Resonance

The Background
Have you ever heard about opera singers being able to shatter wine glasses just with their voice? Or wondered why the washing machine may suddenly jump about violently for a short while as it speeds-up or slows-down its spin speed?

They are both examples of resonance, where objects move significantly more when shaken at one particular frequency (be it by having a particular note sung at them or a motor spinning at a particular speed) than if they are shaken the same amount at other frequencies.

Everything around us has a “Natural Frequency”, a frequency at which it would vibrate if plucked or hit with a hammer for instance. A tuning-fork is a classic example, where its natural frequency is carefully chosen to help people tune musical instruments. But guitar strings being plucked and rulers being twanged over the edge of a desk all have their own particular frequency at which they will vibrate.

A playground has its own natural frequency, and when you push someone on the swing, and your pushes are exactly at that natural frequency, the result is clear – you see their swings get bigger and bigger and bigger. But when you push at other frequencies, very little happens. Sometimes you might be pushing when they are not within reach. Sometimes you might be pushing them forwards as they are swinging back towards you, and you actually slow them down. It is only when the frequencies match that all your energy goes into making them swing higher and higher with each push.

Buildings are no exception, and can have particularly large responses to vibration at particular frequencies. Sometimes a lorry idling at some traffic lights (low frequency engine noise) will cause the windows in a nearby house to rattle loudly, when the same lorry driving past (higher frequency engine noise) has no effect. A marching army is often told to “break-step” before crossing a bridge, so that each soldier is out of step with the others and the impact on the bridge desk is at a much higher frequency and lower amplitude than if they were all marching together in step. Perhaps you saw on the news in June 2000 the opening of the Millennium Bridge in London? With 2,000 people all walking along it at once, they fell into step with each other and it is now known as the “Wobbly Bridge”.

Engineers who design stadiums have to think very carefully about the way the spectators might start to sway or jump up-and-down on the upper-floor seating-decks. If this happens when a goal is celebrated it tends not to last for very long and is everyone is not usually jumping in-time with one another. However, modern stadiums are not just used for sports, and are designed to play host to many different events including music concerts. In this case, people are very likely to be jumping up-and-down, and very likely to be in time with each other since they have the music to act as a guide.

Stadium seating is designed to have a natural frequency of higher than 6Hz. People at music concerts will generally be jumping to music with a much lower frequency, meaning there will be enough of a difference between the two such that the stadium will not resonate.

The Input Problem

Analyse your favourite music to find out what frequency you are likely to be jumping at when you next go to a live concert. You can do this by counting the number of beats in a given length of time. Maybe you can borrow an oscilloscope and use it to measure the frequency direct (it might be better to play the music on an iPod at double-speed and then half the result). Or perhaps you can use the beat-finder function in free software "Audacity" (available from audacity.sourceforge.net). Make sure you convert all "beats-per-minute" values to Hertz.

Compare your results with those of your classmates. Who listens to the fastest music? Who the slowest? Is there a broad range of frequencies in your tastes, or do your friends tend to listen to music of similar frequency to yourself? Do all the different methods of measuring give the same answer if used on the same piece of music? If not, which do you think is most accurate and why?

The Output Problem

What sort of resonant frequencies do the structures around your school have? Is there an upper floor to your assembly hall you could jump on? Or a balcony or link-bridge corridor? Maybe you can measure a footbridge in your town / city?

The frequencies are usually measured by incorporating accelerometers at strategic points where the movement is expected to be greatest and then dropping a big weight on them (on the structures, not the accelerometers). Many modern smart-phones have accelerometers built-in which might be able to measure the vibrations. But it might take a coordinated jump by a few people at once to make the structure move enough to be measured.

If you can get permission, why not try to visit your local sports stadium and measure the resonant frequency of their seating tiers?

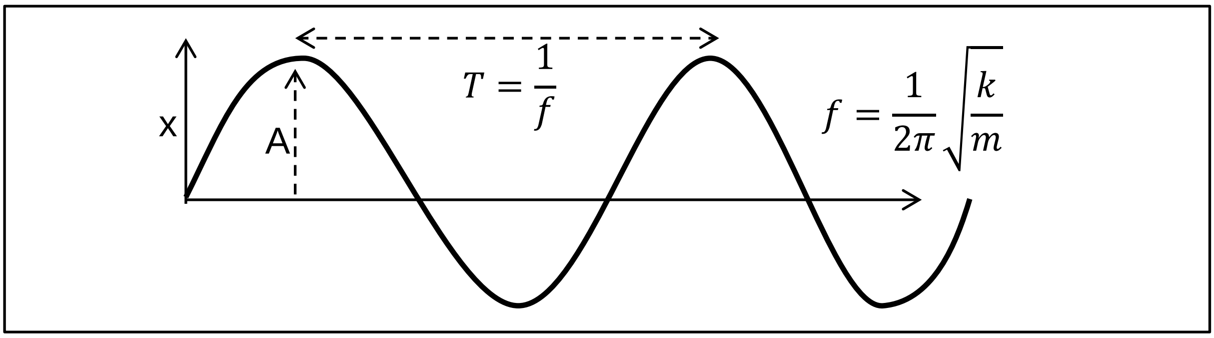
How do the resonant frequencies of the structures compare with those of your music? Are the structures likely to suffer from resonance?

Obviously you'll have to be sensible and not break anything. The resonant response of structures can grow very large under repeated loading such as jumping. If there are any signs that the structure is undergoing significant movement STOP.

Teachers' Notes

Modern musical keyboards can often be programmed to play rhythms at a specific number of beats-per-minute. Maybe once they have determined the frequency of their favourite music, you could test them by getting them to vote for the best speed at which a particular song should be played and seeing how it compares. The "Audacity" software mentioned above can speed-up or slow-down music – or ask the school band to play something fast and slow.

If we assume vibration is sinusoidal (which is a safe bet) then there is a relationship between the amplitude of the movement, the velocity, and the acceleration:



By differentiating the equation of position with respect to time, we get:

$$x(t) = A\sin(2\pi ft)$$
$$\dot{x}(t) = 2\pi fA\cos(2\pi ft)$$
$$\ddot{x}(t) = -(2\pi f)^2 A\sin(2\pi ft)$$

Since the maximum value of the sin function is one, and the maximum amplitude of the displacement is "A" metres, then the maximum velocity will be "$2\pi fA$"m/s and the maximum acceleration will be "$4\pi^2 f^2 A$" m/s^2, which changes with the square of the frequency. Even at low amplitude displacements, you'd be surprised how this can give an acceleration of greater than gravity if the frequency is high.

If your school is short of structures to wobble, twanging rulers on the desks can still be quite informative. The formula for natural frequency I show in my talk is for a one-degree-of-freedom problem:

$$frequency = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

where k is the stiffness (in units of force-per-length) and m is the mass. You can probably measure the stiffness of the ruler by applying weights and measuring the displacement. The effective one-degree-of-freedom mass is trickier since the ruler's mass is evenly distributed along its length. But if you place a relatively large weight at the end of the ruler (a large ball of blu-tack for example) then the mass of the ruler itself is negligible in comparison and can be safely neglected. Students could explore the relationship of mass to frequency by applying more blu-tack. They could explore the relationship of stiffness by changing the length of the ruler or taping two rulers together. The frequency of vibration can be measured using a strobe light, microphone-and-oscilloscope or by videoing it and watching it back in slow-motion.

Other Activities

Other than jumping to music, the other area where engineers need to take resonance into account is in the design of stairs, especially light metal-framed stairs. It is interesting to compare the footfall frequency of pupils running up- and down- stairs. When running down, the top foot has to move further (it is lifted off the top step and then placed way down on the step below) before impacting on the stair, so the frequency of impact is relatively low. When running upstairs, the lower foot is lifted up and then almost immediately impacts on the step above, so the impacts are closer together and at a higher frequency. Students could measure this difference directly by timing themselves running up and down a suitable stairwell.

References

[1] There is lots of discussion on the web about breaking a wine glass by singing, including videos on YouTube, but the most "reliable" source is probably this one in Scientific American:
http://www.scientificamerican.com/article.cfm?id=fact-or-fiction-opera-singer-can-shatter-glass