Exercise sheet 5 for Math 263: ODEs for Engineers Matt Roberts
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1. A mass of 2kg is attached to a spring of spring constant 8. There is a frictional damping force of coefficient 4. The mass is released from the stationary position $y(0) = -1$. The position $y(t)$ of the mass at time t thus satisfies

$$2y'' + 4y' + 8y = 0, \quad t > 0, \quad y(0) = -1, \quad y'(0) = 0.$$

   Find $y(t)$. (You may wish to sketch the graph of y.)

2. We showed in class that $D - a = e^{ax} D e^{-ax}$. Prove, by induction, that

$$(D-a)^n = e^{ax} D^n e^{-ax} \quad \text{for all } n \geq 1.$$

   (That is, assume that $(D-a)^k = e^{ax} D^k e^{-ax}$ and use this to show that $(D-a)^{k+1} = e^{ax} D^{k+1} e^{-ax}$. The principle of induction then tells us that the statement is true for all k.)

3. Find the general solution to $y^{(5)} + y^{(4)} + 3y^{(3)} + 3y'' = 0$.

4. (a) Check that $y_1 = x \sin x$ and $y_2 = x \cos x$ are solutions to

$$y'' - \frac{2}{x} y' + \left(1 + \frac{2}{x^2}\right) y = 0, \quad x > 0.$$

   (b) Calculate $W(\pi/2)$. Does there exist $x > 0$ such that $W(x) \geq 0$? How do you know?

   (c) ($\star$) Now calculate $W(0)$. Does this contradict a theorem from the course? Can you explain what is happening?

If you spot any errors, please inform me: `matthew.roberts@mcgill.ca`