MA263 Sheet 11 solutions. MR

(1) $A - 2I = \begin{pmatrix} 1 & 2 \\ -2 & 3 \end{pmatrix} - 2\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ -2 & 1 \end{pmatrix}$

$\det(A - 2I) = -1 + 4 = 3.$

(2) $A - \lambda I = \begin{pmatrix} 1-\lambda & 2 \\ -2 & 3-\lambda \end{pmatrix}$. $\operatorname{Tr} A = 4$, $\det A = 7$.

$$P(\lambda) = \lambda^2 - 4\lambda + 7 = (\lambda - 2)^2 + 3$$

which has roots $\lambda_1 = 2 + \sqrt{3}i$, $\lambda_2 = 2 - \sqrt{3}i$. These are the eigenvalues of A; the eigenvectors are

$$\underset{\sim}{v}_1 = \begin{pmatrix} 1 \\ \frac{\lambda_1 - a_{11}}{a_{12}} \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{1}{2} + \frac{\sqrt{3}}{2}i \end{pmatrix} \text{ and } \underset{\sim}{v}_2 = \begin{pmatrix} 1 \\ \frac{\lambda_2 - a_{11}}{a_{12}} \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{1}{2} - \frac{\sqrt{3}}{2}i \end{pmatrix}.$$

(3) For the general solution, we set $\alpha = 2$, $\omega = \sqrt{3}$, $\sigma = \frac{\alpha - a_{11}}{a_{12}} = 1/2$ and $\tau = \frac{\omega}{a_{12}} = \frac{\sqrt{3}}{2}$. Then

$$\underset{\sim}{x} = e^{2t}\left(C_1 \cos\sqrt{3}t - C_2 \sin\sqrt{3}t\right)\begin{pmatrix} 1 \\ 1/2 \end{pmatrix}$$

$$+ e^{2t}\left(C_1 \sin\sqrt{3}t + C_2 \cos\sqrt{3}t\right)\begin{pmatrix} 0 \\ \sqrt{3}/2 \end{pmatrix}.$$