

Figure 1: Nine iterations from a Metropolis-Hastings sampler for $\theta \mid x \sim N(0, 1)$ where the proposal distribution is normal with mean θ and chosen variance $\sigma^2 = 1$ and the starting point is $\theta^{(0)} = 1$. The top plot shows the target distribution, N(0, 1), the proposal distribution for $\theta^{(9)}$ which is N(-0.216, 1), the proposal $\theta^* = -1.779$ and the current observed density. The move is rejected; the bottom plot is the trace plot of $\theta^{(t)}$.

t	1	2	3	4	5	6	7	8	9
$ heta^*$	1.319	-0.118	0.392	-1.089	-1.947	-0.716	-0.251	-0.216	-1.779
Accept/Reject?	Accept	Accept	Accept	Accept	Reject	Accept	Accept	Accept	Reject
$ heta^{(t)}$	1.319	-0.118	0.392	-1.089	-1.089	-0.716	-0.251	-0.216	-0.216



Figure 2: 100 iterations from a Metropolis-Hastings sampler for $\theta \mid x \sim N(0, 1)$ where the proposal distribution is normal with mean θ and chosen variance $\sigma^2 = 1$ and the starting point is $\theta^{(0)} = 1$. The first plot shows the trace plot of $\theta^{(t)}$, the second the observed density against the target density and the third the autocorrelation plot. The sample mean is 0.3435006 and the sample variance is 0.5720027. 3% of points were observed to be greater than 1.96 and the acceptance rate is 0.66.



Figure 3: 5000 iterations from a Metropolis-Hastings sampler for $\theta \mid x \sim N(0, 1)$ where the proposal distribution is normal with mean θ and chosen variance $\sigma^2 = 1$ and the starting point is $\theta^{(0)} = 1$. The first plot shows the trace plot of $\theta^{(t)}$, the second the observed density against the target density and the third the autocorrelation plot. The sample mean is 0.01340934 and the sample variance is 0.984014. 2.76% of points were observed to be greater than 1.96 and the acceptance rate is 0.7038.



Figure 4: Nine iterations from a Metropolis-Hastings sampler for $\theta | x \sim N(0, 1)$ where the proposal distribution is normal with mean θ and chosen variance $\sigma^2 = 0.36$ and the starting point is $\theta^{(0)} = 1$. The top plot shows the target distribution, N(0, 1), the proposal distribution for $\theta^{(9)}$ which is N(-0.416, 0.36), the proposal $\theta^* = 0.923$ and the current observed density. The move is accepted; the bottom plot is the trace plot of $\theta^{(t)}$.

t	1	2	3	4	5	6	7	8	9
$ heta^*$	0.786	-0.280	-1.092	-0.335	-1.169	-0.987	0.234	-0.416	0.923
Accept/Reject?	Accept	Accept	Reject	Accept	Reject	Accept	Accept	Accept	Accept
$ heta^{(t)}$	0.786	-0.280	-0.280	-0.335	-0.335	-0.987	0.234	-0.416	0.923



Figure 5: 100 iterations from a Metropolis-Hastings sampler for $\theta \mid x \sim N(0, 1)$ where the proposal distribution is normal with mean θ and chosen variance $\sigma^2 = 0.36$ and the starting point is $\theta^{(0)} = 1$. The first plot shows the trace plot of $\theta^{(t)}$, the second the observed density against the target density and the third the autocorrelation plot. The sample mean is -0.04438661 and the sample variance is 0.5414882. 0% of points were observed to be greater than 1.96 and the acceptance rate is 0.83. Notice that, by comparing with Figure 2, reducing σ^2 , that is we are proposing smaller jumps, increases the acceptance rate but reduces the mi ing.



Figure 6: 5000 iterations from a Metropolis-Hastings sampler for $\theta \mid x \sim N(0, 1)$ where the proposal distribution is normal with mean θ and chosen variance $\sigma^2 = 0.36$ and the starting point is $\theta^{(0)} = 1$. The first plot shows the trace plot of $\theta^{(t)}$, the second the observed density against the target density and the third the autocorrelation plot. The sample mean is -0.008158827 and the sample variance is 0.9792388. 2.42% of points were observed to be greater than 1.96 and the acceptance rate is 0.7894. Notice that, by comparing with Figure 3, reducing σ^2 , that is we are proposing smaller jumps, increases the acceptance rate but reduces the mi ing.