MA30118 - Exponential smoothing handout

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The data below show the monthly percentage of all shipments that were received on time over the past 12 months.

80 82 84 83 83 84 85 84 82 83 84 83

1. Construct an exponential forecast using $\alpha = 0.2$ and calculate the forecast error for each month. Take $\hat{X}_2 = M_1 = 80$.

Month, t	X_t	$\hat{X}_t = M_{t-1} = \alpha X_{t-1} + (1-\alpha)M_{t-2}$	$X_t - \hat{X}_t$
1	80		
2	82	$M_1 = 80$	$82 - 80 = 2$
3	84	$M_2 = 0.2(82) + 0.8(80) = 80.4$	$84 - 80.4 = 3.6$
4	83	$M_3 = 0.2(84) + 0.8(80.4) = 81.12$	$83 - 81.12 = 1.88$
5	83	$M_4 = 0.2(83) + 0.8(81.12) = 81.496$	$83 - 81.496 = 1.504$
6	84	$M_5 = 0.2(83) + 0.8(81.496) = 81.7968$	$84 - 81.7968 = 2.2032$
7	85	$M_6 = 82.23744$	2.76256
8	84	$M_7 = 82.789952$	1.210048
9	82	$M_8 = 83.0319616$	-1.0319616
10	83	$M_9 = 82.82556928$	0.17443072
11	84	$M_{10} = 82.86045542$	1.13954458
12	83	$M_{11} = 83.08836434$	-0.08836434

The original time series and smoothed series are shown in Figure 1.

2. What is the forecast for future months?

$$\hat{X}_{12+n} = M_{12} = 0.2(83) + 0.8(83.08836434) = 83.07069147.$$

3. What is the value of the mean square deviation?

$$\begin{aligned} MSD &= \{2^2 + 3.6^2 + 1.88^2 + 1.504^2 + 2.2032^2 + 2.76256^2 + 1.210048^2 + (-1.0319616)^2 + \\ &\quad 0.17443072^2 + 1.13954458^2 + (-0.08836434)^2\}/11 \\ &= \{4 + 12.96 + 3.5344 + 2.262016 + 4.85409024 + 7.631737754 + 1.464216162 + \\ &\quad 1.064944744 + 0.030426076 + 1.29856185 + 0.007808257\}/11 \\ &= 39.10820108/11 \\ &= 3.55529108. \end{aligned}$$

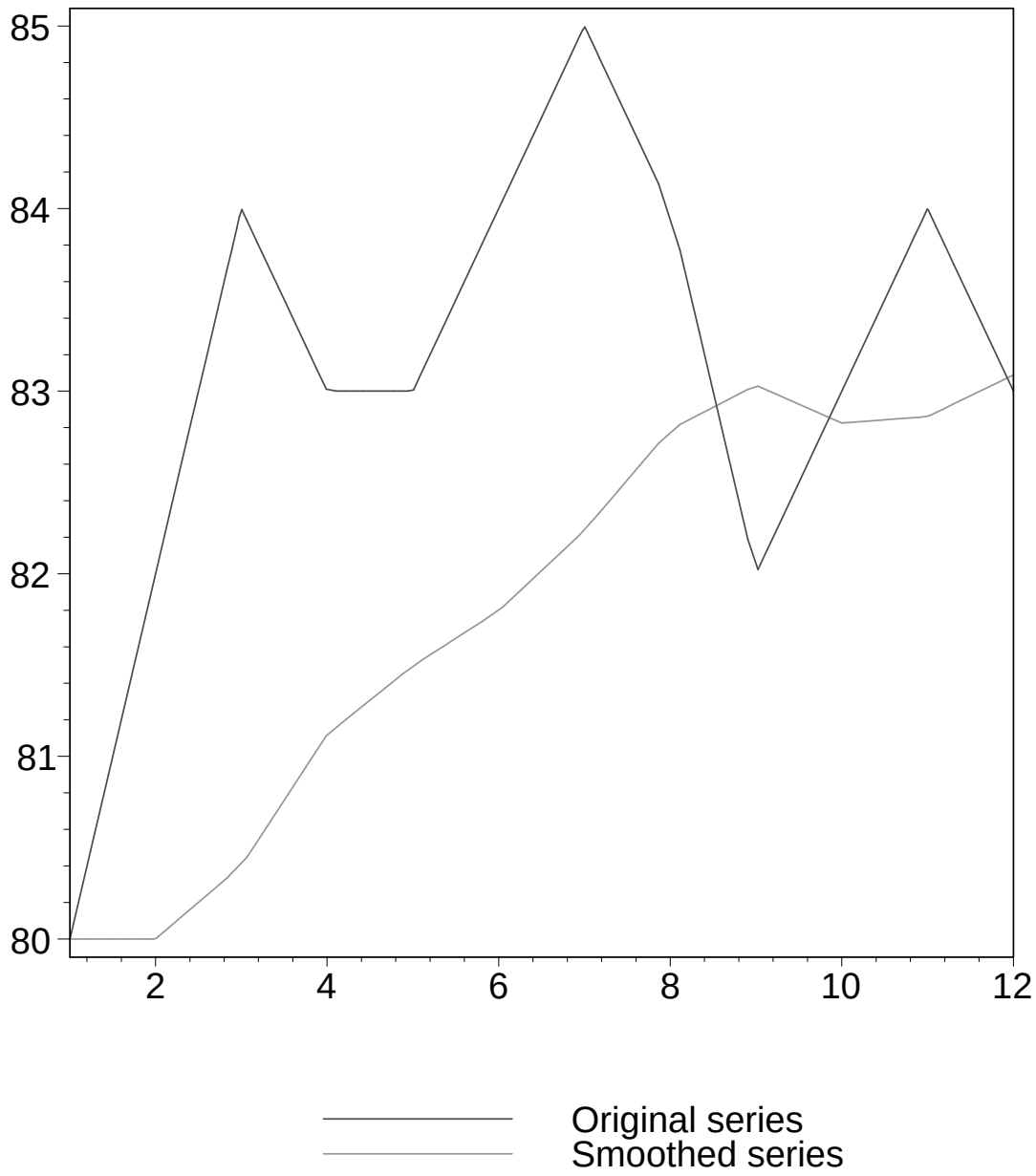

Figure 1: The time series and smoothed series for the shipments.