Previously on MA40189:

- statistical decision problem: $[\Theta, \mathcal{D}, \pi(\theta), L(\theta, d)]$
  - solve $[\Theta, \mathcal{D}, f(\theta), L(\theta, d)]$ for immediate decision
  - solve $[\Theta, \mathcal{D}, f(\theta \,|\, x), L(\theta, d)]$ for decision having observed the sample x
- Bayes risk $\rho^*(\pi)$ minimises expected loss

$$\rho(\pi, d) \;=\; \int_\theta L(\theta, d)\pi(\theta)\, d\theta$$

- Bayes rule d^* decision which achieves Bayes risk

Today on MA40189:

- risk of the sampling procedure: to decide whether or not to sample
- for each possible sample, specify which decision to make

$$\begin{aligned}\rho(f(\theta), \delta) \;&=\; \int_x \int_\theta L(\theta, \delta(x)) f(\theta, x)\, d\theta dx \\ &=\; \int_x \Big\{ \int_\theta L(\theta, \delta(x)) f(\theta \,|\, x)\, d\theta \Big\} f(x)\, dx \\ &=\; \int_x E\{L(\theta, \delta(x)) \,|\, X\} f(x)\, dx\end{aligned}$$

- risk of the sampling procedure is

$$\rho_n^* \;=\; E[E\{L(\theta, \delta^*(x)) \,|\, X\}]$$

- final example: estimate the parameter, θ, of a Poisson distribution
- $L(\theta, d) = \theta(\theta - d)^2$, $\theta \sim Gamma(\alpha, \beta)$, $X_i \,|\, \theta \sim Po(\theta)$