Derivatives: part I
Derivatives

- Derivatives are financial products whose value depends on the value of underlying variables.

- The main use of derivatives is to reduce risk for one party. The diverse range of potential underlying assets and pay-off alternatives leads to a huge range of derivatives available to be traded in the market. Derivatives can be based on different types of assets such as:
  - Commodities
  - Equity
  - Bonds
  - Interest rates
  - Exchange rates
  - Indexes
  - Consumer price index
  - weather conditions etc.
Types of derivatives

• Forwards: One party agrees to buy an asset on a date in the future at a fixed price. There is no element of optionality about the deal.

• Futures: Essentially the same as a forward, except that the deal is made through an organized and regulated exchange rather than being negotiated directly between the two parties. Note that futures are guaranteed against default.

• Swaps: Agreement made between two parties to exchange payments on regular future dates, where the payment legs are calculated on a different basis. Swaps are OTC deals.

• Options: Gives the holder the right to buy/sell an asset by a certain date at a fixed price.
Types of derivatives

• Credit default swaps (CDS)

• Collateralized debt obligations (CDO)
The participants

- **Dealers:** Derivative contracts are bought and sold by dealers who work for major banks. Some contracts are traded on exchanges, other are OTC (over-the-counter) transactions.
  
  - Sales staff speak to clients about their requirements
  - Experts assemble solutions to those problems using combinations of products
  - Traders manage the risks involved by running the bank’s derivatives books
  - Risk managers keep an eye on the overall level of risk
  - “Quants” devise the tools required to price the products

- **Hedgers:** Corporations, banks and governments all use derivative products to hedge or reduce their exposures to market variables.
• Speculators: Derivatives are very well suited to speculating on the prices of commodities, financial assets and market variables. Generally speaking, it is much less expensive to create a speculative position using derivatives than by actually trading the underlying asset. As a result, potential returns are much greater.

• Arbitrageurs: An arbitrage is a deal that produce risk-free profits by exploiting mispricing in the markets. For instance, it occurs when a trader can purchase an asset cheaply in one location and simultaneously arrange to sell it at a higher price.
Swaps
Swaps

• Definition: Agreement made between two parties to exchange payments on regular future dates, where the payment legs are calculated on a different basis.

• The most common type of swaps is an interest rate swap.

• Other type include currency swap, asset swap, commodity swap etc.

• The most common IRS is the fixed/floating swap, referred to as plain vanilla deal.

• For IRS, the purpose is to transform a fixed-liability into a floating-rate liability and vice-versa.
• Characteristics:

  – One party agrees to pay a fixed rate of interest applied to the notional principal on regular future dates

  – The other party agrees to make a return payment based on a variable rate of interest applied to the same notional payment

  – The notional principal is fixed at the outset

  – The notional principal is never exchanged, it is used to calculate payments
The floating rate that is used most often is calculated with reference to the LIBOR (London Interbank Offered Rate).

If the floating rate is the LIBOR itself, then the swap is called a LIBOR flat swap.

Consider Bank A with ratings AAA, and Bank B with ratings BBB. Cost of borrowing:

<table>
<thead>
<tr>
<th></th>
<th>Bank A</th>
<th>Bank B</th>
<th>Advantage of A over B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed-rate</td>
<td>10%</td>
<td>12%</td>
<td>2%</td>
</tr>
<tr>
<td>Floating rate</td>
<td>LIBOR+0.25%</td>
<td>LIBOR+0.75%</td>
<td>0.5%</td>
</tr>
</tbody>
</table>
• Bank A has a comparative advantage in fixed-rate loans, Bank B has a comparative advantage in floating-rate loans.

• Bank A's motive is to exploit the comparative advantage to make gain from trade

• A swap is therefore feasible if Bank A prefers to have a floating rate and Bank B prefers to have a fixed rate.
Bank A (AAA) → Swap bank (10.5% LIBOR) → Bank B (BBB) (10.75% LIBOR)

Company I (10%) → Swap bank

Bank B (BBB) → Company II (LIBOR +0.75%)
• Total borrowing costs:

Bank A: Borrows fixed at 10%
Receives from swap bank  (10.5%)
Pays to swap bank LIBOR
Total LIBOR-0.5%

Bank B: Borrows floating at LIBOR+0.75%
Receives from swap bank  (LIBOR)
Pays to swap bank 10.75%
Total 11.5%

SWAP bank: Pays A (10.5%)
Receives from A LIBOR
Pays B LIBOR
Receives from B 10.75%
Additional issues:

• Companies do not directly gain from the swap. However, they could negotiate a better return on their deposits.

• Typical swap quote: 45-55 for a 3-year swap. This means that the bank is willing either to pay a fixed rate of 45 basis point over the 3-year bond yield and receive LIBOR, or to receive a fixed rate of 55 basis point over the 3-year bond yield and pay LIBOR.
Risks involved in swaps

• Credit risk: risk that the other party will default on its obligation. While credit as such is not extended, there is the risk that the transaction does not take place. This risk declines as the swap approaches maturity.

• Market risk: risk that market interest rates diverge from the rates agreed in the swap, leading to a position loss for one party. The risk does not necessarily decline as the swap approaches maturity.

• The risks are not unrelated. For instance, a swap might be showing a position gain but the other party might then default.

• The swap bank faces a spread risk, i.e. the risk that the difference between the swap price and the futures prices moves in an adverse direction.
Hedging with swaps

- Consider the following swap between A and B:

  - The current LIBOR is 3.75%.
  - The notional is £100 million
  - The maturity is 10 years, i.e. there are 10 payments
  - Hence, the first year, A pays £5m and receives £3.75m
• Why would A agree to such a deal?
  – This type of payment structure is typical of the situation in which the market anticipates future increases in interest rates
  – In that case A could gain from the swap
  – However, hedging is the most obvious reason for this swap

• Suppose A is a company that has borrowed £100million from a bank with 10 years maturity. The rate of interest is reset every year to LIBOR+0.75%.

• The swap transaction makes company A’s net payment independent of the LIBOR
<table>
<thead>
<tr>
<th>LIBOR</th>
<th>Loan rate</th>
<th>Loan interest (£m)</th>
<th>Swap fixed payment (£m)</th>
<th>Swap receipt (£m)</th>
<th>Net payment (£m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4%</td>
<td>4.75%</td>
<td>-4.75</td>
<td>-5</td>
<td>4</td>
<td>-5.75</td>
</tr>
<tr>
<td>5%</td>
<td>5.75%</td>
<td>-5.75</td>
<td>-5</td>
<td>5</td>
<td>-5.75</td>
</tr>
<tr>
<td>6%</td>
<td>6.75%</td>
<td>-6.75</td>
<td>-5</td>
<td>6</td>
<td>-5.75</td>
</tr>
<tr>
<td>7%</td>
<td>7.75%</td>
<td>-7.75</td>
<td>-5</td>
<td>7</td>
<td>-5.75</td>
</tr>
</tbody>
</table>
Problems

• A well-known company with a top credit rating will pay LIBOR plus a margin if it borrows money from a bank. It would like cheaper financing.

• A money market investor has made a deposit that is due to mature but is concerned that interest rates are falling and the returns on re-investing the cash will be poor.

• An investor owns a fixed coupon bond but believes that interest rates are likely to rise and hence the value of bond will fall. He could sell the bond but feels that the problem is short term and wishes to retain the bond in his portfolio.
Problems II

- A money market investor will earn sub-LIBOR return by depositing funds with a bank. LIBOR is the bank’s lending rate; it will pay out less on income deposits. The investor would like a higher return.

- A mortgage-lending bank funds itself on a floating-rate basis but wishes to create fixed-rate loans. If it does so it runs the risk that interest rates will rise and it will pay more in funding than it receives in interest on the mortgage loans.
Equity swaps

• An equity swap is the OTC alternative to equity index and single stock futures. It is an agreement between two parties to:
  – to exchange payment at regular intervals
  – over an agreed period of time
  – where at least one payment leg depends on the value of a share, or a stock market index

• A typical equity swap occurs when a company owns a block of shares in another firm which it would like to monetize. However it also wishes to retain economic exposure to changes in the value of the shares for some period. It then sells the shares and enters into an equity swap in which it receives the return on the shares paid in cash.
Monetizing corporate cross-holdings

- Example: A company owns £100m in another company. It sells the shares and enters in a swap. The notional is the shares value.
- The bank pays the return on the block of shares (capital gains + dividends) on a quarterly basis.
- The company pays the LIBOR (e.g. 4%) on a quarterly basis.
- After 3 months, if the values of the shares is £102m, the company pays £1m and receives £2m.
- If three months later the value of the shares drops to £99m, the company pays £1.02m+£3m=£4.02m.
Other applications of equity swaps

• Equity swaps are useful for an investor who wishes to gain exposure to a basket of foreign shares but faces restrictions on ownership.
• Banks also gain:

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Investor

Bank

LIBOR+0.3%

shares return

Borrows at LIBOR

purchase shares
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Credit default swaps

• A CDS is a form of insurance against default on a loan or a bond. The two parties are:
  – the buyer of protection
  – the seller of protection

  ![Diagram showing buyer and seller of protection]

  premium
  
  Buyer of protection ————> Seller of protection
  
  payment contingent on credit event

• Events that trigger the contingent payment: bankruptcy, insolvency, failure to meet payment obligation when due, credit rating downgrade.
• The periodic premium paid on a credit default swap is related to the credit spread of the referenced asset (i.e. the asset that is to be protected).

• The credit spread is the difference between is the return on the asset and the risk free return (Treasury bonds).
Options
Call option: gives the holder to right (but not the obligation) to buy shares at some time in the future at a fixed price.

Put option: gives the holder to right (but not the obligation) to sell shares at some time in the future at a fixed price.

European option: Exercised only at maturity date

American option: Can be exercised at any time up to maturity
Factors affecting the price of options (=c)

- Underlying stock price S
- Exercise price X
- Variance of the returns of the underlying asset $\sigma^2$
- Time to maturity T

\[
\frac{\partial c}{\partial S} > 0, \quad \frac{\partial c}{\partial X} < 0, \quad \frac{\partial c}{\partial \sigma^2} > 0, \quad \frac{\partial c}{\partial T} > 0
\]

The riskier the underlying assets, the greater the probability that the stock price will exceed the exercise price.

The longer the maturity, the greater the probability that the stock price will exceed the exercise price.
Value of options at expiration

Buying a call option

Buying a put option
Selling a call option

\[ \Delta W \]

\[ S \]

Selling a put option

\[ \Delta W \]

\[ S \]
Bounding the value of a call

What is the value of options when bought before the expiration date?

Value of the call prior to expiration date

Upper bound = \( S \)

Lower bound = \( S - \text{exercise price} \)
Combinations of options and the put-call parity

Buying a put + buying the underlying stock = protective put.

Equivalent to buying a call + buying a zero-coupon bond.

⇒ These two strategies must have the same cost.

PUT-CALL PARITY:

Price of underlying stock + price of put = price of call + present value of exercise price
Covered-call strategy:

Buy a stock and write a call simultaneously.

Straddle

Buy a call plus buy a put:
Strips and straps

Strip: combining one call with two puts.
Strap: combining one put with two calls.
NPV formula cannot be used because the appropriate discount rate is unknown.

Suppose that the current market price of a stock is $50. Next year price will be either $40 or $60. Imagine that investors can borrow at 10%.

What is the value of a call option on that stock? We need to examine two strategies:
1. Buy a call
2. Buy one-half a share of stock and borrow $18.18 (implying a repayment of $20 at the end of the year.)
Future payoffs:

<table>
<thead>
<tr>
<th>Initial transactions</th>
<th>If stock price is $60</th>
<th>If stock price is $40</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buy a call</td>
<td>$60-$50=$10</td>
<td>$0</td>
</tr>
<tr>
<td>Buy one-half share</td>
<td>0.5*$60=$30</td>
<td>0.5*$40=$20</td>
</tr>
<tr>
<td>Borrow $18.18 at 10%</td>
<td>-$18.18*1.1=-$20</td>
<td>-$20</td>
</tr>
<tr>
<td>Total</td>
<td>$10</td>
<td>$0</td>
</tr>
</tbody>
</table>
The two strategies have the same payoffs, they must then have the same cost.

The cost of the second strategy is:

- Buy one-half share of stock: \(0.5 \times 50 = 25\) 
- Borrow $18.18: \(-18.18\) 
- Total: \(25 - 18.18 = 6.82\)

The call option must be priced at $6.82.
Determining the Delta:

The call price at the end of the year will be either $10 or $0. The call price has a potential swing of $10. The stock price has a potential swing of $20.

$$\text{Delta} = \frac{\text{swing of call}}{\text{swing of stock}} = \frac{$10}{$20} = 0.5$$
Risk neutral valuation

Note that the option value is independent on the probability that the stock goes up or down!

Reason: The current stock prices already captures the optimistic and pessimistic views about the future stock price.

Suppose that the return on the stock has to be equal to the risk-free rate 10%.

\[ 10\% = (\text{probability of a rise}) \times 20\% + (1 - \text{probability of a rise}) \times (-20\%) \]

\[ \rightarrow \text{probability of a rise is } 3/4 \]
By applying these probabilities, we also get the call price:

\[
\text{Value of a call} = \frac{\frac{3}{4} \times $10 + \frac{1}{4} \times $0}{1.10} = $6.82
\]

With risk aversion things are much more complex (Black-Scholes model).
Another method to determine the price

Suppose that there exists a call option with exercise price of 50:

- Stock price
  - 50
  - 48.5
- Bond price
  - 1.06
- Call option
  - ???
  - 0
The option payoffs can be replicated by a combination of the stock and a bond:
Denote A the number of shares and B the number of bonds which exactly replicate the option’s payoffs.

This gives two equations:

\[55A + 1.06B = 5\]
\[48.5A + 1.06B = 0\]

Hence,

\[A = 0.7692\]
\[B = -35.1959\]

The option price must be equal to the cost of replicating its payoffs:

\[
\text{Call option price} = 0.7692 \times 50 - 35.1959 = 3.2656
\]