ES50106
Financial investment management

- Exercises -

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Lecture 1 - Utility theory

1. You are an investor with a CRRA utility function having risk aversion 2. You are faced with a decision to invest your total wealth of £1,000,000 into a riskless asset which generates a return of 5% or into a risky asset which either generates a return of 30% or of -20% with equal probability.

   How much of your wealth would you invest into the risky asset?

2. Explain the certainty equivalent and its relevance in decision-making.

Lecture 2 - Portfolio selection theory

1. You are an investor with a risk aversion of 1 and have to decide your optimal investment into assets that have expected returns $\mu = [0.1, 0.2]'$ and a covariance matrix $\Sigma = \begin{bmatrix} 0.04 & 0.025 \\ 0.025 & 0.0625 \end{bmatrix}$.

   If you have £1,000,000, how much would you invest into the two assets?

2. Explain the two-fund separation-theorem and its implication for portfolio selection.

Lecture 3 - Myopic portfolio choice

1. You have a market with 4 assets, having a mean of $[0.12 \ 0.15 \ 0.05 \ 0.20]'$
   and covariance matrix $\begin{bmatrix} 0.0400 & 0.0350 & 0.0000 & 0.0300 \\ 0.0350 & 0.0625 & 0.0000 & 0.0450 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0300 & 0.0450 & 0.0000 & 0.0900 \end{bmatrix}$.
   If your risk aversion is 3 what is your optimal portfolio?

2. Under which conditions is the portfolio choice of long-term investors who can re-balance their portfolios myopic?

Lecture 4 - Consumption and portfolio choice

1. What serves as a budget constraint when we simultaneously consider consumption and investment decisions?

2. How is consumption affected by in increasing return of the portfolio invested in?

3. If an investor is very risk averse, why would he still hold the risky asset?

Lecture 5 - Portfolio choice with labor income

1. You are a civil servant who has a very secure government job paying £36,000 p.a. You are 27 years old and are sure to retire at 70 with a pension paying 40% of you income for another 20 years. Currently your only asset is cash to the value of £100,000 which you inherited from a relative. You consider your optimal investment of this amount into the stock market and cash holding. If you expect the stock market to rise by 8% p.a. with a volatility of 25% p.a. and the risk free rate is 6% p.a., how much would you invest into the stock market? How would this allocation have changed 20 years later, assuming you have not accumulated more financial wealth. How can you explain the difference?

   Assume an absolute risk aversion of 3.

2. Why do you hedge against the risk in your labor income using the stock market?

Lecture 6 - Tactical asset allocation

1. You are an investor with a risk aversion of 5 considering your portfolio into the assets with a long term return of $[0.08\ 0.10]'$, a covariance matrix of $\begin{bmatrix} 0.0225 & 0.015 \\ 0.015 & .04 \end{bmatrix}$ and you expect for the coming time period asset 1 to perform 2% better than the long-term average and asset 2 3% worse. What would be you strategic, tactical and total portfolio?

2. What effect does an increasing risk aversion have on the strategic and tactical portfolio?

Lecture 7 - Performance measurement

1. How strong would your average optimal bias towards the stock market be if the volatility of the stock market is 20%, the signal received has a correlation of 0.6 with the actual return and a variance of 0.25. The expected return of the stock market is 0.12 and the risk free rate 0.05. Your absolute risk aversion is 3.

2. How can you explain this large bias in question 1.?

Lecture 8 - Tactical bets with multiple signals

1. Suppose you are receiving three signals affecting the expected return of the asset. The signals are presented such that they are normalized and all exhibit a correlation of 0.7. You observe the values of the signals to be 0.19, 1.81, and -2.61, respectively. Assuming that trading any of these signals would generate an α of 4%, what would the optimal tactical bet for this asset be for an investor with absolute risk aversion of 1?

2. Explain your result on the optimal aggressiveness from question 1.

3. If the average volatility of signals is 0.8 and the average correlation between them 0.7, how many signals would you ideally follow if your risk aversion is 3 and it costs you 0.1 to follow each signal?

4. How do you interpret "aggressiveness"?

Lecture 9 - The Black-Litterman approach

1. You are invested into 3 assets that have a long-run expected return of [0.10 0.08 0.12]' with variances 0.04, 0.0625, and 0.09, respectively. The correlation between assets 1 and 2 is 0.7, between assets 1 and 3 it is 0.8 and between assets 2 and 3 it is 0.9. You are ceratin that asset 1 will outperform asset 3 by 3%. What would be your optimal tactical asset allocation if you risk aversion is 3?

2. Why does the expected return of asset 2 in question 1 change although no views on its performance are held?

Lecture 10 - Risk Management

1. You have an equally weighted portfolio consisting of 3 assets who have a volatility of 0.2, 0.25 and 0.3, respectively and a correlation of 0.7 between assets 1 and 2, 0.8 between assets 1 and 3, and 0.9 between assets 2 and 3. Your total investment is £100,000 and you are using the 95% VaR. How would you change your portfolio if you were only changing the weights of 2 assets and wanted to achieve a VaR of no more than £35,000?

2. Using the same portfolio and information as in question 1, you want to make sure that at the end of 1 time period your total wealth does not fall below £85,000. How much would you invest into the portfolio described and how much into a risk free asset that yields 5%?

3. How would you determine which confidence level to choose for your VaR calculations?