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Geometry optimization of space frame structures for joint modularity

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Abstract

This paper outlines a method for the geometry optimization of space frame structures for joint uniformity. Joints are one of the main drivers for the constructability of space frames, as they represent a high percentage of the overall material and fabrication cost. They are studied in relation to the geometrical complexity of the design surface and the fabrication process applied. A computational workflow is proposed for their geometrical optimization, which comprises of three steps: the comparison and identification of varying joint configurations within a structure, their clustering into a minimum number of groups that satisfies a given tolerance, and finally their geometrical optimization for joint uniformity. The efficiency of the proposed workflow is validated through a variety of examples and comparisons. Developed in a intuitive user-interface environment, it allows designers to carry out early design studies to minimize the construction cost of their proposals and enhance the application of informed space frame designs in practice.

Keywords: geometry optimization; space frame structures; joint; fabrication process; construction;

1. Introduction

Space-frames are materially efficient truss structures with a high structural performance and the ability to approximate freeform designs[1]. The use of prefabricated and modular elements in their construction provides a low cost and time efficient assembly process. These characteristics have rendered them highly popular in a variety of applications, ranging from large span structures to free-form designs. For their efficient design the challenge lies in the layout of their members [2].

The increasing demand for free-form architecture in recent years [3] has increased the application of space-frames in projects of complex geometries. In effect, this has introduced a level of complexity into their construction process, as changes in curvature generate non-uniform space frame configurations. The geometrical characteristics of their elements - member length and joint angles - are therefore different. This requires a level of customization, which increases the complexity, duration and therefore the overall cost of construction. The joints in particular have the most crucial effect, typically representing 30-50% of the total fabrication cost [4] and up to 50% of the material required for construction [5]. The focus of this paper is therefore on the joint configuration of space frame structures approximating complex geometries, the degree of customization required and the effect this has on the overall construction process.

Different methods have been developed for the optimization of space frame structures for joint cost. Asadpoure et al [6] uses a smooth and differentiable cost objective, considering the cost as a function of the member mass. Nevertheless, fabrication and construction costs do not scale linearly with material weight [7]. Havelia [8], in particular, used a cost-driven topology and sizing optimization to show that heavier yet modular structures can be more cost-efficient, if fabrication and erection costs are taken into account. Ranalli et al [9] simultaneously perform cost-based and sizing optimization of two-dimensional truss structures. With a catalogue of potential joint connections and inputs from local

Table 1: The different processes that can be followed for the fabrication of a space-frame structure of a changing curvature and their respective degree of automation and angle tolerance.

Surface curvature, *k*	**Fabrication process**	**Process automation**	**Angle tolerance**
	Continuous	Fixed	Absolute
k = f(x)	In groups	Programmable	Any in groups
	Discrete	Flexible	Any

suppliers and fabricators, they set up a library which enables the optimization process. The locally sourced inputs required for costing and the scale of problems addressed restrict a potential wider application on real-scale problems.

The aim of this paper is to study the relationship between the change in curvature of a design surface and the respective construction complexity of space frame structures as a factor of joint customization. A compuational framework is proposed for the optimization of space frame structures that incorporates freeform geometries and constructibility criteria. To address this challenge, a classification of surfaces in relation to their curvature is firstly proposed, followed by a thorough study of the possibilities and limitations of existing fabrication techniques.

1.1 Surface curvature

In the context of joint geometry a constant or zero curvature allows the joint angles to remain uniform throughout the space frame structure. Focus is therefore placed on surfaces of changing curvature, including surfaces that can be analytically described, freeform NURBS surfaces and surfaces generated wth non-gemetric methods, after the application of a force. For a detailed classification of surfaces according to their degree of curvature and generation method readers are directed to [10], [11] and [12].

1.2 Fabrication processes

The methods used to fabricate joints define how quickly they can be produced and the degree of customization available. In this section, different fabrication processes are described in relation to their degree of automation and the angle tolerance allowed for between their connecting members (Table 1).

1.2.1 Continuous fabrication

Continuous fabrication produces large quantities of standardized elements in an automated process and in a short period of time [13]. Material is transformed through methods such as casting, drilling or CNC cutting, using a laser, water or plasma cutter, in order to obtain the final form of the joint [14]. An example of such a joint is shown in Figure 1a. Continuous processes are not considered customizable [13], allowing only for an initial set up of the joint configuration before the production begins. This favors standardized products and high production rates. Joints produced with continuous processes allow for a small tolerance between the angles of their members. The value of this tolerance is specific to each joint type and is restricted to a few degrees [14].

1.2.2 Discrete fabrication process

Developments in joint fabrication have led to the use of additive manufacturing for the production of steel joints [15]. This forms a discrete fabrication process, in which joints are produced individually [13], as shown in Figure 1b. The flexible automation [13] ensures a high degree of customization and allows for any tolerance between the members' angles. This freedom enables the incorporation of multiple optimization criteria within the design of individual joints, further improving their

a. b. c.

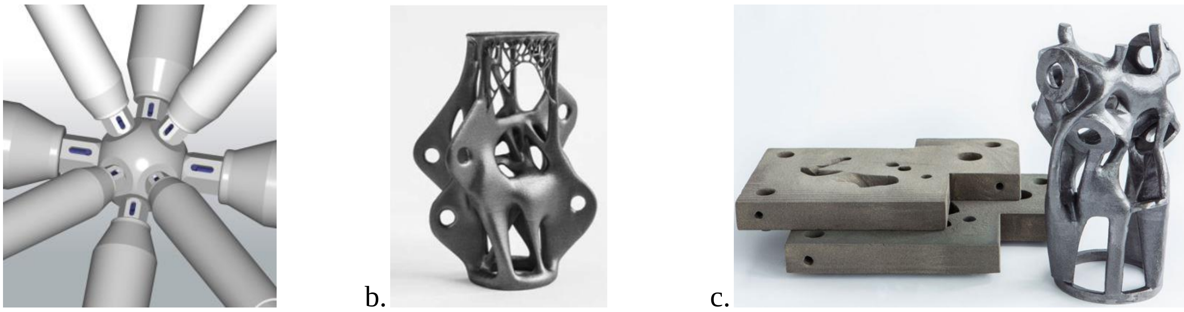


Figure 1: a) A spherical ball joint system produced with a continuous fabrication process [http://www.mero.de/]. b) A 3d-printed metal joint, optimized for geometry and topology [15]. c) A metal joint cast in a 3d printed sand mold [16].

directly linked to the capacity of existing printing hardware, this process is subject to size limitations of the final product. In addition, the fact that this process is under development makes a bespoke certification of the end product's material properties necessary, thus further increasing the overall production time and cost. Despite these challenges, research on the practical application of additive manufacturing in the construction industry is continuously evolving and it is considered as one of the fastest growing fabrication techniques.

1.2.3 Fabrication in groups

An alternative method for the use of additive manufacturing in the fabrication of joints has been developed, triggered by the challenges mentioned above. Leading the research is ARUP, which is investigating the application of additive manufacturing for the printing of sand molds, in which metal joints are then cast [16] (Fig. 1c). There are no size limitations for the end product, as individual components can be connected to form larger molds. Each sand–printed mold can be used for a number of casts, until it deteriorates too much and needs recycling. The production of groups of identical elements has significant impacts of the overall process. First of all, it introduces a programmable automation [13], since the time-expensive method of additive manufacturing is set up and applied only for the printing of the molds at specific intervals in the process. Moreover, given that casting is an established fabrication process, no certification of the final products' properties is required. Finally, a level of customization is introduced, since different groups of joints can accommodate any tolerance between their members' angles. Within the same group, the tolerance is fixed, as in continuous processes.

1.3 Objective

Taking into account the fabrication methods described above, it is evident that the quantity of joints and the level of customization required is a key driver in identifying the optimum fabrication process for any design. Since these factors are project specific, a global evaluation or ranking is not possible. Nevertheless, the flexibility of the "fabrication in groups" process, which allows for some level of standardization and of customization, makes it worthy of further investigation. Standardized and repeating joints reduce the fabrication time and facilitate erection, hence minimizing cost and providing a safer working environment [4]. At the same time, the degree of customization incorporated between different joint groups allows the final space frame structure to approximate complex freeform surfaces. Focusing on this scenario, this paper studies the geometrical optimization of space frame structures for joint uniformity. The goal is to develop a computational workflow to explore the relationship between the change in curvature, the tolerance of the joint and the degree of customization required. This provides direct feedback to the designers on the fabrication complexity of a project in early design stages.

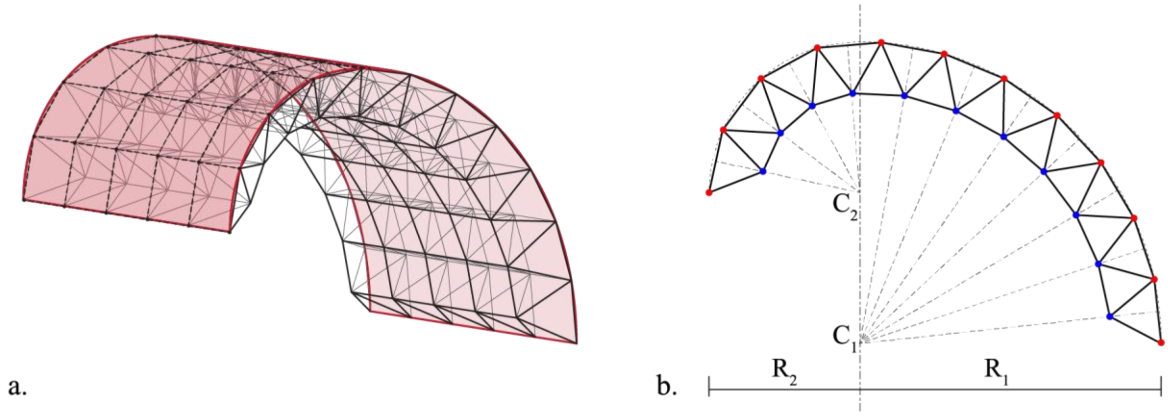


Figure 2: a) The target surface geometry and the starting grid are the inputs for this methodology. b) The experiment setup for a space frame structure of changing curvature. The top layer vertices V_b (red) are considered fixed to approximate the target surface, while the bottom layer vertices V_b (blue) are free to move.

2. Methodology

The methodology for the optimization requires a target architectural surface, *S*, and an initial mesh, *M*, as inputs (Fig. 2a). There are no restrictions regarding the topology and uniformity of the mesh. Methods for generating meshes on free-form surfaces are described in [11]. The space frame structure is generated from the mesh according to the method described in [17]. It comprises of a set of vertices, V_i, $i = 1 \ldots n$, connected by a set of edges, E_i, $i = 1 \ldots m$. The top layer vertices, $V_t \subset V$, are considered fixed, in order to approximate the architectural surface, while the bottom layer vertices, $V_b \subset V$, are free to move. The tolerance between the angles of different vertices, *c*, is given as an input, according to the joint type used in a specific project (Fig. 2b).

Goal of this workflow is to optimize the geometry of the bottom layer of vertices to improve joint uniformity, and comprises of three stages: 1) the angle calculation and comparison for the different joints within a structure, 2) the grouping of the joints into the minimum number of different groups that satisfies a given tolerance, *c* and 3) the geometrical optimization of the spatial coordinates of the bottom vertices V_b to improve joint uniformity. This workflow has been implemented in Grasshopper, the parametric modelling environment of Rhino 3D, with custom components coded in Visual Studio as a plug in for Grasshopper. The popularity of this software in the industry, and the intuitive user interface, will enhance the application of this tool in practice.

2.1 Angle calculation

Different methods have been developed for the geometrical comparison of joints in grid structures. Stephan et al [12] identify three angles to characterize joints, the vertical, horizontal and twist, calculated in relation to a reference design surface. However, the goal of this study is the comparison of different joints in regards to their angles in space, without the need for a reference surface. To achieve this, the member order for the angle calculations needs to be defined according to a local coordinate system, independent of any global geometry. A principal component analysis is therefore carried out to define the best-fit plane of each joint, P_i, $i = 1 \ldots n$ (Fig. 3a). The member forming the largest angle with the plane is chosen as the starting member, and the angle between the starting member and each of the other members is calculated, taking the members in a clockwise order when looking down onto the best-fit plane from the end of it (Fig. 3b). Two joints are considered to be the same type, if all these angles are identical and in the same order.

A set of different joint types *k* is therefore defined, $1 \leq k \leq n$, and the different angle types, *a...v*, are identified for each valence, v_i, $i = 1 \ldots n$ (Fig 3b, 4a). The angle between different vertices is calculated for each angle type and between all possible combinations of vertices, $k*(k-1)/2$. The absolute value of this process is stored in a matrix for each angle type, as shown in Figure 4b.

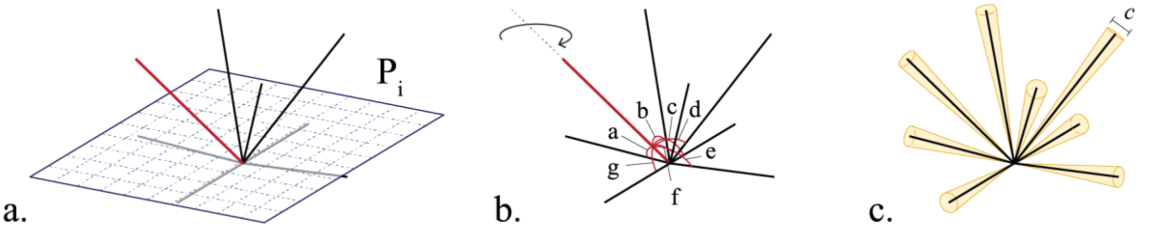


Figure3: a) Calculation of the best-fit plane and definition of the starting member for the angle calculation, according to the members' angle with the plane, b) clockwise calculation of the angles and identification of the different angle types and c) the angle tolerance as defined for each vertex.

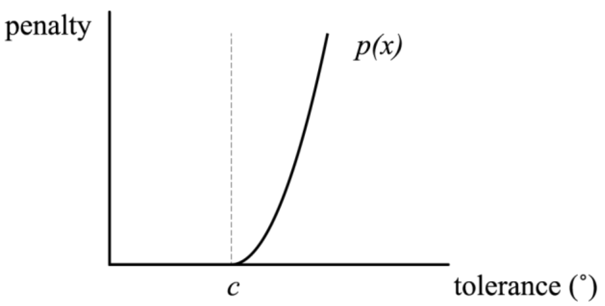
a.

$$\left[\underbrace{\begin{bmatrix} a_1 \\ b_1 \\ c_1 \\ \cdots \\ g_i \end{bmatrix}}_{vertex\ 1} \cdots \underbrace{\begin{bmatrix} a_n \\ b_n \\ c_n \\ \cdots \\ g_n \end{bmatrix}}_{vertex\ n} \right]$$

b.

	1	2		k-2	k-1	k
k	$\lvert a_1-a_k \rvert$	$\lvert a_2-a_k \rvert$	…	$\lvert a_{k-2}-a_k \rvert$	$\lvert a_{k-1}-a_k \rvert$	-
k-1	$\lvert a_1-a_{k-1} \rvert$	$\lvert a_2-a_{k-1} \rvert$	…	$\lvert a_{k-2}-a_{k-1} \rvert$	-	
k-2	$\lvert a_1-a_{k-2} \rvert$	$\lvert a_2-a_{k-2} \rvert$		-		
	…	…				
3	$\lvert a_1-a_3 \rvert$	$\lvert a_2-a_3 \rvert$				
2	$\lvert a_1-a_2 \rvert$	-				
1	-					

Figure 4: a) The angles of the different angles types, as stored for each vertex. b) The angle difference between all possible combinations of vertices for angle type *a*. Respective values are stored for each angle type.



$$p(x) = 0; \qquad x \leq c;\ c \geq 0; \qquad (1a)$$

$$p(x) = (x-c)^2; \qquad x > c; \qquad (1b)$$

Figure 5: The graphical representation and formulation of the penalty function p(x). The tangential configuration and the steep curvature ensure a quick conversion rate.

2.2 Grouping - penalty

This step identifies the number of different joints, *joint index*, that are required for the construction of the space frame structure. This is achieved by the evaluation of the angle differences stored in the matrices (Fig. 4b). Two vertices V_1 and V_2 are considered in the same joint group if the angle difference between their members is smaller than the tolerance *c* for all angle types, $|a_{2i}-a_{1i}| \leq c$, $i=1...v$ (Fig. 3c). If the vertices have different valences ($v_1<v_2$, say) they are considered of the same group if and only if the angles A_1 of V_1 are an ordered subset of the angles A_2 of V_2 ($A_1 \subseteq A_2$) or if their difference is smaller than the tolerance for all angle types. This means joint type 2 could be used in position 1, with parts of its connections unused. Readers are directed to [18] for further reading on this topic. A custom component is therefore developed, which iteratively evaluates the angle matrices and calculates the joint index j, $j \leq k \leq n$, which is required for the construction of the space frame structure. A penalty function, p(x) is defined, which penalizes each angle difference that is above the tolerance limit *c*. A total penalty value is hence defined for each angle type, Σp_a, Σp_b, ..., Σp_h. A pilot study on the form of the penalty function determined that a parabolic expression gave good convergence, due to its tangential configuration and the steep conversion rate (Fig.5).

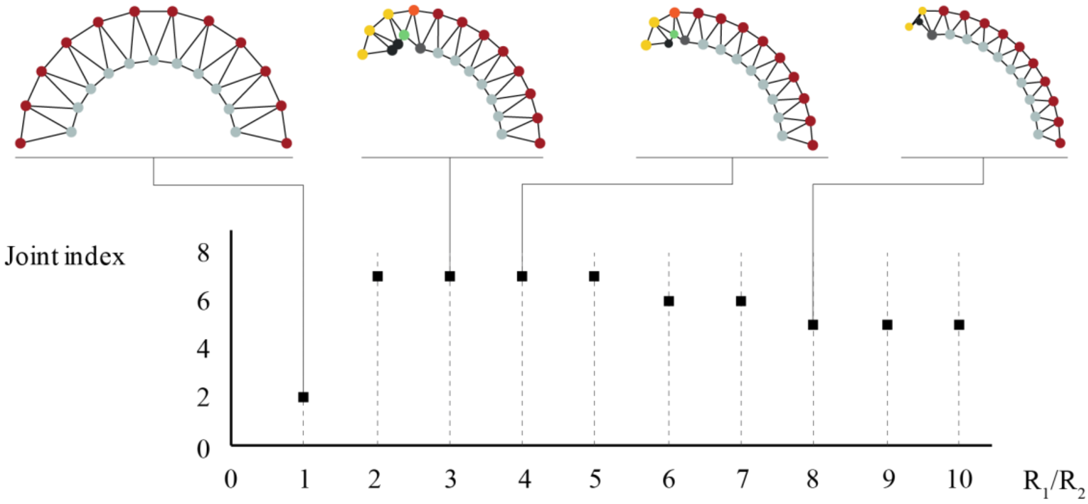


Figure 6: The effect of changing curvatures on joint index and the corresponding geometries for relationships $R_1/R_2 = 1$, $R_1/R_2 = 4$ and $R_1/R_2 = 8$ respectively.

2.3 Geometry optimization

Goal of the optimization is to determine the spatial coordinates of the bottom layer vertices V_b, in order to minimize the number of different joints in the space frame structure (the joint index j). Nevertheless, the integer nature of the joint index and the limited domain of potential values, $j \in [1, n]$, challenge the convergence of the optimization. The objective function is therefore formulated as a combination of the joint index, j, and the penalty function, as shown in Equation 2. Variables of the optimization are the spatial coordinates of the bottom layer vertices. Practical considerations can be incorporated in the definition of the variables, such as depth limitations or a restriction of their movement only along the vector normal to the target surface. The optimization is performed via genetic algorithm using the inbuilt evolutionary algorithm tool of Grasshopper, Galapagos [19].

$$\text{minimize} \qquad \sum_{x=a}^{h} j * p(x); \qquad\qquad m \in [1, \text{n}] \quad (2)$$

2.4 Experiments

2.4.1 Changing curvature

The first set of experiments aims to identify the way in which the change in curvature affects the joint index. Two arcs of different curvatures are considered for this reason, as shown in Fig 2. The curvature of one arc is constant throughout the experiments, (R1 = 25m), while the curvature of the other arc (R2) is varied and the effect of their ratio R1/R2 on joint index is studied. The number of subdivisions and the joint tolerance remain constant at 11subdivisions and c=3˚ respectively. The top layer vertices V_t are fixed in translation to approximate the design curve, while the bottom layer vertices, V_b, are free to move along the bisector line joining them to the centre of curvature of their respective arcs (Fig. 2b). The starting positions of the bottom vertices were assigned randomly and a pilot study was run to identify the best values for the genetic algorithm variables to ensure convergence.

The results of these experiments are demonstrated in Fig. 6. The configuration with both arcs having the same curvature is the optimum result, as expected, with a joint index of 2, one for the top layer and one for the bottom layer vertices. The tool also achieves the optimum solutions for the remaining relationships of R1/R2, which vary between 5 and 7. This difference can be attributed to the grid configuration. In cases of a smooth curvature change ($2 \leq R_1/R_2 \leq 7$), the angle difference is distributed between three vertices in the middle of the structure. However, in cases of a tight transition ($R_1/R_2 \geq 8$),the part of the structure with a tight curvature comprises only of a single bay, therefore the angle difference is distributed between fewer vertices.

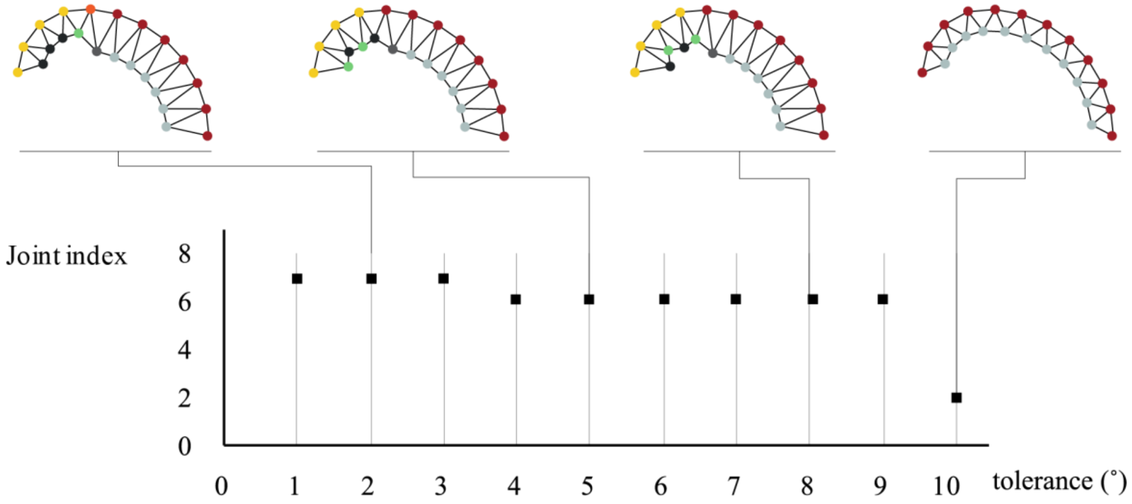


Figure 7: a) The effect of tolerance limits on the joint index, b) The geometrical configuration of the space frame structure for tolerance values of 2,4 and 10 respectively.

The starting grid layout is therefore an equally important as the degree of change in curvature for the joint index of a structure. At the same time, these experiments highlight the efficiency of the tool to converge to the optimum result.

2.4.2 Changing tolerance

The effect of the tolerance on the joint index of a space-frame was afterwards studied. Using the same set up as above, and with an R_1/R_2 ratio of 2, the structure was geometrically optimized for a series of different allowable tolerances, $c \in [1,10]$. As shown in Fig. 7, for all the values of tolerances tested, there are only three values of joint indices. Even though the distribution of these three values would change for a different curvature, they provide insightful information from a fabrication point of view. The top and bottom layer of the two curvatures have different angel configuration and their difference is accommodated in the central part of the structure. When the tolerance c is tight ($c \leq 3$), this is distributed between three vertices in the center of the structure, leading to a joint index of 7. However, when the tolerance is larger ($c \geq 4$), two vertices can accommodate this change and reduce the joint index 6. The number of different joint configurations for a given design curvature is therefore restricted to three specific values, while any change in tolerance within these zones would not affect the fabrication process.

5. Conclusions

The proposed computational workflow achieves space-frame structures geometrically optimized for joint uniformity in a practical and efficient manner. This method can provide direct insight into the fabrication complexity of a given structure at early design stage. Future research by the authors will include the incorporation of structural performance criteria on larger scale problems in order to provide insight into the structure's performance. In regards to the description of the problem, alternative methods of formulating the optimization problem will be studied, in an attempt to minimize the number of design variables. This would facilitate graphical representation of the design space and hence provide a deeper understanding of each problem's solution.

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References

[1] J. Chilton, *Space Grid Structures*. London: Architectural Press, 2000.

[2] M.P. Bendedsoe and O. Sigmund, *Topology Optimization Theory, Methods and Applications*. Springer, 2004.

[3] M. Veltkamp, "Free Form Structural Design: Schemes, systems and prototypes of structures for irregular shaped buildings," Ph.D. dissertation, Technische Universiteit Delft, 2007.

[4] SCI P358, "Joints in steel construction: Simple joints to Eurocode 3 (2014 reprint)", 2014.[Online]. Available:https://www.steelconstruction.info/Simple_connections#Costs. [Accessed April 16, 2018].

[5] S. Narayanan, *Space Structures: Principles and Practice*. Brentwood, USA: Multi- Science Publishing Co, Ltd., 2006.

[6] A. Asadpoure, J. K. Guest, and L. Valdevit, "Incorporating fabrication cost into topology optimization of discrete structures and lattices," *Struct Multidisc Optim*, pp. 385–396, 2015.

[7] B. Paulson, "Designing to reduce construction cost," *J. Constr. Div.*, vol. 102, no. 4, pp. 587–592, 1976.

[8] P. Havelia, "A ground structure method to optimize topology and sizing of steel frame structures to minimize material, fabrication and erection cost," Dissertation for degree of eng., Dept. Civil and Environ. Eng, Stanford Univ., Stanford, CA,2016.

[9] F. Ranalli, F. Flager, and M. Fischer, "A Ground Structure Method to Minimize the Total Installed Cost of Steel Frame Structures By," *World Acad. Sci. Eng. Technol. Int. J. Civ. Environ. Eng.*, vol. 12, no. April, pp. 115–123, 2018.

[10] A. Gheorghiu and V. Dragomir, *Geometry of Structural Forms*. Essex, UK: Applied Science Publishers Ltd., 1978.

[11] H. Pottmann, A. Asperl, M. Hofer and A. Kilian, *Architectural geometry*. Exton, PA: Bentley Institute Press, 2007.

[12] S. Stephan, J. Sánchez-Alvarez, and K. Knebel, "Stabwerke auf Freiformflächen," *Stahlbau*, vol. 73, no. 8, pp. 562–572, 2004.

[13] S. Kalpakjian, S.R. Schmid, *Manufacturing Engineering and Technology* 7ª ed. Cambridge, UK: Pearson, 2013.

[14] R. Gonzalez Bravo, "Aplicaciones industriales para la construcción de estructuras laminares metálicas," Ph.D. dissertation, Dept. de Construccion y tecnologia arquitectonicas, Escuela Tecnica Superior de Arquitectura de Madrid, Madrid, Spain, 2015.

[15] S. Galjaard, S. Hofman, N. Perry, and S. Ren, "Optimizing Structural Building Elements in Metal by using Additive Manufacturing," *Int. Assoc. Shell Spat. Struct.*, August, 2015.

[16] P. Niehe, "Sand printing makes complex casted structural parts affordable," 2017. [Online].Available: https://www.arup.com/news-and-events/news/sand-printing-makes-complex-casted-structural-parts-affordable. [Accessed March 25, 2018].

[17] A. Koronaki, P. Shepherd, and M. Evernden, "Layout optimization of space frame structures," in *Proc. IASS Annu. Symp. 2017 Interfaces Archit. Eng. Sci., Hamburg, Germany, September 25-28, 2017,* Hamburg: Annette Bogle, Manfred Grohmann (eds), 2017.

[18] M. Church, "Optimization of double-layer grid structures", Research dissertation, Dept. of Arch. and Civil Eng., University of Bath, UK, 2018.

[19] D. Rutten, "Evolutionary Principles applied to Problem Solving."[Online]. Available at: http://www.grasshopper3d.com/profiles/blogs/evolutionary-principles. [Accessed April 15, 2018].