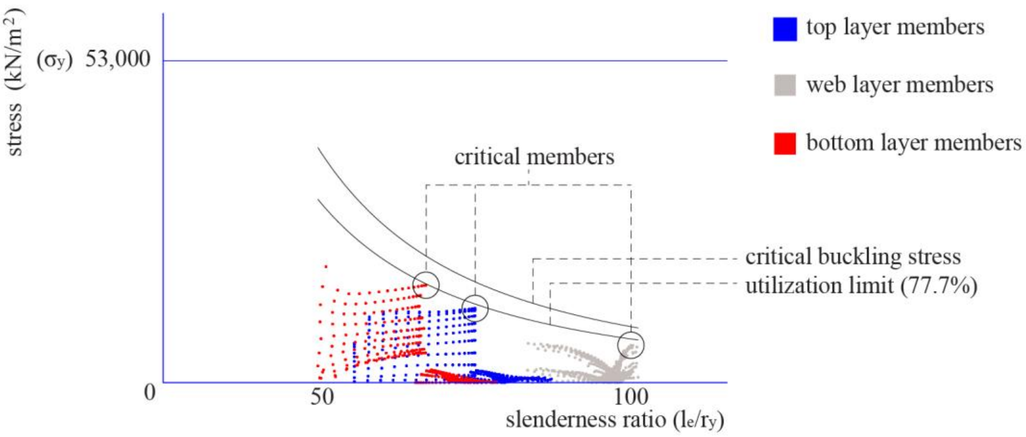


Figure 8: Slenderness ratio graph for the dual topology, showing the utilization for each layer's members

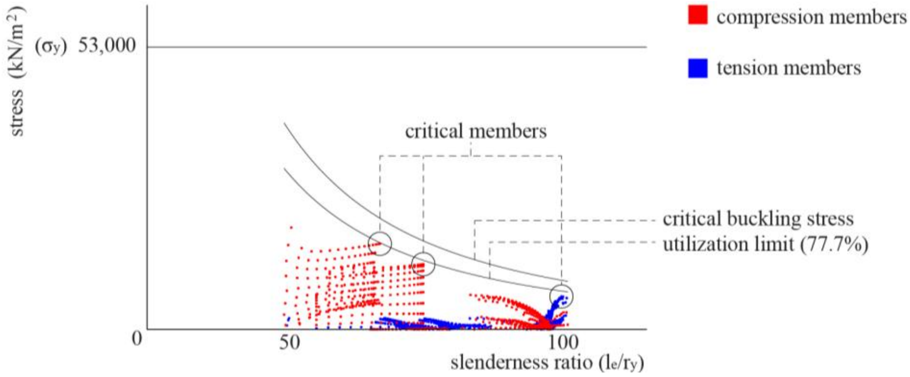


Figure 9: Slenderness ratio graph for the dual topology, showing the utilization for tension and compression members

5. Future work

Future work of the authors is therefore oriented towards the exploration of optimization techniques in which modularity is preserved. Setting more geometrical and structural variables, such as the depth of the structure, the scale of the modules applied and exploration of different material properties, could provide insightful information, while maintaining the feature of modularity and its inherent advantages mentioned above.

Acknowledgements

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