

MAF Corporate Finance.

Real Options: Game-theory Exercise 1.

Consider an economy consisting of two risk-neutral firms, firm 1 and firm 2. The risk-free rate is zero.

Date 0: Firm 1 has a new investment opportunity, project 1, requiring investment $I > 0$. Firm 1 decides whether to make this investment. If the firm takes the project, there is a probability of date 1 income, as described next.

Date 1: If the firm invested in project 1 at date 0, then a state of nature is revealed at date 1, as follows. With probability P , the state of nature is g (good), while with probability $1-P$, the state of nature is b (bad). If the state of nature is good, the project will provide cashflow $R > I$. If the state of nature is bad, the project will provide zero cashflow.

Date 2: If the date 1 state of nature is good, a rival firm, firm 2, appears with a given probability. This probability is p if firm 1 had invested in project 1 at date 0, and $q > p$ if firm 1 had not invested in project 1 at date 0. If the date 1 state of nature is bad, the rival firm 2 does not appear. The state of nature that occurs at date 2 persists throughout the remainder of the game.

Date 3: A second project, project 2, requiring investment I , becomes available (this is known about at date 0). If firm 2 appeared at date 2, we enter the product market competition sub-game as follows. Firm 2 moves first, deciding whether to invest in project 2 at cost $F < I$. Then firm 1 observes firm 2's decision, and decides whether to also invest, at cost I . If firm 2 didn't appear at date 2, firm 1 makes the investment decision alone.

If both firms invest at date 3, then, in the case of the good state of nature, project 2 will provide cashflow $R(1-\gamma)$ to each firm. If the state of nature is bad, the project will provide zero cashflow to each firm. If only one firm invests at date 3, the project provides that firm with cashflow $R > I$ in the good state of nature, and zero cashflow in the bad state of nature.

Date 4: the game ends, and payoffs occur.

Required:

- a) Draw the game-tree, appropriately labelled.
- b) Calculate the equilibrium of the game under the assumption that $R > I > R(1-\gamma) > F$.

PTO for recommended reading:

Recommended Reading:

“A Real Options and Game-theoretic Approach to Corporate Investment Strategy Under Competition.” Han Smit and L.A. Ankum, *Financial Management*, Autumn 1993.

“Real Options and Games: Competition, alliances and other applications of valuation and strategy.” Han Smit and Lenos Trigeorgis, *Review of Financial Economics* 15 , 2006, pages 95 -112.

“Overconfidence, Investment Policy, and Executive Stock Options.” Gervais, Heaton and Odean, SSRN database.