Debt Signalling Game.

Consider a two-period debt-signalling game, consisting of 2 firms, firm G (good) and firm B (Bad). Each firm is run by a risk-neutral manager. Each firm has a project in place. If the project succeeds, it will produce an income of $R > 0$. If the project fails, it will produce zero income. For firm G, the probability of success is $p > q$, while, for firm B, the probability of success is q. The risk-free rate is zero. In the absence of any signals, the firms are observationally equivalent to the financial market, which assigns an equal probability of each firm being type G or B (prior beliefs).

The sequence of events is as follows:

At date 0, the manager of each firm simultaneously decides whether to issue low or high debt; $d \in \{0, D \in (0,R)\}$. The market observes the debt signals and updates its beliefs, and values each firm accordingly. Specifically, the market updates its beliefs (Bayesian updating => posterior beliefs) as follows. If both firms issue the same level of debt, the market is unable to update its beliefs, and continues to assign equal probability to each firm being of either type. If one firm issues high debt, and the other firm issues low debt, the market believes that the firm that issues high debt is type G, and the firm that issues low debt is type B.

At date 1, both firms' projects achieve success or failure. If the firm can pay the debt-holders, it does so. In the case that the firm is unable to pay the debt-holders, the manager of that firm faces financial distress costs $F > 0$.

Each manager has the following payoff function;

$$\Pi_i = \alpha V_0 - (1-\Delta) f,$$

Where $\alpha \in (0,1]$ is the manager's share of date 0 market value, V_0 is the date 0 market value of the firm, $\Delta \in \{p, q\}$ for the good or bad firm respectively, and $f \in \{0, F\}$ if the debt-holders get re-paid, or do not get re-paid, respectively. Hence, $(1-\Delta) f$ is the manager's expected date 1 financial distress costs.

Required:

1. Derive the managerial payoffs for the 4 combinations of debt level.
2. Draw the normal form game.
3. Derive the best response functions.
4. Solve for the perfect Bayesian equilibria of the game, in the following cases;

i) $\alpha(\frac{p-q}{2})R > (1-q)F > (1-p)F$. ii) $(1-q)F > \alpha(\frac{p-q}{2})R > (1-p)F$.

iii) $(1-q)F > (1-p)F > \alpha(\frac{p-q}{2})R$.

5. Provide the economic intuition behind these results.