Previously on MA40189:

- Beta-Binomial updating
- prior $\theta \sim Beta(\alpha, \beta)$
  
  $$f(\theta) = \frac{1}{B(\alpha, \beta)} \theta^{\alpha-1}(1 - \theta)^{\beta-1}$$

- likelihood $X | \theta \sim Bin(n, \theta)$
  
  $$f(x | \theta) = P(X = x | \theta) = \binom{n}{x} \theta^x (1 - \theta)^{n-x}$$

Today on MA40189:

- Posterior $\propto$ Prior $\times$ Likelihood
  
  $$f(\theta | x) = c \theta^{\alpha+x-1}(1 - \theta)^{\beta+n-x-1}$$
  
  for some constant $c$ not involving $\theta$

- posterior $\theta | x \sim Beta(\alpha + x, \beta + n - x)$

- the prior and posterior are from the same family: this is a conjugate update

- formally define conjugacy

- with respect to the Binomial likelihood, the Beta distribution is a conjugate family

- revisit example of tossing coins and drawing pins
  
  - role of the prior in the posterior
  
  - strong and weak priors