Previously on MA40189:

- prediction about future observations of data

$$f(z \,|\, x) \;=\; \int_\theta f(z \,|\, x, \theta) f(\theta \,|\, x)\, d\theta$$

- if X and Z are conditionally independent given θ then only need the distributions $Z \,|\, \theta$ and $\theta \,|\, x$ and

$$f(z \,|\, x) \;=\; \int_\theta f(z \,|\, \theta) f(\theta \,|\, x)\, d\theta$$

Today on MA40189:

- example of Binomial-Beta distribution
  - prior for $\theta \sim Beta(\alpha, \beta)$
  - observations, $X \,|\, \theta \sim Bin(n, \theta)$, $Z \,|\, \theta \sim Bin(m, \theta)$
  - distribution of $Z \,|\, x$ is Binomial-Beta.
- if X and Z are conditionally independent given θ then

$$\begin{aligned} E(Z \,|\, X) &= E(E(Z \,|\, \theta) \,|\, X) \\ Var(Z \,|\, X) &= Var(E(Z \,|\, \theta) \,|\, X) + E(Var(Z \,|\, \theta) \,|\, X) \end{aligned}$$

- observations that classicists judge as independent (and identically distributed) judged as exchangeable by Bayesians
- $X_1, \ldots, X_n$ finitely exchangeable if

$$f(x_1, \ldots, x_n) \;=\; f(x_{\pi(1)}, \ldots, x_{\pi(n)})$$

  for all permutations π defined on $\{1, \ldots, n\}$