Definition of the geometry of the British Museum Great Court roof

Part I Definition of geometry as built

Chris J K Williams

Department of Architecture and Civil Engineering, University of Bath, UK

Introduction

Figure 1. Great Court roof, architect: Foster + Partners, engineer: Buro Happold, steel and glass contractor: Waagner-Biro

Figure 1 shows the Great Court roof. Its geometrical definition consisted of two parts, the shape of the surface and the pattern of steel members upon that surface.

Surface geometry

Techniques such as non-uniform rational B-spline (NURBS) surfaces could have been used to define the roof surface. However with boundaries of constant height that are rectangular and circular in plan, it was easy and more convenient to use a simpler approach. This particularly applied because of the need to generate the grid of members on the surface and to have a singularity in surface curvature at the corners. Had more recent advances like subdivision surfaces been available, this logic would probably still apply, particularly since most of the programming work was in the definition of the member topology, or pattern of triangles on the surface, and this would be the same regardless of the surface definition technique.

The reason for the singularity of curvature is that the rectangular boundary is on sliding supports to avoid horizontal thrust on the existing building. Hence the roof can only be restrained horizontally at the corners where the resultant thrust can be balanced by tensions in the rectangular edge beam. Functions without a singularity in curvature must be horizontal at a horizontal corner, and are therefore like a ‘ski jump’ where the thrust has to change direction rapidly as the corner is approached, thus causing structural problems. But a cone lying on its side can have a slope even though it intersects two horizontal lines crossing at right angles.

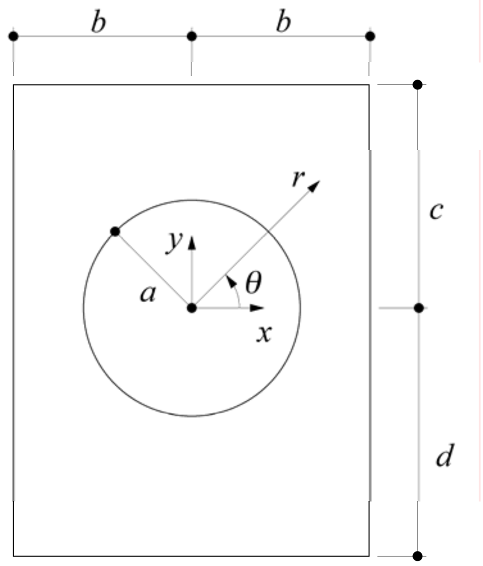

Figure 2. Plan geometry

The plan geometry is shown in figure 2 and the height of the roof was calculated using a sum of the functions drawn in figures 3, 4 and 5, slightly modified to satisfy planning requirements etc.

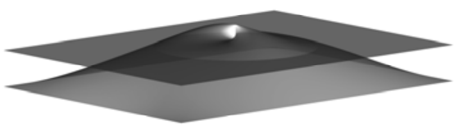

Figure 3. Level change function η

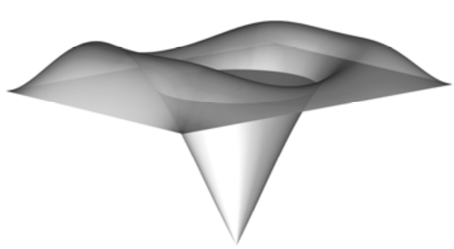

Figure 4. Function without singularity α

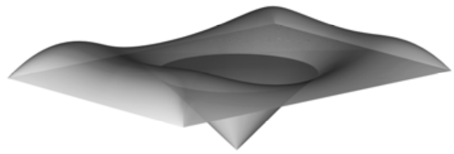

Figure 5. Function with singularity ζ

The functions are:

$$\eta = \frac{\left(1-\frac{x}{b}\right)\left(1+\frac{x}{b}\right)\left(1-\frac{y}{c}\right)\left(1+\frac{y}{d}\right)}{\left(1-\frac{ax}{rb}\right)\left(1+\frac{ax}{rb}\right)\left(1-\frac{ay}{rc}\right)\left(1+\frac{ay}{rd}\right)},$$

$$\alpha = \left(\frac{r}{a}-1\right)\left(1-\frac{x}{b}\right)\left(1+\frac{x}{b}\right)\left(1-\frac{y}{c}\right)\left(1+\frac{y}{d}\right)$$

and

$$\zeta = \frac{\left(\frac{1}{a}-\frac{1}{r}\right)(b-x)(b+x)(c-y)(d+y)}{\left[\begin{array}{l}(b+x)(d+y)\sqrt{(b-x)^2+(c-y)^2}\\ +(b+x)(c-y)\sqrt{(b-x)^2+(d+y)^2}\\ +(b-x)(d+y)\sqrt{(b+x)^2+(c-y)^2}\\ +(b-x)(c-y)\sqrt{(b+x)^2+(d+y)^2}\end{array}\right]}.$$

η, α and ζ are all zero when $x=b$, $x=-b$, $y=c$ and $y=-d$. α and ζ are zero and $\eta=1$ when $r=a$. Thus the requirement that the boundaries are horizontal was satisfied. η was used to give the change in height between the circular and rectangular boundaries.

If $x = -b + \varepsilon \cos\phi$ and $y = -d + \varepsilon \sin\phi$ where ε is small,

$$\zeta \approx \frac{1}{a}\left(1 - \frac{a}{\sqrt{b^2 + d^2}}\right)\frac{\varepsilon \cos\phi \sin\phi}{\cos\phi + \sin\phi + 1}.$$

This is a cone with its apex at the corner. Thus there is a singularity in curvature at the corner.

Grid upon the surface

Figure 6 is a drawing of the structural steel grid and figure 7 shows the roof surface faceted by the resulting triangular panels. The triangular grid was chosen for two reasons: firstly a triangular structural grid is most efficient and secondly to avoid the need to produce flat quadrilateral panels or curved glass.

The grid was 'relaxed' on the surface to avoid discontinuities in geodesic curvature. This was done by moving each node to a point on the surface equal to the weighted average of its neighbours. The weighting functions were chosen to control the maximum size of glass panel. All the panels are different, except for their mirror image on the opposite side of the north-south axis. The relaxation was done on a finer grid than the actual steel members and took many hundreds of cycles. Speed of convergence was improved using Alister Day's dynamic relaxation.

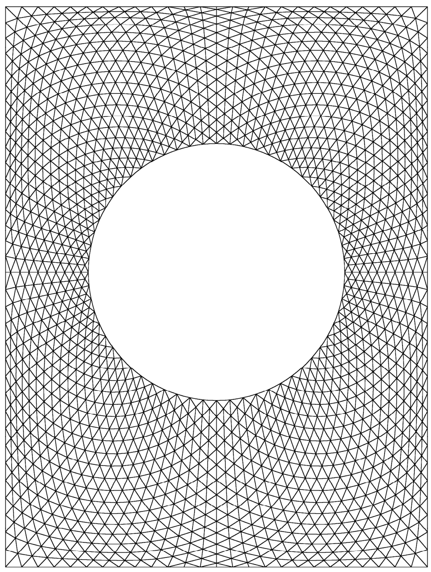

Figure 7. Steel grid

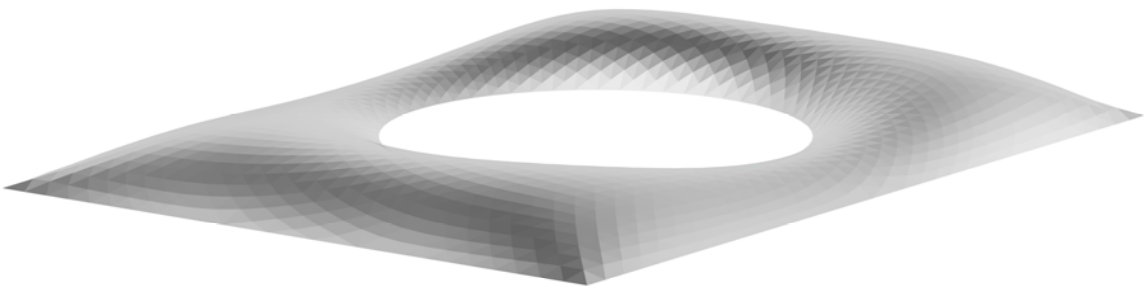

Figure 7. Faceted surface

Part II Exploration of alternative scheme using subdivision surfaces

Paul Shepherd

Department of Architecture and Civil Engineering, University of Bath, UK

Introduction

The Digital Architectonics Research Group in the Centre for Advanced Studies in Architecture (CASA) of the University of Bath is involved in a research project sponsored by Informatix Inc. to investigate the advantages of using Subdivision Surface techniques for generating and optimising architectural designs.

Subdivision surfaces begin with a coarse mesh representation of a surface. Through a recursive process of splitting each mesh face into a number of smaller faces, and adjusting the coordinates of the newly created vertices (and possibly the original vertices also), a finer and smoother mesh representation is produced, as shown in Figure 1. This can be seen as a mesh-smoothing process, whereby each subsequent level of recursion results in a finer, smoother surface representation. The recursion process can be continued indefinitely and with careful choice of vertex positioning, the mesh can converge onto an underlying limit-surface with provably smooth properties (G2 continuity).

This smoothness leads to aesthetically pleasing surfaces which show promise as representations of building envelopes. The variable level of detail lends itself well to the application of optimisation algorithms, which can manipulate the original control mesh and lead to more efficient geometries in terms of structural and environmental performance criteria.

One disadvantage from a building design point of view is that, by ensuring a smooth continuous surface, control over the exact position of the surface at any one point is surrendered, and in particular it is difficult to specify the surface boundary. This would not be

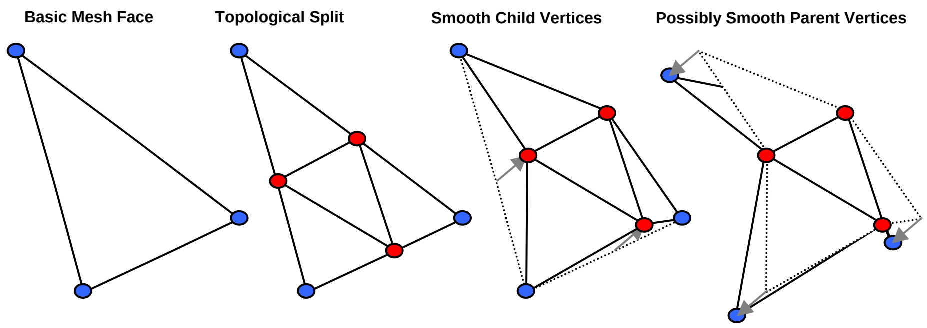

Figure 1 Subdivision of a single face

acceptable in most architectural designs, and in particular the consequences for a design of an atrium roof such as the British Museum Great Court can be seen especially in the corners as shown below in Figure 2.

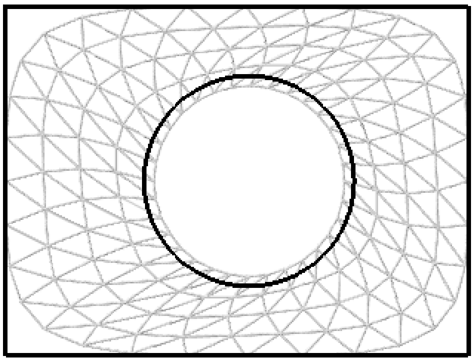

Figure 2 Unconstrained subdivision

This problem has been overcome by adopting a constrained subdivision scheme, whereby each vertex around the edge is snapped back to the constraining boundary after each subdivision step. The price paid for this extra control is a loss of smoothness around the boundary. However, since this effect is localised to the boundary and it results in a practical system of defining optimal surfaces for buildings, it is seen as a good compromise.

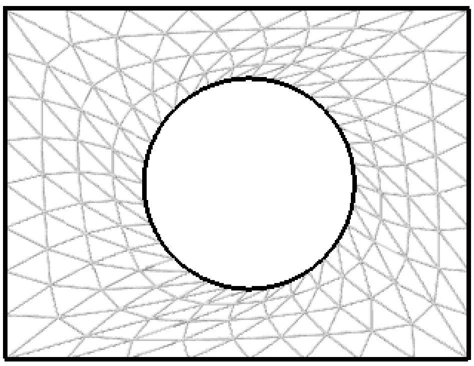

Figure 3 Constrained subdivision

Once a suitable surface has been found, different options for applying a structural grid can be explored. Whilst software tools for draping grids over the subdivision limit surface or for iteratively optimising a grid for structural performance via dynamic relaxation or simulated annealing are being developed, the most obvious choice is to use the subdivision mesh itself. As can be seen from Figures 4 & 5, this naturally leads to a smooth mesh with significant repetition of member lengths. Grids of various densities can instantly be generated corresponding to various levels of subdivision from the initial mesh.

By adopting a subdivision surface modelling approach for the Great Court roof, many different options can be quickly generated and tested against many different criteria such as solar gain, acoustic performance and structural efficiency. Research is ongoing, which aims to integrate automatic multi-objective optimisation routines into a subdivision framework, to create software tools for architects and engineers to explore creative design options whilst having immediate, up-front information on the likely performance of each option.

Figure 4 Render of Great Court using a subdivision surface grid (courtesy of John Tredinnick)

Figure 5 Roof as-built (left) and proposed subdivision roof (right) (courtesy of John Tredinnick)