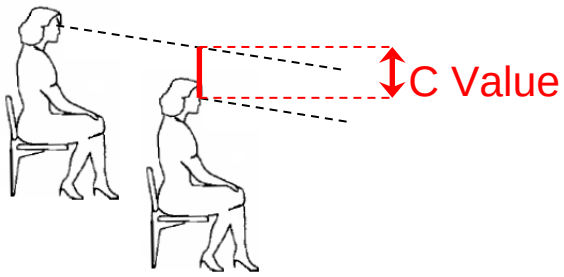
Sightlines

The Background

Have you been lucky enough to get any tickets for the Olympic games in London? Have you thought about the kind of view you will get? How would you feel if you went to see the games and all you could see was the back of the head of the person sat in front?

The architects and engineers who design stadiums have to think very carefully about the way the seats are laid out in order to squeeze in as many seats as possible without compromising on the quality of the view. They use an algebraic equation to determine the quality of view, known as the "C Value", which is the vertical distance in mm between your line of sight and the eyes of the person in front.



An architect would decide on the C Value they wish to have based on the table below.

C Value (mm)	Description
60	Need to look between heads in front
90	Can see well with head tilted backwards
120	Optimal viewing standard
150	Can see well even if over spectators with hats

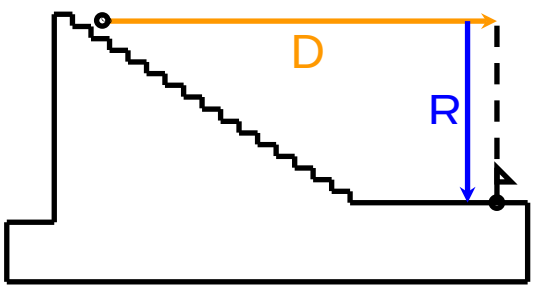
They would then use this equation to find out the required height of the steps between each row of seats:

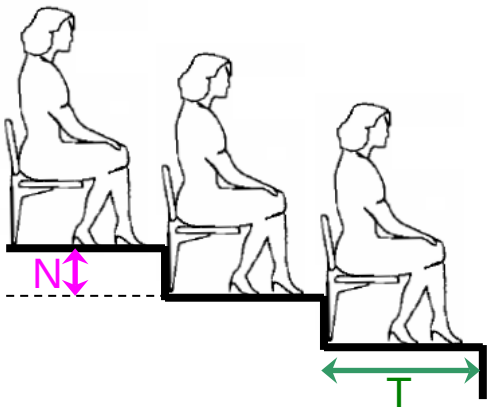
$$N = \frac{(R + C) \times (D + T)}{D} - R$$

Where…

R= is the vertical height between the eye and the point of focus
D= is the horizontal distance between the eye and the point of focus
N= is the seating step height
T= is seating step width

… as shown below.





The Problem

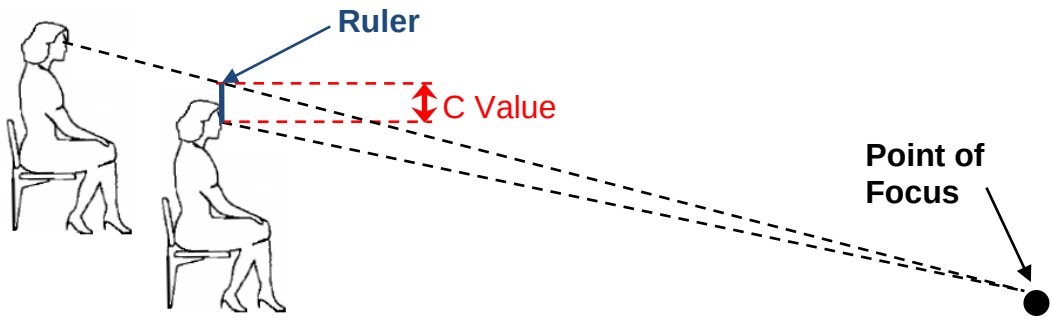
Find a suitable room (Lecture room, theatre, assembly hall, sports hall) in your school. It needs to have stepped seating and an obvious "stage" or other area where activities take place. If your school doesn't have such a place, you can use any set of stairs, and place an object some way in front of the bottom step to represent the edge of a playing field.

From your experience of the view quality, what do you expect the C value to be? Can you only see between the heads of the people in front, or could you see over the top of them even if they wore a hat?

Architects usually start by knowing the C value they want to provide, and calculate the step heights accordingly. You are starting from already built seating and back-calculating the C value, so you will need to rearrange the equation to give the value of C and measure the dimensions of the steps yourself. Were you about right in your estimation?

Measure the step geometry at different places along a row and at different rows. Is the C Value the same everywhere? If not, why not?

Now you can measure the C Value directly by getting two people of about the same height to sit one in front of the other. By using a long piece of string to represent the sightline you should be able to measure the vertical distance between the string and the eyes of the person in front. Does it give the same value as you got by measuring the stair geometry? If not, why not? Which is likely to be most accurate?

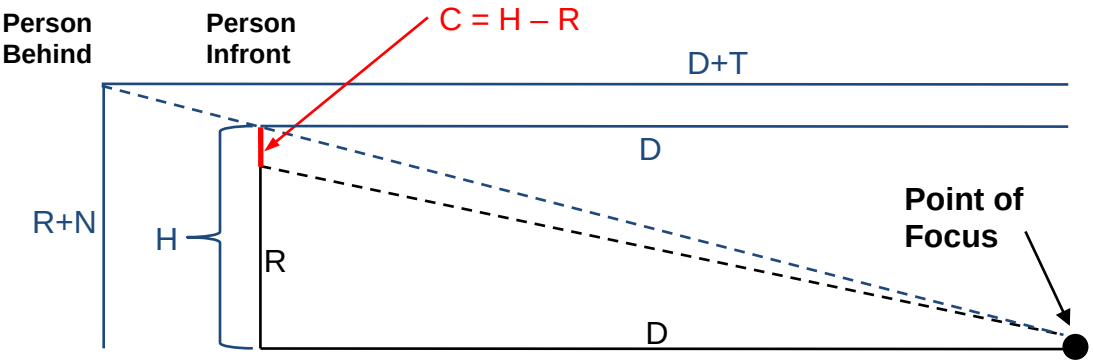


Can you derive the equation yourself using trigonometry?

If you can get permission, why not try to visit your local sports stadium and measure their sightlines? How would you expect them to compare with your school?

Teachers' Notes

The equation for "C" can be found by rearranging the equation given above, or derived from trigonometry by considering "H", the height of the person behind's sightline as it passes over the person in front:



The gradient of the person behind's view is the vertical distance from the focus (R+N) divided by the horizontal distance from the focus (D+T). This same gradient sampled at the person in front is the vertical distance H divided by the horizontal D. Since these two gradients are the same, we can set them equal to each other and rearrange to give H.

$$\frac{H}{D} = \frac{(R+N)}{(D+T)} \implies H = D \times \frac{(R+N)}{(D+T)}$$

The vertical distance between the two sightlines, "C", is therefore simply this distance "H" minus the height "R":

$$C = H - R = \frac{D(N+R)}{D+T} - R$$

Applying the equation for a fixed C value to many rows of seats would result in a curved seating block, since the value of "N" increases as D increases (assuming a constant value for T and each time adding onto R the value of the previous row's N). However, seating blocks are made from similar concrete units and people are at risk of tripping up as they climb the steps if they are not regularly spaced. So each block of seating usually has a constant, average value of N (and T), meaning the view at the front is of slightly better quality than that at the back.

The C Values found by measuring in school will be sensitive to the chosen Focal Point. In a sports ground the closest focal point is obviously the touch-line of the pitch. But in a lecture-hall setting it may be that the closest point of focus was not chosen to be the front of the stage, but a lectern or a point in the middle of the stage.

Other Activities

How might the cost of a seat rise with quality? When Arsenal first opened their new Emirates Stadium, the cost of a match ticket rose in line with C Value as the table below shows. But the correlation is not perfect since some areas, such as behind the goal, have restricted views of other parts of the pitch.

Emirates Upper Tier	C Value	2007 Match Price
Corner	60	£60
Wing Back	75	£55
Next to Centre Back	75	£61
Centre Back	75	£70
		£62 Average
Wing Front	80	£60
Next to Centre Front	80	£70
Centre Front	80	£94
		£75 Average
Goal Back	100	£55
Goal	125	£66

Unfortunately the designers can't give every seat a C Value of 150 because there wouldn't be enough headroom below the floor above. Nor can the stadium owners charge £94 for every single seat as there wouldn't be 60,000 fans who could afford to watch. So the space has to be used wisely and the right balance struck between providing enough cheaper, but lower quality seats, and making sure there are enough premium quality seats to earn the club more revenue.

There are some interesting differences in stadium design around the world. The table below shows the minimum recommended seating widths and depths (legroom) according to different countries' regulations (taken from Reference #2 below). Discuss why different countries might have different regulations. Are we physically different, or just have different expectations of comfort? Is it due to the types of sports played in the stadiums?

Country	Minimum Recommended Seat Width (mm)	Minimum Recommended Seat Depth (T) (mm)
England	460	700
USA	450	762
Germany, Norway, Sweden & Netherlands	500	800
Austria & Switzerland	450	750
Italy	450	600

There are also interesting discussions to be had on wider themes of stadium layout such as the number of and ratio between male & female toilets. There are typically more "men" present during a sports match, but they also take less time to visit the toilet. Some stadiums have changeable signage so that toilets can be reallocated from male to female and back depending on the event (Football World Cup vs Take That Concert for example?).

References

[1] This is the book that engineers refer to when designing stadiums. It contains lots of interesting facts and figures but is probably not worth buying for school:
http://www.amazon.co.uk/safety-grounds-Britain-Department-Culture/dp/0117020745

[2] This was written to accompany (and explain in plain English) the above government regulations. It is more accessible but still quite dry and technical:
http://www.amazon.co.uk/Stadia-Geraint-John-CISRM-MILAM/dp/075066844X/

